

$$7(2x+5) \quad 2. -4x(3x-5)$$

$$(4x+35) \quad (-12x^2+20x)$$

$$(2x+3)(x-6) \quad 6. (2x+5)^2 \quad 7. (x-2)(x^2-3x+7)$$

$$2x^2-12x+3x-18 \quad (2x+5)(2x+5) \quad x^3-3x^2+7x$$

$$(2x^2-9x-18) \quad 4x+... \quad -2x^2+6x-14$$

$$ax-ay \quad 9. 3x^2+6x$$

$$a(x-y) \quad 3x(x+2)$$

$$x^2+y-30 \quad 13. x^2-36$$

$$(x+6)(x-5) \quad (x-6)(x+6)$$

$$3x^2+29x+14 \quad 17. 3x^3+12$$

$$x+2 \quad x+7 \quad 3x \quad x^2$$

$$x(x+...)$$

$$ax+bx+4a+4b \quad 21. x$$

$$x(a+b)+4(a+b) \quad x^2$$

$$(a+b)(x+4) \quad (x-$$

$$x^2-2x-15=0 \quad (x-2)(x-5)(x+5)$$

$$(x-5)(x+3)=0 \quad 25. x(x+5)=24$$

$$x=5 \quad x=-3 \quad x^2+5x-24=0$$

উচ্চতর গণিত

নবম-দশম শ্রেণি



জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

$$24. x^2-2x-15=0 \quad (x-2)(x-5)(x+5)$$

$$(x-5)(x+3)=0 \quad 25. x(x+5)=24$$

$$x=5 \quad x=-3 \quad x^2+5x-24=0$$

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড

৬৯-৭০, মতিঝিল বাণিজ্যিক এলাকা, ঢাকা

কর্তৃক প্রকাশিত

[প্রকাশক কর্তৃক সর্বস্বত্ব সংরক্ষিত]

পরীক্ষামূলক সংস্করণ

প্রথম প্রকাশ : অক্টোবর- ২০১২

পাঠ্যপুস্তক প্রণয়নে সমন্বয়ক

মোঃ নাসির উদ্দিন

মোঃ রজব আলী মিয়া

কম্পিউটার কন্সাল্ট্যান্ট

লেজার স্ক্যান লিমিটেড

প্রচ্ছদ

সুদর্শন বাহার

সুজাউল আবেদীন

চিত্রাঙ্কন

তোহফা এন্টারপ্রাইজ

ডিজাইন

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড

সরকার কর্তৃক বিনামূল্যে বিতরণের জন্য

মুদ্রণ :

প্রসঙ্গ-কথা

শিক্ষা জাতীয় জীবনের সর্বতোমুখী উন্নয়নের পূর্বশর্ত। আর দ্রুত পরিবর্তনশীল বিশ্বের চ্যালেঞ্জ মোকাবেলা করে বাংলাদেশকে উন্নয়ন ও সমৃদ্ধির দিকে নিয়ে যাওয়ার জন্য প্রয়োজন সুশিক্ষিত জনশক্তি। ভাষা আন্দোলন ও মুক্তিযুদ্ধের চেতনায় দেশ গড়ার জন্য শিক্ষার্থীর অন্তর্নিহিত মেধা ও সম্ভাবনার পরিপূর্ণ বিকাশে সাহায্য করা মাধ্যমিক শিক্ষার অন্যতম লক্ষ্য। এছাড়া প্রাথমিক স্তরে অর্জিত শিক্ষার মৌলিক জ্ঞান ও দক্ষতা সম্প্রসারিত ও সুসংহত করার মাধ্যমে উচ্চতর শিক্ষার যোগ্য করে তোলাও এ স্তরের শিক্ষার উদ্দেশ্য। জ্ঞানার্জনের এই প্রক্রিয়ার ভিতর দিয়ে শিক্ষার্থীকে দেশের অর্থনৈতিক, সামাজিক, সাংস্কৃতিক ও পরিবেশগত পটভূমির প্রেক্ষিতে দক্ষ ও যোগ্য নাগরিক হিসেবে গড়ে তোলাও মাধ্যমিক শিক্ষার অন্যতম বিবেচ্য বিষয়।

জাতীয় শিক্ষানীতি-২০১০ এর লক্ষ্য ও উদ্দেশ্যকে সামনে রেখে পরিমার্জিত হয়েছে মাধ্যমিক স্তরের শিক্ষাক্রম। পরিমার্জিত এই শিক্ষাক্রমে জাতীয় আদর্শ, লক্ষ্য, উদ্দেশ্য ও সমকালীন চাহিদার প্রতিফলন ঘটানো হয়েছে, সেই সাথে শিক্ষার্থীদের বয়স, মেধা ও গ্রহণ ক্ষমতা অনুযায়ী শিখনফল নির্ধারণ করা হয়েছে। এছাড়া শিক্ষার্থীর নৈতিক ও মানবিক মূল্যবোধ থেকে শুরু করে ইতিহাস ও ঐতিহ্য চেতনা, মহান মুক্তিযুদ্ধের চেতনা, শিল্প-সাহিত্য-সংস্কৃতিবোধ, দেশপ্রেমবোধ, প্রকৃতি-চেতনা এবং ধর্ম-বর্ণ-গোত্র ও নারী-পুরুষ নির্বিশেষে সবার প্রতি সমমর্যাদাবোধ জাগ্রত করার চেষ্টা করা হয়েছে। একটি বিজ্ঞানমনস্ক জাতি গঠনের জন্য জীবনের প্রতিটি ক্ষেত্রে বিজ্ঞানের স্বতঃস্ফূর্ত প্রয়োগ ও ডিজিটাল বাংলাদেশের রূপকল্প-২০২১ এর লক্ষ্য বাস্তবায়নে শিক্ষার্থীদের সক্ষম করে তোলার চেষ্টা করা হয়েছে।

নতুন এই শিক্ষাক্রমের আলোকে প্রণীত হয়েছে মাধ্যমিক স্তরের প্রায় সকল পাঠ্যপুস্তক। উক্ত পাঠ্যপুস্তক প্রণয়নে শিক্ষার্থীদের সামর্থ্য, প্রবণতা ও পূর্ব অভিজ্ঞতাকে গুরুত্বের সঙ্গে বিবেচনা করা হয়েছে। পাঠ্যপুস্তকগুলোর বিষয় নির্বাচন ও উপস্থাপনের ক্ষেত্রে শিক্ষার্থীর সৃজনশীল প্রতিভার বিকাশ সাধনের দিকে বিশেষভাবে গুরুত্ব দেওয়া হয়েছে। প্রতিটি অধ্যায়ের শুরুতে শিখনফল যুক্ত করে শিক্ষার্থীর অর্জিতব্য জ্ঞানের ইজিত প্রদান করা হয়েছে এবং বিচিত্র কাজ, নমুনা প্রশ্ন সংযোজন করে মূল্যায়নকে সৃজনশীল করা হয়েছে।

জ্ঞান-বিজ্ঞানের বিচিত্র গবেষণায় ‘উচ্চতর গণিত’ বিষয়টির প্রয়োগ বিশ্বব্যাপী। বিশেষ করে পদার্থবিদ্যা, জ্যোতির্বিদ্যা ও মহাকাশ গবেষণায় উচ্চতর গণিতের প্রয়োগ অপরিহার্য। এছাড়া প্রাত্যহিক জীবনে বিচিত্র পরীক্ষা-নিরীক্ষা ও গবেষণায় উচ্চতর গণিত তাৎপর্যপূর্ণ অবদান রাখছে। একবিংশ শতকের বিজ্ঞানভিত্তিক বিশ্বের চ্যালেঞ্জ মোকাবেলায় উচ্চতর গণিত অধ্যয়ন অতীব গুরুত্বপূর্ণ। এসব দিক বিবেচনায় রেখে মাধ্যমিক স্তরে ‘উচ্চতর গণিত’ শীর্ষক পাঠ্যপুস্তকটি প্রণয়ন করা হয়েছে। এ ক্ষেত্রে সর্বদাই শিক্ষার্থীদের বোধগম্যতাকে গুরুত্ব দিয়ে সহজ-সুন্দরভাবে পাঠ্যপুস্তকটি প্রণয়ন করার চেষ্টা করা হয়েছে।

একবিংশ শতকের অজীকার ও প্রত্যয়কে সামনে রেখে পরিমার্জিত শিক্ষাক্রমের আলোকে পাঠ্যপুস্তকটি রচিত হয়েছে। কাজেই পাঠ্যপুস্তকটির আরও সমৃদ্ধিসাধনের জন্য যেকোনো গঠনমূলক ও যুক্তিসঙ্গত পরামর্শ গুরুত্বের সঙ্গে বিবেচিত হবে। পাঠ্যপুস্তক প্রণয়নের বিপুল কর্মযজ্ঞের মধ্যে অতি স্বল্প সময়ে পুস্তকটি রচিত হয়েছে। ফলে কিছু ভুলত্রুটি থেকে যেতে পারে। পরবর্তী সংস্করণগুলোতে পাঠ্যপুস্তকটিকে আরও সুন্দর, শোভন ও ত্রুটিমুক্ত করার চেষ্টা অব্যাহত থাকবে। বানানের ক্ষেত্রে অনুসৃত হয়েছে বাংলা একাডেমী কর্তৃক প্রণীত বানানরীতি।

পাঠ্যপুস্তকটি রচনা, সম্পাদনা, চিত্রাঙ্কন, নমুনা প্রশ্নাদি প্রণয়ন ও প্রকাশনার কাজে যারা আন্তরিকভাবে মেধা ও শ্রম দিয়েছেন তাঁদের ধন্যবাদজ্ঞাপন করছি। পাঠ্যপুস্তকটি শিক্ষার্থীদের আনন্দিত পাঠ ও প্রত্যাশিত দক্ষতা অর্জন নিশ্চিত করবে বলে আশা করি।

প্রফেসর মোঃ মোস্তফা কামালউদ্দিন

চেয়ারম্যান

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

সূচিপত্র

অধ্যায়	বিষয়বস্তু	পৃষ্ঠা
প্রথম অধ্যায়	সেট ও ফাংশন	১
দ্বিতীয় অধ্যায়	বীজগাণিতিক রাশি	৪১
তৃতীয় অধ্যায়	জ্যামিতি	৬৫
চতুর্থ অধ্যায়	জ্যামিতিক অঙ্কন	৮১
পঞ্চম অধ্যায়	সমীকরণ	৯২
ষষ্ঠ অধ্যায়	অসমতা	১১৩
সপ্তম অধ্যায়	অসীম ধারা	১২৫
অষ্টম অধ্যায়	ত্রিকোণমিতি	১৩৩
নবম অধ্যায়	সূচকীয় ও লগারিদমীয় ফাংশন	১৮১
দশম অধ্যায়	দ্বিপদী বিস্তৃতি	২০৯
একাদশ অধ্যায়	স্থানাঙ্ক জ্যামিতি	২২৫
দ্বাদশ অধ্যায়	সমতলীয় ভেক্টর	২৫৬
ত্রয়োদশ অধ্যায়	ঘন জ্যামিতি	২৭০
চতুর্দশ অধ্যায়	সম্ভাবনা	২৮৬
	উত্তরমালা	২৯৬

c0g Aa'vq

tmU I dvskb

(Set and Function)

tmU c0gK avi Yv gva'vqK exRMvYtZ Avtj vPbv Kiv ntqtQ | G Aa'vtq gva'vqK exRMvYtZi AwZwi³ mel qe' Avtj vPbv Kiv ntj v :

Aa'vq tkfI wk'v_#v

- mveR tmU, DctmU, c+k tmU I kw³ tmU MVb Ki tZ cvi te |
- wevfbaetmUi msthvM, tQ` I ASt- wbyQ Ki tZ cvi te |
- tmU c0uqvi agvevj i ths³K c0vY Ki tZ cvi te |
- mgZj tmU eYbv Ki tZ cvi te Ges Gi gva'tg Amxg tmU avi Yv e'vL'v Ki tZ cvi te |
- tmU msthvMi kw³ tmU wbyQi m' e'vL'v Ki tZ cvi te Ges t'fbiP' I D`vni tYi mrvth' Zv hvPvB Ki tZ cvi te |
- tmU c0uqv c0qm Kti RxebrfweK mgm'v mgvavb Ki tZ cvi te |
- tmU mrvth' w' t' kb I dvskb Gi avi Yv e'vL'v Ki tZ cvi te |
- dvsktbi t'Wtgb I tiA wbyQ Ki tZ cvi te |
- GK-GK dvskb, mveR dvskb I GK-GK mveR dvskb D`vni tYi mrvth' e'vL'v Ki tZ cvi te |
- weci xZ dvskb e'vL'v Ki tZ cvi te |

1.1 tmU

ev'e RMZ ev vPsv- RMtZi e' i th'Kv'bv m'ba'ni Z msMh'K tmU ejv nq | thgb, *Mathematics* k'wJ *a, c, e, h, i, m, s, t* A'ji t'j vi m'ba'ni Z msMh' | ZvB GwJ *Mathematics* k'fai A'ji mg'ni tmU Ges c0Z'KwJ A'ji H tmU Dcv'vb | tmU t'K Avgiv Bst'wR eo nv'Zi A'ji w' t'q c'Kvk KwI Ges Gi Dcv'vb t'j v e'U'bi { } gvtS Ave x Kti Dcv'vb t'j v t'K Avj v'v Kivi Rb' Kgv e'envi Kiv nq | A_#

$$M = \{a, c, e, h, i, m, s, t\}$$

Av'iv Kt'qKwJ D`vni Y :

(K) 1g `kwJ AFYvZtK msL'vi tmU 'F' v'v evY'Z ntj v : $F = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(L) mBv'ni w' b_ t'j vi tmU D v'v w' t' k'Z ntj , Avgiv wj L t'Z cw'v

$D = \{k'v'evi, i'we'vi, t'm'ge'vi, g'z'evi, e'p'evi, e'p'v'v'Z'evi \text{ I } i'p'evi \}$

$A_{ev} D = \{x : x \text{ ntj v mBv'ni w' } b_ \text{ t'j vi } b'v'g \}$

KvR : Zvj K'v c'x'vZ t'j L :

(K) eQ' i i g'v' t'j vi tmU |

(L) `w'Y Gw'k'qvi t' k_ t'j vi tmU |

(M) `v'fweK msL'vi tmU |

(N) evs'j v' t'ki mi Kw'v cvK' t'j vi tmU |

mweR tmU

mweR tmU (*Universal Set*) Avtj vPbvi Rb" wbtPi tmU, tj v wetePbv Kwi

$$P = \{x : x \text{ abvZK cYmsL} \vee \text{Ges } 5x \leq 16\}$$

$$Q = \{x : x \text{ abvZK cYmsL} \vee \text{Ges } x^2 < 20\}$$

Ges $R = \{x : x \text{ abvZK cYmsL} \vee \text{Ges } \sqrt{x} \leq 2\}$ hv tKej abvZK cYmsL" vi Y Kti |

GLb $U = \{x : x \text{ abvZK cYmsL} \vee \text{tmU}\}$ wetePbv Kwi |

Zvntj P, Q Ges R ntj v U Gi DctmU Ges U tK ej v nq mweR tmU |

wbw" tmU tK Avtj vPbvaxb mKj tmU i mweR tmU ej v nq |

DctmU (*Subset*)

$$P = \{1,2,3\}, Q = \{1,2,3,4\} \text{ Ges } R = \{1,2,3,4\}$$

tmU wetePbv Kiti t" Lv hvq P Gi cZmU Dcv" vb R Gi Dcv" vb, A_ " $x \in P \Rightarrow x \in R$.

P tmU w tK R tmU i DctmU ej v nq Ges tj Lv nq $P \subseteq R$.

Abjfc fite Q tmU i cZmU Dcv" vb R tmU i Dcv" vb A_ " , $x \in Q \Rightarrow x \in R$

mZi vs Q tK R tmU i DctmU ej v nq Ges tj Lv nq $Q \subseteq R$.

$P \cap Q$ tmU w tK R tmU i DctmU nI qv m tEjI Gt" i gta" cv_ " w" gvb |

GLv tB, Dtj L" th, $n(P) = 3$ Ges $n(R) = 4$, thLv tB $n(S) \neq 0$ S tmU i Dcv" vb mSL" v |

P tK R Gi cKZ DctmU ej v nq Ges tj Lv nq $P \subset R$.

thtKv tBv tmU A Gi Rb"

$$(i) A \subseteq A$$

(i) $\Phi \subseteq A$ (duKv tmU Φ thtKv tBv tmU i DctmU)

hw" A tmU, mmxg tmU B Gi DctmU nq i.e. $A \subseteq B$ ZLb $n(A) \leq n(B)$

hw" A tmU, mmxg tmU B Gi cKZ DctmU i.e. $A \subset B$ ZLb $n(A) < n(B)$.

"be" : $\not\subseteq$ wPy A DctmU bq Ges $\not\subset$ Gi A cKZ DctmU bq |

c tK tmU (*Complement Set*)

wetePbv Kiv hvK $U = \{x : x \text{ abvZK cYmsL}\}$ Ges $P = \{1,2,3\}$ | tmU $P' = \{x : 5x > 16\}$ msAwqZ

Kiv ntj v hvi tKv tBv Dcv" vb P tmU tB | mZi vs $P' = \{4,5,6, \dots\}$ Ges GtK ej v nq c tK tmU |

Z" c, $Q = \{1,2,3,4\}$ tmU i Rb" c tK tmU $Q' = \{5,6,7, \dots\}$.

hw" U mweR tmU nq, Zte P tmU i c tK tmU $P' = \{x : x \notin P, x \in U\}$.

D"vniY 1 | t" I qv AvtQ $U = \{x : x \text{ cYmsL}, 0 < x \leq 10\}$, $A = \{x : 2x > 7\}$ Ges $B =$

$\{x : 3x < 20\}$ GLvb t tK (a) tmU $A \cap A'$ (b) tmU $B \cap B'$ Gi Dcv" b, tj v Zvwj Kv c xwZ tZ

cKik Ki |

†KvbwU mZ" ev wq_ "v ej : i) $A' \subseteq B$, ii) $B' \subseteq A$, iii) $A \not\subseteq B$
 mgvavb : $U = \{1,2,3,4,5,6,7,8,9,10\}$

$$(a) A = \{x : 2x > 7\} = \{4,5,6,7,8,9,10\}$$

$$\therefore A' = \{1,2,3\}$$

$$(b) B = \{x : 3x < 20\} = \{1,2,3,4,5,6\}$$

$$\therefore B' = \{7,8,9,10\}$$

$\therefore A' \subseteq B$ mZ", $B' \not\subseteq A$ wq_ "v Ges $A \not\subseteq B$ mZ"

kw³ tmU (*Power Set*)

†Kv†bv tmU A Gi mKj Dc†tm†Ui tmU†K A Gi kw³ tmU ev cvl qvi tmU ej v nq Ges G†K $P(A)$ Øvi v c†Kvk Kiv nq|

thgb, $A = \{1,2,3\}$ ntj, A Gi kw³ tmU,

$$P(A) = \{\Phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

j ¶Yxq th, $P(A)$ Gi Dcv`vb_†j v c†Z†KB tmU A Gi Dc†tmU|

`be" : $B \in P(A)$ ej †j e††Z nte $B \subseteq A$, †Kv†bv Av†j vPbvq mweR tmU U aiv ntj, H Av†j vPbvq we†ewPZ c†Z†K tmU $P(U)$ Gi m`m"|

hw` †Kv†bv tm†Ui Dcv`vb mnxg nq, aiv hvK H tm†U n msL†K Dcv`vb Av†Q, Zvntj D^3 tmU†Ui kw³ tm†U 2^n msL†K Dcv`vb _vK†e |

KvR :

1| † l qv Av†Q $U = \{1,2,3,4,5,6,7,8,9,10\}$

wb†Pi tmU_†j v Zvwj Kv c×wZ†Z c†Kvk Ki :

$$(a) A = \{x : 5x > 37\}$$

$$(b) B = \{x : x + 5 < 12\}$$

$$(c) C = \{x : 6 < 2x < 17\}$$

$$(d) D = \{x : x^2 < 37\}$$

2| † l qv Av†Q $U = \{x : 1 \leq x \leq 20, x \in \mathbb{Z}^+\}$.

wb†Pi tmU_†j v Zvwj Kv c×wZ†Z c†Kvk Ki :

$$(a) A = \{x : x, 2 \text{ Gi } \text{wYZK}\} \quad (b) B = \{x : x, 5 \text{ Gi } \text{wYZK}\}$$

$$(c) C = \{x : x, 10 \text{ Gi } \text{wYZK}\}$$

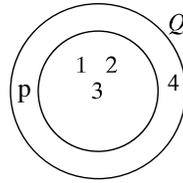
c†E Z†_i Av†j v†K wb†Pi †Kvb_†j v mZ" ev wq_ "v ej

$$C \subset A, B \subset A, C \subset B$$

3| hw` $A = \{a, b, c, d, e\}$ nq, Z†e $P(A)$ wbY† Ki |

tfbwpĀ (Venn Diagram)

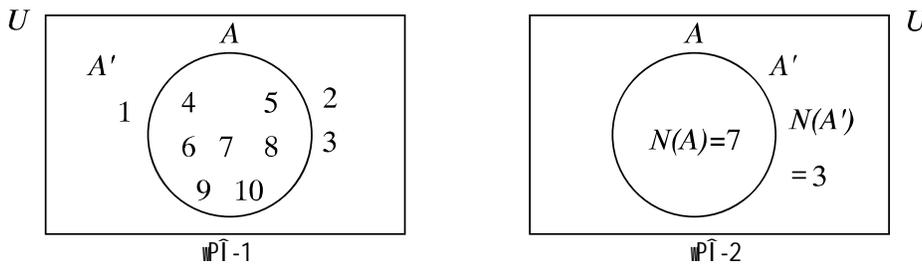
wPĀ $P = \{1,2,3\}$ Ges $Q = \{1,2,3,4\}$ tmĀUi gĀ" mĀúKĤnj v $P \subseteq Q$.



tKvĀbv tmĀUi GKwĀK DcĀtmĀUi gĀ" GiĀc mĀúKĤnj v R KiĀZ th RĀwĀZK wPĀ eĀenvi Kiv nq, ZvB tfbwpĀ |

mvaviYZ AvqZĀĀĀ Āvivi mweĀ tmU eSvĀbv nq | eĀEvKvi ev wĀ fRvKvi tĀĀĀ DcĀtmU tevSvĀZ eĀenvi Kiv nq | wĀĀPi tfbwpĀĀ wPĀ-1 G mweĀ tmU $U = \{x : x \text{ cYmsL}^v, 0 < x \leq 10\}$,

tmU $A = \{x : 2x > 7\}$ Ges $A' = \{x : 2x \leq 7\}$ tĀ LvĀbv ntĀj v |



cĀZĀtmĀUi msLĀv ZwĀj Kvex Kivi cwi eĀZĀĀĀ 2 Gi AbjĀc KĀi cĀZ tmĀUi DcvĀ vbĀ, tĀj v wĀj LĀZ cwi | hLb AvĀgĀv wĀj wL $n(A)$: A wĀ AvĀgĀv AbĀgĀv Kwi th, A mĀxĀg tmU |

hwĀ U mweĀ tmU Ges A thĀKvĀbv tmU ZLb wĀj LĀZ cwi $n(A) + n(A') = n(U)$

DĀvniY 2 | tĀ l qv AvĀĀ $U = \{x : 2 \leq x \leq 30, x \in \mathbb{Z}^+\}$ Ges $P = \{x : x \text{ ntĀj v } 30 \text{ Gi DrcvĀ K}\}$

- (a) P tmĀUi DcvĀ vbĀ, tĀj v ZwĀj Kv cĀwĀZĀZ wĀj L
- (b) P' tmĀUi eYĀvĀ vĀ
- (c) $n(P')$ wĀYĀ Ki

mgvavb :

- (a) $P = \{2, 3, 5, 6, 10, 15, 30\}$
- (b) $P' = \{x : x, 30 \text{ Gi DrcvĀ K bq}\}$
- (c) $n(P') = n(U) - n(P)$
 $= 29 - 7$
 $\therefore n(P') = 22$

tmĀUi msĀthvM

BstĀi wR eYĀĀj v wĀĀq mweĀ tmU l ĀĀw DcĀtmU h_vĀĀĀ

$E = \{e, n, g, l, i, s, h\}$

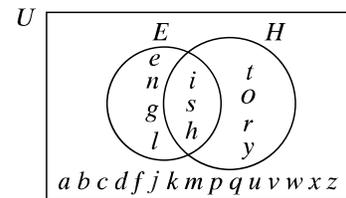
Ges $H = \{h, i, s, t, o, r, y\}$

(a) tfbwpĀĀ mweĀ tmU U, E Ges H tĀK wPwĀZ Ki

(b) $\text{tmU } E \cup H = \{x : x \in E \vee x \in H\}$ Gi

Dcv`vb, tj v Zvwj Kv c×wZ†Z cKvk Ki

mgvavb : (a) tfb wPÎ



(b) tfb wPÎ ntZ cvB, $\{x : x \in E \vee x \in H\}$

$$= \{e, n, g, l, i, s, h, t, o, r, y\}$$

j ¶ Kwî : $\text{tmU } \{x : x \in E \vee x \in H\} = \{e, n, g, l, i, s, h, t, o, r, y\}$

E Ges H tm†Ui mKj Dcv`vb w†q MmYZ tmU hv†K msthvM tmU ej v nq Ges $E \cup H$ cZ†Ki gva`tg cKvk Kiv nq

$$A_{\text{P}}, E \cup H = \{x : x \in E \vee x \in H\}$$

D`vni Y 3 | mweK tmU I `Bw Dc†mU †` I qv ntj v

$$U = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{x : x \text{ tgSvj K msL`v}\}$$

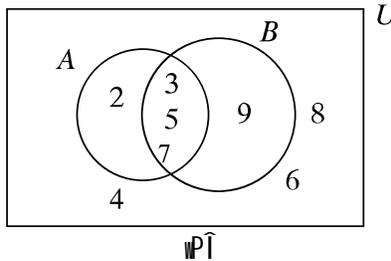
$$B = \{x : x \text{ we†Rvo msL`v}\}$$

(a) $A, B \mid A \cup B$ tm†Ui Dcv`vb, tj v Zvwj Kv c× Ki :

(b) tfb wPÎ $A \cup B$ †` Lvl

(c) $\text{tmU } A \cup B \mid \text{tmU } A \cup B'$ Gi Dcv`vb, tj v Zvwj Kv c×wZ†Z cKvk Ki |

mgvavb : (a) $A = \{2, 3, 5, 7\}, B = \{3, 5, 7, 9\}$ Ges $A \cup B = \{2, 3, 5, 7, 9\}$



$$(c) \quad A \cup B = \{x : x \in A \vee x \in B\} = \{2, 3, 5, 7, 9\}$$

$$(A \cup B)' = \{4, 6, 8\}$$

tm†Ui tQ`

BstiwR eYgvj vi A¶i, tj v mweK tmU Ges `Bw Dc†mU

$$A = \{e, n, g, l, i, s, h\} \text{ Ges } B = \{h, i, s, t, o, r, y\} \text{ msÁwqZ Kwî |}$$

Zvntj tmU $\{x : x \in A \text{ Ges } x \in B\} = \{i, s, h\}$. hv A Ges B tm†Ui mKj mvariY Dcv`vb w†q

MmYZ | Gfv†e MmYZ tmU†K $A \mid B$ tm†Ui tQ` tmU ej v nq Ges $A \cap B$ wj †L cKvk Kiv nq | $A_{\text{P}},$

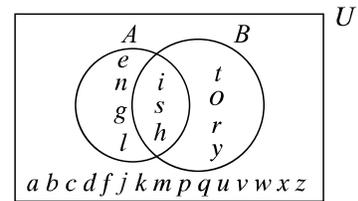
$$A \cap B = \{x : x \in A \text{ Ges } x \in B\}$$

Abjfcvte Avgiv cvB,

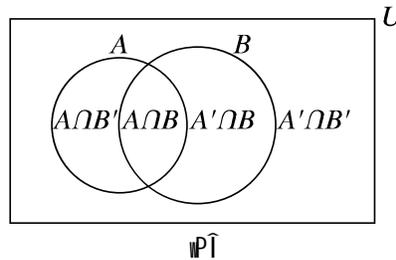
$$A \cap B' = \{x : x \in A \text{ Ges } x \in B'\} = \{e, n, g, l\}.$$

$$A' \cap B = \{x : x \in A' \text{ Ges } x \in B\} = \{t, o, r, y\}.$$

$$A' \cap B' = \{x : x \in A' \text{ Ges } x \in B'\} \\ = \{a, b, c, d, f, j, k, m, p, q, u, v, w, x, z\}.$$



wbtpi tfbwptf Dcpi i tmU, tjv t' Lvfbv ntjv :



D`vni Y 4 : t` lqv AvtQ U = {1,2,3,4,5,6,7,8,9}, A = {2,4,6,8}, B = {4,8} Ges C = {1,3,5,6}

tfbwptf Askb Ki (a) A \cap B Ges A \cap B'

(b) B \cap C Ges B' \cap C'

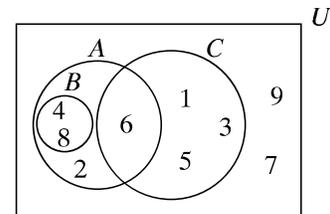
mgvavb : (a) thtnZl B \subseteq A

$$A \cap B = B = \{4, 8\}$$

$$A \cap B' = A = \{2, 6\}$$

$$(b) B \cap C = \{\}$$

$$B' \cap C' = B = \{2, 7, 9\}$$



D³ D`vni Y t`_tk cvB B \cap C = \{\} AZGe tmU B \perp C tk ejv nq wbtfn` tmU|

$$A \perp B \text{ tmU} \Leftrightarrow A \cap B = \Phi$$

D`vni Y 5 | U = {p, q, r, s, t, u, v, w}, A = {p, q, r, s}, B = {r, s, t} | C = {s, t, u, v, w}

(a) A \cap B, B \cap C Ges C \cap A Gi Dcv` vb, tjv Zvwj Kv c \times \cap Z \cap wj wce \times Ki Ges tfbwptf t' Lv|

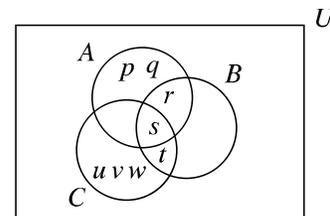
(b) A \cap B \cap C Gi Dcv` vb, tjv Zvwj Kv c \times \cap Z \cap c \times \cap v k Ki :

mgvavb : (a) A \cap B = \{r, s\}

$$B \cap C = \{s, t\}$$

$$C \cap A = \{s\}$$

$$(b) A \cap B \cap C = \{r, s\} \cap C = \{r, s\} \cap \{s, t, u, v, w\} \\ = \{s\}$$



D`vni Y 6 | t` lqv AvtQ U mweR tmU Ges A \cap B = \Phi wfbawfbetfbwptf wbtpi tmU, tjv Av"Qw` Z Ki:

(a) $A \cap B$

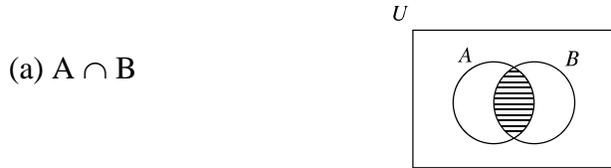
(b) $A' \cap B$

(c) $A \cap B'$

(d) $A' \cap B'$

†`LvI th, $n(A \cap B) + n(A' \cap B) + n(A \cap B') + n(A' \cap B') = n(U)$

mgvavb :

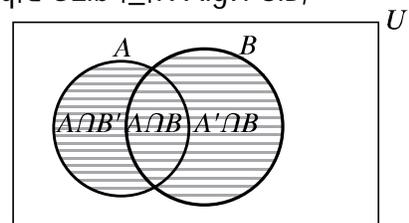


†fbiP†I mwieR tmU U Gi cÖZiU Dc†mU Gi m`m` msL`v† Lv†bv n†q†Q GLvb †_†K Avgiv cvB,

$$n(A \cap B) + n(A' \cap B) + n(A \cap B') + n(A' \cap B') = n(U)$$

mwieR tmU U Gi th†Kv†bv `Biu Dc†m†Ui †¶†I† tj Lv hvq

$$n(A \cap B) + n(A' \cap B) + n(A \cap B') + n(A' \cap B') = n(U)$$

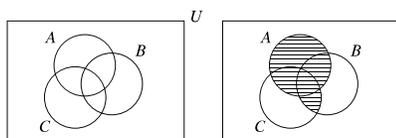


D`vniY 7 | †fbiP†I Mvp K†i †`LvI

(a) $A \cap (B \cup C)$

(b) $A \cup (B \cap C)$

mgvavb :



D`vniY 8 | $U = \{2,3,4,5,6,7,8,9,10\}$, $A = \{x : x \text{ †RvomsL`v}\}$

Ges $B = \{x : 7 < 3x < 25\}$

(a) $A, B, A \cap B, A \cup B$ Ges $A \cap B'$ Gi Dcv`vb_†j v Zwj Kv c×wZ†Z tj L |

(b) $x \in A$ Ges $x \notin B$

(c) $x \notin A$ Ges $x \notin B$

mgvavb : (a) $A = \{2,4,6,8,10\}$

$A \cap B = \{4,6,8\}$

$B = \{3,4,5,6,7,8\}$

$A \cup B = \{2,3,4,5,6,7,8,10\}$

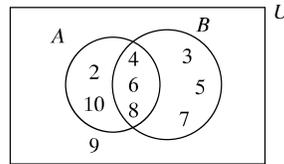
$A \cap B' = \{2,10\}$

(b) $x \in A$ Ges $x \notin B$

$\Leftrightarrow x \in A$ Ges $x \in B'$

$\Leftrightarrow x \in A \cap B'$

$\therefore x = 2, 10$



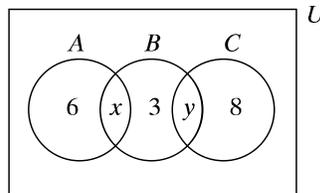
(c) $x \notin A$ Ges $x \notin B$

$\Leftrightarrow x \in A'$ Ges $x \in B'$

$\Leftrightarrow x \in A' \cap B' = \{9\}$

$\therefore x = 9$

D`vniY 9 | t`fbvP`f` mmeR` tmU U Gi c`ZmU Dc`tm`Ui Dc`v`vb msL`v t` Lv`bv n`q`q`Q | GLv`vb D`f`j`L` th, $U = A \cup B \cup C$.



(a) t` l`qv Av`q` n(B) = n(C) Ges GLv`vb t`_`f`K x Gi gvb w`b`Y`q` Ki |

(b) t` l`qv Av`q` n(B \cap C) = n(A \cup B') Ges GLv`vb t`_`f`K y Gi gvb w`b`Y`q` Ki

(a) n(U) KZ ?

mgvavb : (a) $n(B) = n(C)$

$$x + 3 + y = y + 8$$

$$x = 5$$

(b) $n(B \cap C) = n(A \cup B')$

$$y = 6$$

(c) $n(U) = 6 + x + 3 + y + 8$

$$= 6 + 5 + 3 + 6 + 8$$

$$= 28$$

1 | t` l`qv Av`q` th, $U = \{1,2,3,4,5,6,7,8,9\}$ Ges $A = \{x : x, 3 \text{ Gi } \text{YxZK}\}$. t` Lv`l` th,

KvR :

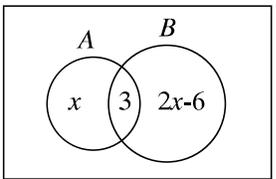
1 | $\hat{t} \text{ l qv AvtQ th, } U = \{1,2,3,4,5,6,7,8,9\}$ Ges $A = \{x : x, 3 \text{ Gi } \text{YxZK}\}$. $\hat{t} \text{ Lvl th,}$
 (a) $A \cup A' = U$ (b) $A \cap A' = \Phi$

2 | $\hat{t} \text{ l qv AvtQ } U = \{3,4,5,6,7,8,9\}$, $A = \{x : x \text{ tgšwj K msL'v}\}$ Ges $B = \{x : x \text{ tRvo msL'v}\}$ |
 $\hat{t} \text{ fbuP} \hat{t} \text{ i mrvvth' tmU } A$ Ges $A \cap B$ Gi Dcv`vb, tj vi Zwj Kv ^Zwi Ki |
 $\hat{t} \text{ Lvl th, (a) } A' \cap B' = \{9\}$ (b) $A \subseteq B'$ Ges $A \subseteq A'$.

3 | $\hat{t} \text{ fbuP} \hat{t} \text{ A } \cap B$ tm \hat{t} Ui Dcv`vb, tj v $\hat{t} \text{ Lv} \hat{t} \text{bv ntj v |}$
 $\hat{t} \text{ l qv AvtQ, } n(A) = n(A' \cap B)$ Zvntj
 (a) x Gi gvb $\text{wbY} \hat{t} \text{ Ki}$
 (b) $n(A) \mid n(B)$ Gi gvb $\text{wbY} \hat{t} \text{ Ki |}$

4 | $U = \{p, q, r, s, t, u, v, w\}$, $A = \{p, q, r, s\}$
 $B = \{r, s, t\}$ Ges $C = \{s, t, u, v, w\}$
 (a) $n(A \cup B) = \text{KZ?}$
 (b) $(A \cup B)'$ Ges $A \cup B \cup C$ Gi Dcv`vb, tj vi Zwj Kv ^Zwi Ki |

5 | $\hat{t} \text{ fbuP} \hat{t} \text{ Mvp (Shade) K} \hat{t} \text{ i } \hat{t} \text{ Lvl : (a) } (P \cap Q) \cap R'$ (b) $(A \cap B') \cup C$



tmU c \hat{t} uqv \hat{t} ag \hat{t} ewj

B \hat{t} Zvc \hat{t} e \hat{t} tm \hat{t} Ui msthvM, \hat{t} Q` Ges $\text{wb} \hat{t} \hat{t} \hat{t}$ ` tmU m \hat{t} ut \hat{t} K \hat{t} Av \hat{t} j vPbv Kiv ntq \hat{t} Q | GLv \hat{t} b G \hat{t} ` i ag \hat{t} ewj m \hat{t} ut \hat{t} K \hat{t} Av \hat{t} j vPbv Kiv ntj v :

msthvM I \hat{t} Q` tm \hat{t} Ui ag \hat{t} ej x :

c \hat{t} Z \hat{t} v 1 | $\text{weibgq } \text{wbqg} ((Commutative law))$

g \hat{t} b Kwi , $A = \{1,2,4\}$ Ges $B = \{2,3,5\}$ ` $\text{B} \hat{t} \text{U tmU |}$ Zvntj

$$A \cup B = \{1,2,4\} \cup \{2,3,5\}$$

$$= \{1,2,4,3,5\}$$

$$B \cup A = \{2,3,5\} \cup \{1,2,4\}$$

$$= \{2,3,5,1,4\}$$

th \hat{t} nZi
 $A \cup B = \{x : x \in A \text{ ev } x \in B\}$

th \hat{t} nZi $A \cup B$ Ges $B \cup A$ G c \hat{t} KZ c \hat{t} q | GKB Dcv`vb, tj v $\text{we} \hat{t} \text{ gvb |}$

AZGe, $A \cup B = B \cup A$

GKBf \hat{t} e Avevi , $A = \{a, b, c\}$ Ges $B = \{b, c, a\}$ $\text{wb} \hat{t} \hat{t} \hat{t}$ $\hat{t} \text{ Lv} \hat{t} \text{bv hvq } A \cup B = B \cup A$

m \hat{t} vavi YZ th \hat{t} Kv \hat{t} bv ` $\text{B} \hat{t} \text{U tmU } A$ Ges B $\hat{t} \text{ q} \hat{t} \hat{t}$ $\hat{t} \text{ Lv} \hat{t} \text{bv hvq}$

$A \cup B = B \cup A$

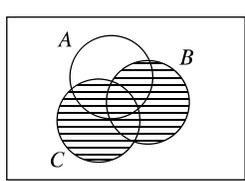
Gu \hat{t} B msthvM tm \hat{t} Ui $\text{weibgq } \text{weia |}$
 \therefore tm \hat{t} Ui msthvM tm \hat{t} Ui $\text{weibgq } \text{weia } \text{tg} \hat{t} \text{b P} \hat{t} \text{j |}$

ḥbe : Abj jcfvte tQ` cġµqvq wewbgq wewa

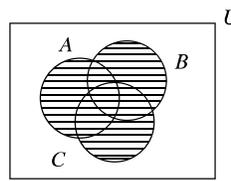
$$A \cap B = B \cap A$$

cġZÁv 2 | mnḥvRb wbgq (Associative law)

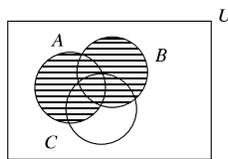
G wbgqU eḥvi Rb` ḥfbwPĪ e`envi Kiv nḥj v | awi $A, B \mid C$ wZbwU tmU



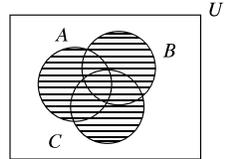
PĪ-a (i)
 $B \cup C$ nḥj v Mvp AskUKz



PĪ-a (ii)
 $A \cap (B \cup C)$ nḥj v Mvp AskUKz



PĪ-b (i)
 $A \cup B$ nḥj v Mvp AskUKz



PĪ-b (ii)
 $(A \cup B) \cup C$ nḥj v Mvp AskUKz

ḥfbwPĪ a(ii) Ges b(ii) ḥḥK GUv cwī®vi ḥ, $A \cap (B \cup C) = (A \cap B) \cup C$

G wbgqUB $A = \{a, b, c, d\}$, $B = \{b, c, f\}$ Ges $C = \{c, d, g\}$ wZbwU tmU wbtq eḥvi ḥPón Kwī
GLvḥb $B \cup C = \{b, c, f\} \cup \{c, d, g\}$

$$= \{b, c, f, d, g\}.$$

Ges $A \cap (B \cup C) = \{a, b, c, d\} \cap \{b, c, f, d, g\}$

$$= \{a, b, c, d, f, g\} \dots \dots \dots (i)$$

GLb, $A \cap B = \{a, b, c, d\} \cap \{b, c, f\}$

$$= \{a, b, c, d, f\}$$

Ges $(A \cap B) \cup C = \{a, b, c, d, f\} \cup \{c, d, g\}$

$$= \{a, b, c, d, f, g\} \dots \dots \dots (ii)$$

(i) | (ii) nḥZ Avgiv cvB, $A \cap (B \cup C) = (A \cap B) \cup C$

mvavi YZ, ḥḥKvḥbv wZbwU tmU $A, B \mid C$ Gi Rb`

$$A \cap (B \cup C) = (A \cap B) \cup C$$

∴ tmḥUi mḥḥvM cġµqv mnḥvRb wbgq tgḥb Pḥj |

Abj jcfvte tQ` cġµqv mnḥvRb wbgq tgḥb Pḥj

$$A \cap (B \cap C) = (A \cap B) \cap C$$

c0ZÁv 3 | $A \cup A$: Gi Rb" awi $A = \{2,3,5\}$

$$A \cup A = \{2,3,5\} \cup \{2,3,5\}$$

$$= \{2,3,5\}$$

$$= A$$

GKBFvte $A = \{x, y, z\}$ wbtq t` Lvfbv hvq th, $A \cup A = A$

∴ wmvš-: thtKvfbv tmU A Gi Rb"

$$\boxed{A \cup A = A}$$

GKBFvte wbtR Ki : $A \cap A = A$

c0ZÁv 4 | hw` $A \subset B$ ZLb $A \cup B = B$.

awi, $A = \{1,2,3\}$ Ges $B = \{x | x \in N, 1 \leq x \leq 5\}$ `BwU tmU |

$$\therefore A = \{1,2,3\} \text{ Ges } B = \{1,2,3,4,5\}$$

$$\therefore A \subset B.$$

$$\text{GLb } A \cup B = \{1,2,3\} \cup \{1,2,3,4,5\}$$

$$= \{1,2,3,4,5\}$$

$$= B.$$

Gfvte, hw` $A \subset B$ ZLb $A \cup B = B$ Ges hw` $B \subset A$ ZLb $A \cup B = A$.

GKBFvte wbtR Ki : $A \subset B$ ZLb $A \cap B = A$ Ges hw` $B \subset A$ ZLb $A \cap B = B$

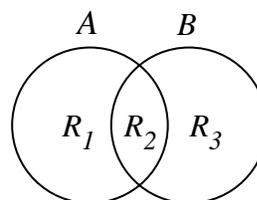
c0ZÁv 5 | $A \subset (A \cup B)$: gtb Kwi, A Ges B `BwU tmU | cvtki wPÍ j q Kwi R_1 Ges R_2 Gj vKv

A tmU i Ašf | Avevi, R_2 Ges R_3 Gj vKv B Gi Ašf |

mZivs, R_1, R_2 Ges R_3 Gj vKv $A \cup B$ Gi Ašf |

wKš' R_1 Ges R_2 AĀj R_1, R_2 Ges R_3 Gj vKvi AšmZ

i.e. $A \subset (A \cup B)$.



wmvš-: thtKvfbv tmU $A \cap B$ Gi Rb"

$$\boxed{A \subset (A \cup B)} \text{ Ges } \boxed{B \subset (A \cup B)}$$

`be" : GKBFvte wbtR Ki : thtKvfbv tmU A Ges B Gi Rb" $(A \cap B) \subset A$ Ges $(A \cap B) \subset B$

c0ZÁv 6 | $A \cup U = U$ Ges $A \cup \Phi = A$ Avgiv Rwb, $A \subset U$ Ges $\Phi \subset A$ (4) bs agfbhvqx,

$$A \cup U = U \text{ Ges } A \cup \Phi = A$$

KvR :

1 | $A \cup B$ wbyq Ki hLb

$$A = \{x | x \text{ cYmsL'v, } -2 \leq x < 1\} \text{ Ges } B = \{x | x \text{ tgsWj K msL'v, } 24 \leq x \leq 28\}$$

- 2| $A \cup U$ wbyq Ki thlvb $U = \{x | x \text{ cYmsL'v, } -2 < x < 3\}$ Ges
 $A = \{x | x \in z, -1 < x \leq 1\}$
- 3| hw $A = \{2,3,5\}, B = \{a,b,c\}, C = \{2,3,5,7\}$ Ges
 $D = \{a,b,c,d\}$ nq, Zte cgvY Ki th, $(A \cup B) \subset (C \cup D)$
- 4| $A = \{a,b,c\}$ Ges $B = \{b,c,d\}$ Gi Rb" hvPvB Ki $A \cap B = B \cap A$.
- 5| hw $A = \{1,3,5,7\}, B = \{3,7,8\}$ Ges $C = \{7,8,9\}$ nq, Zte f`Lvl th,
 $(A \cap B) \cap C = (B \cap C) \cap A$.

c0ZÁv 7| e)Ub wbyq (*Distributive Law*)

A, B, C thtKvbtv tmU ntj, f`Lvl th,

$$(K) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$(L) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

cgvY : gtb Kwí, $x \in A \cup (B \cap C)$

Zvntj $x \in A$ A_ev $x \in B \cap C$

$$\Rightarrow x \in A \text{ A_ev } (x \in B \text{ Ges } x \in C)$$

$$\Rightarrow (x \in A \text{ A_ev } x \in B) \text{ Ges } (x \in A \text{ A_ev } x \in C)$$

$$\Rightarrow x \in A \cup B \text{ Ges } x \in A \cup C$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

$\therefore A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ (i)

Avevi gtb Kwí, $x \in (A \cup B) \cap (A \cup C)$

Zvntj, $x \in A \cup B$ Ges $x \in A \cup C$

$$\Rightarrow (x \in A \text{ A_ev } x \in B) \text{ Ges } (x \in A \text{ A_ev } x \in C)$$

$$\Rightarrow x \in A \text{ A_ev } (x \in B \text{ Ges } x \in C)$$

$$\Rightarrow x \in A \text{ A_ev } x \in B \cap C$$

$$\Rightarrow x \in A \cup (B \cap C)$$

$\therefore (A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ (ii)

mYZivs (i) | (ii) ntZ cvl qv hvq $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(L) GKBFvte wbtR Ki |

KvR :

(i) e)Ub wevai mfwU cgvY Ki | thlvb -

$A = \{1,2,3,6\}, B = \{2,3,4,5\}$ Ges $C = \{3,5,6,7\}$

(ii) cgvYwU tfbwPti i gva'tg f`Lvl

wm×vS: tmfUi msthvM | tQ` cgvYwU `BvUi c0Z'KwU AcivUi tcvf'fZ eEb wbyq tgb Ptj |

cĀZÁv 8 | ĩv gi M'vĵbi mĤ (De Morgans law) :

mmeĤ tmU U Gi thĵKvĵbv DcĵtmU $A \mid B$ Gi Rb" (K) $(A \cup B)' = A' \cap B'$
(L) $(A \cap B)' = A' \cup B'$

cġvY (K) : gĵb Kwi , $x \in (A \cup B)'$

Zvntĵ , $x \notin A \cup B$

$\Rightarrow x \notin A$ Ges $x \notin B$

$\Rightarrow x \in A'$ Ges $x \in B'$..

$\Rightarrow x \in A' \cap B'$

$\therefore (A \cup B)' \subset A' \cap B'$

Avevi gĵb Kwi , $x \in A' \cap B'$

Zvntĵ , $x \in A'$ A_ev $x \in B'$

$\Rightarrow x \notin A$ A_ev $x \notin B \Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)'$

$\therefore A' \cap B' \subset (A \cup B)'$

mĵi vs $(A \cup B)' = A' \cap B'$ cġvYZ |

(L) Abjĵcĵvĵe wĵĵR Ki :

cĀZÁv 9 | mmeĤ tmU U Gi thĵKvĵbv DcĵtmU $A \mid B$ Gi Rb" $A \setminus B = A \cap B'$

cġvY : gĵb Kwi , $x \in A \setminus B$

Zvntĵ $x \in A$ Ges $x \notin B$

$\Rightarrow x \in A$ Ges $x \in B'$

$\therefore x \in A \cap B'$

$\therefore A \setminus B \subset A \cap B'$

Avevi gĵb Kwi , $x \in A \cap B'$

Zvntĵ , $x \in A$ Ges $x \in B'$

$\Rightarrow x \in A$ Ges $x \notin B$

$\therefore x \in A \setminus B$

$\therefore A \cap B' \subset A \setminus B$

mĵi vs, $A \setminus B = A \cap B'$

cĀZÁv 10 | thĵKvĵbv tmU A, B, C Gi Rb"

(K) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(L) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

cġvY : (K) msÁbvĵvĵi

$A \times (B \cap C)$

$= \{(x, y) : x \in A, y \in B \cap C\}$

$$\begin{aligned}
 &= \{(x, y) : x \in A, y \in B \text{ Ges } y \in C\} \\
 &= \{(x, y) : (x, y) \in A \times B \text{ Ges } (x, y) \in A \times C\} = \{(x, y) : (x, y) \in (A \times B) \cap (A \times C)\} \\
 &A \times (B \cap C) \subset (A \times B) \cap (A \times C)
 \end{aligned}$$

Avevi $(A \times B) \cap (A \times C)$

$$\begin{aligned}
 &= \{(x, y) : (x, y) \in A \times B \text{ Ges } (x, y) \in A \times C\} \\
 &= \{x, y\} : x \in A, y \in B \text{ Ges } x \in A, y \in C\} \\
 &= \{(x, y) : x \in A, y \in B \cap C\} \\
 &= \{(x, y) : (x, y) \in A \times B \cap C\}
 \end{aligned}$$

$\therefore (A \times B) \cap (A \times C) \subset A \times (B \cap C)$

A_# $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(L) Abjfcfvte vbtR Ki |

11 | tmU c#μqv msμvš-Avtiv KwZcq c#ZÁv :

- (K) A th#Kv#bv tmU ntj $A \subset A$
- (L) duKv tmU Φ th#Kv#bv tmU A Gi Dc#tmU
- (M) A I B th#Kv#bv tmU ntj $A = B$ nte hw` I tKej hw` $A \subset B$ Ges $B \subset A$ nq |
- (N) hw` $A \subset \Phi$ nq, Z#e $A = \Phi$
- (O) hw` $A \subset B$ Ges $B \subset C$ Z#e, $A \subset C$
- (P) A I B th#Kv#bv tmU ntj, $A \cap B \subset A$ Ges $A \cap B \subset B$
- (Q) A I B th#Kv#bv tmU ntj $A \subset A \cup B$ Ges $B \subset A \cup B$

c#yY : (L) : g#b Kwí $\Phi \notin A$, m#Zivis msÁvbjvnti Ggb x Av#Q thb $x \in \Phi$ | wKŠ' $\Phi \notin A$
 th#nZikb` tm#U Av#`Š tKv#bv Dcv`vb tbB |

$\therefore \Phi \notin A$ mZ` bq

$\therefore \Phi \in A$

(N) t` I qv Av#Q, $A \subset \Phi$ Avevi Avgiv Rvwb, $\Phi \subset A$ m#Zivis $A = \Phi$ [c#ZÁv M t_#K]

(Q) tm#Ui msthv#Mi msÁvbjvnti A tm#Ui mKj Dcv`vb $A \cup B$ tm#U _v#K | m#Zivis Dc#tm#Ui
 msÁvbjvnti $A \subset A \cup B$ | GKB hw#tZ $B \subset A \cup B$

`be` : K, M, O I P c#ZÁv, t#j v vbtR Ki |

KvR : [GLv#b mKj tmU mweR tmU U Gi Dc#tmU vetePbv Ki tZ nte]

1 | t`LvI th : $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

2 | t`LvI th, $A \subset B$ nte hw` Ges tKej hw` vbtg#³ th#Kv#bv GKwU kZ⁹Lv#U :

(K) $A \cap B = A$

(L) $A \cup B = B$

(M) $B' \subset A$

(N) $A \cap B' = \Phi$

(O) $B \cup A' = U$

- 3 | t' Lvl th,
 (K) $A \setminus B \subset A \cup B$
 (L) $A' \setminus B' = B \setminus A$
 (M) $A \setminus B \subset A$
 (N) $A \subset B$ ntj . $A \cup (B \setminus A) = B$
 (O) $A \cap B = \Phi$ ntj , $A \subset B'$ Ges $A \cap B' = A$ Ges $A \cup B' = B'$
- 4 | t' Lvl th,
 (K) $(A \cap B)' = A' \cup B'$
 (L) $(A \cup B \cup C)' = A' \cap B' \cap C'$
 (M) $(A \cap B \cap C)' = A' \cup B' \cup C'$

mgZj I Amxg tmU

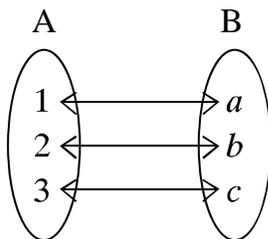
GK-GK wj (*One One Correspondence*)

gtb Kwi , $A = \{a, b, c\}$ wZbRb tj vti tmU Ges $B = \{30, 40, 50\}$ H wZbRb tj vti eqtmi tmU |
 AwakS'gtb Kwi , a Gi eqm 30, b Gi eqm 40 Ges C Gi eqm 50.
 mZi vs ej v hvq th, A tmU mvt_ B tmU GK-GK wj AvtQ |

msAv : hw` A tmU cZw Dcv`vtbi mvt_ B tmU GKw I tKej GKw Dcv`vb Ges B tmU cZw Dcv`vtbi mvt_ A tmU GKw I tKej GKw Dcv`vtbi wj `vcb Kiv nq, Zte A I B Gi gta` GK-GK wj ej v nq | A I B Gi gta` GK GK wj tK maviYZ $A \leftrightarrow B$ wj tL cKvk Kiv nq Ges A tmU tKvb m`m` x Gi m½ B tmU th m`m` y Gi wj Kiv ntqtQ Zv $x \leftrightarrow y$ wj tL eYv Kiv nq |

mgZj tmU (*Equivalent set*)

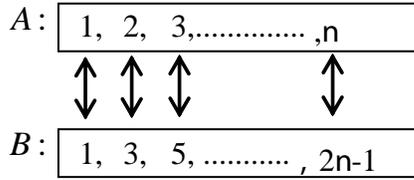
awi , $A = \{1, 2, 3\}$ Ges $B = \{a, b, c\}$ `Bw tmU | wtpi wptI A I B tmU tqi gta` GKw GK-GK wj `vcb Kti t` Lvfbv ntj v :



msAv : thtKvfbv tmU A I B Gi gta` hw` GKw GK-GK wj $A \leftrightarrow B$ eYv Kiv hvq, Zte A I B tK mgZj tmU ej v nq | A I B tK mgZj tevSvZ $A \sim B$ cZxK tj Lv nq | $A \sim B$ cZxK ntj , Gt` i thtKvfbv GKw tK AciwUi mvt_ mgZj ej v nq |

D`vniY 10| t`Lvl th, $A = \{1,2,3,\dots,n\}$ Ges $B = \{1,3,5,\dots,2n-1\}$ tmU0q mgZj , thLvfb n GKwU `vfwek msL`v|

mgvavb : $A \mid B$ tmU `BwU gta` GKwU GK-GK wj wbtgic wPtI t`Lvfbv ntjv :



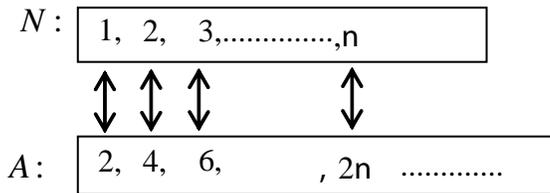
myZivs $A \mid B$ tmU `BwU mgZj |

gše` : Dciti wPwI Z GK-GK wj wUtk $A \leftrightarrow B : k \leftrightarrow 2k-1, k \in A$ 0viv eY0v Kiv hvq|

D`vniY 11| t`Lvl th, `vfwek msL`vi tmU N Ges tRvo msL`vi tmU $A = \{2, 4, 6,\dots,n,\dots\}$

mgZj |

mgvavb : GLvfb, $N = \{1,2,3,\dots;n,\dots\}$ N Ges A Gi gta` GKwU GK-GK wj wbtgic wPtI t`Lvfbv ntjv



myZivs $N \mid A$ mgZj tmU|

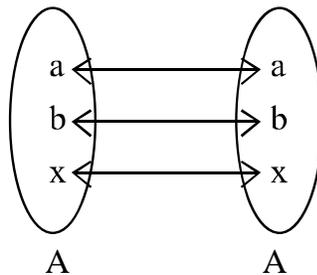
gše` : Dciti wPwI Z GK-GK wj wUtk $N \leftrightarrow A : n \leftrightarrow 2n, n \in N$ 0viv eY0v Kiv hvq|

`be` : duKv tmU Φ Gi wbtRi mgZj aiv nq| $A \sim \Phi, \Phi \sim \Phi$

c0ZAv 1| c0Z`K tmU A Zvi wbtRi mgZj |

c0vY : $A \sim \Phi$ ntj , $A \sim A$ aiv nq|

gfb Kwi , $A \neq \Phi$

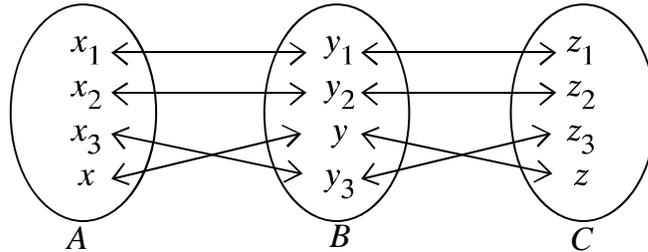


A tmUUi c0Z`K m`m` x Gi mt½ Zvi wbtRtk wj Kiv ntj GK-GK wj $A \leftrightarrow A : x \leftrightarrow x, x \in A$ `vncZ nq|

myZivs $A \sim A$.

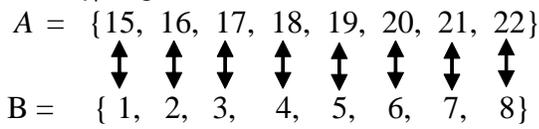
c0ZÁv 2 : hw` A I B mgZj tmU nq Ges B I C mgZj tmU nq, Zte A I C mgZj tmU nte|

c0yvY : thtnZl A ~ B, mZivs A Gi c0Z'K m`m` x Gi m½ B Gi GKwU Abb` m`m` y Gi wjj Kiv hvq| Avevi thtnZl B ~ C, mZivs B Gi GB m`m` y Gi m½ C Gi GKwU Abb` m`m` z Gi wjj Kiv hvq| GLb A Gi m`m` x Gi m½ C Gi G m`m` z Gi wjj Kiv ntj , A I C tmfUi gta` GKwU GK-GK wjj `wcz nq| A_@, A ~ C nq|



mvš-I Abš-tmU (*Finite and Infinite sets*)

A = {15, 16, 17, 18, 19, 20, 21, 22} tmUUi m`m` ,tj v MYbv Kti t`Lv hvq th, A tmfUi m`m` msL`v 8 | GB MYbv KvR A tmfUi m½ B = {1, 2, 3, 4, 5, 6, 7, 8} tmfUi GKwU GK-GK wjj `vcb Kti m`úbe Kiv nq| thgb,



Gifc MYbv Kti th mKj tmfUi m`m` msL`v wba@Y Kiv hvq, Zv` iK mvš-tmU ej v nq| duKv tmUfKI mvš-tmU aiv nq|

msÁv : (K) duKv tmU Φ mvš-tmU Gi m`m` msL`v 0.

(L) hw` tKvfbv tmU A Ges $J_m = \{1, 2, 3, \dots, m\}$ mgZj nq, thLvfb $m \in N$, Zte A GKwU mvš-tmU Ges A Gi m`m` msL`v m |

(M) A tKvfbv mvš-tmU ntj , A Gi m`m` msL`vK $n(A)$ 0viv mwPZ nq|

(N) tKvfbv tmU A mvš-tmU bv ntj , GtK Abš-tmU ej v nq|

`be` 1 | $J_1 = \{1\}, J_2 = \{1, 2\}, J_3 = \{1, 2, 3\}$ BZ`w` c0Z`tKB N Gi mvš- Dc`tmU Ges $n(J_1) = 1, n(J_2) = 2, n(J_3) = 3$ BZ`w` |

ev`weK ct`¶, $J_m \sim J_m$ (GB Abt`Qt` i c0ZÁv 1 `be`) Ges $n(J_m) = m$ |

`be` 2 | i agv` mvš-tmU B m`m` msL`v wbow` Kiv hvq| mZivs $n(A)$ wj Ltj eStZ nte A mvš-tmU|

`be` 3 | A I B mgZj tmU Ges Gt` i gta` GKwU tmU mvš-ntj Aci tmUUi mvš-nte Ges $n(A) = n(B)$ nte|

cĀZĀv 3 | hw` A mvš-řmU nq Ges B, A Gi cĀKZ DcřmU nq, Zře B mvš-řmU Ges $n(B) < n(A)$ nře |

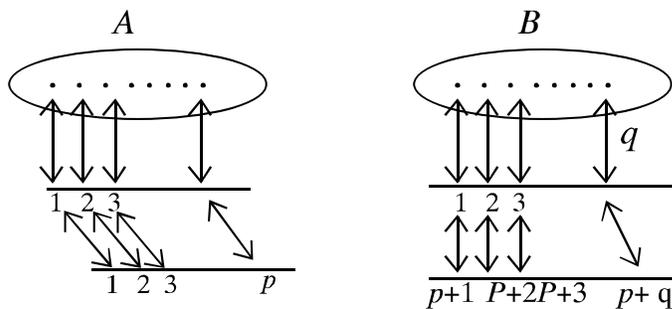
cĀZĀv 4 | A Abš-řmU nře hw` I řkej hw` A Ges A Gi GKĀU cĀKZ DcřmU mgZř nq |

`ře` 5 | N GKĀU Abš-řmU (D`vni Y 11 `ře`)|

mvš-řmU i Dcv` vb mSL`v

mvš-řmU A Gi Dcv` vb mSL`v $n(A)$ řviv mPZ Kiv nřqřQ Ges $n(A)$ vbařřři cĀWZ e`vL`v Kiv nřqřQ |

gřb KwĀ, $n(A) = P > 0, n(B) = q > 0$, řhLvřb $A \cap B = \emptyset$



Dcřři i vPřř eWYZ GK-GK wřj řřřK ř`Lv hvq řh, $A \cup B \sim J_{p+q}$

A_řř, $n(A \cup B) = p + q = n(A) + n(B)$ G řřřK ej v hvq řh,

cĀZĀv 1 | hw` A I B ci`ři vřřř` řmU nq, Zře $n(A \cup B) = n(A) + n(B)$

GB cĀZĀvřK mřcřřvi Y Kři ej v hvq řh, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) \text{ BZ`w` ,}$$

řhLvřb A, B, C, D řmU, řj v ci`ři vřřř` mvš-řmU |

cĀZĀv 2 | řřřKvřbv mvš-řmU A I B Gi Rb` $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

cřyvY : GLvřb, $A \setminus B, A \cap B$ Ges $B \setminus A$ řmU wZbvU ci`ři vřřř` řmU [řřbvPřř `ře`] Ges

$$A = (A \setminus B) \cup (A \cap B)$$

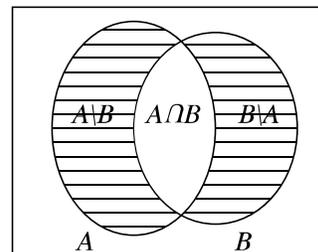
$$B = (B \setminus A) \cup (A \cap B)$$

$$A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$$

$$\therefore n(A) = n(A \setminus B) + n(A \cap B) \dots (i)$$

$$n(B) = n(B \setminus A) + n(A \cap B) \dots (ii)$$

$$n(A \cup B) = n(A \setminus B) + n(A \cap B) + n(B \setminus A) \dots (iii)$$



mživs, (i) bs t₁K cvB, $n(A \setminus B) = n(A) - n(A \cap B)$

(ii) bs t₁K cvB, $n(B \setminus A) = n(B) - n(A \cap B)$

GLb, $n(A \setminus B)$ Ges $n(B \setminus A)$ (iii) bs G emtq cvB,

$$n(A \cup B) = n(A) - n(A \cap B) + n(B) - n(A \cap B) + n(A \cap B)$$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

KvR :

1| vbtg³ c₀Z²K t₁†¹ A | B Gi gta^m m³te^e mKj GK-GK v₁g₁ eY⁰ Ki :

(K) $A = \{a, b\}$ $B = \{1, 2\}$.

(L) $A = \{a, b, c\}$ $B = \{a, b, c\}$

2| Dcti c₀k₀ev² c₀Z²K GK-GK v₁g₁ Ki †Yi Rb² $F = \{(x, y) : x \in A, y \in B\}$ Ges $x \leftrightarrow y$ tmU₁ Z_vv₁ Kv c₀x₁Z₁Z eY⁰ Ki |

3| gtb Kw¹ $A = \{a, b, c, d\}$ Ges $B = \{1, 2, 3, 4\}$ | $A \times B$ Gi GK₁U DctmU F eY⁰ Ki | hvi Ašf³ μgtRvo₁ †j vi c₀g ct¹ i m¹/₂ v₀Z₁xq ct¹ i v₁g₁ Kiv n₁t₁j , A | B Gi GK₁U GK-GK v₁g₁ v₁mcZ nq thLv₁tb, $a \leftrightarrow 3$ |

4| t¹ Lvl th, $A = \{1, 2, 3, \dots, n\}$ Ges $B = \{1, 2, 2^2, \dots, 2^{m+1}\}$ tmU v₁β₁U mgZ₁j |

5| t¹ Lvl th, $S = \{3^n : n = 0 \text{ A}_ev n \in N\}$ tmU₁ N Gi mgZ₁j |

6| Dcti c₀k₀ev² S tm₁t₁Ui GK₁U c₀KZ DctmU eY⁰ Ki hv S Gi mgZ₁j |

7| t¹ Lvl th, mKj v₁et₁Rvo v₁f₁v₁ek msL₁vi tmU $A = \{1, 3, 5, 7, \dots\}$ BZ¹v¹ Abš¹-tmU |

kw³ tmU

gva¹v₁gK exRM₁v₁†Z G ms₁μ₁v₁š¹-v₁e¹ v₁vi Z A₁t₁j v₁P₁v₁ Kiv n₁t₁q₁t₁Q | GLv₁tb i¹aykw³ tm₁t₁Ui D¹v₁niY t¹ l qv n₁t₁j v :

D¹v₁niY 12 | hv¹ $A = \{1, 2, 3\}$ Ges $B = \{2, 3, 4\}$ nq, Zte t¹ Lvl th, $P(A) \cap P(B) = P(A \cap B)$

mgvavb : GLv₁tb, $A = \{1, 2, 3\}$ Ges $B = \{2, 3, 4\}$

mživs, $P(A) = \{\Phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Ges $P(B) = \{\Phi, \{2\}, \{3\}, \{4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{2, 3, 4\}\}$

$$\therefore P(A) \cap P(B) = \{\Phi, \{2\}, \{3\}, \{2, 3\}\}$$

GLb, $A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\}$

$$= \{2, 3\}$$

$$\therefore P(A \cap B) = \{\Phi, \{2\}, \{3\}, \{2, 3\}\}$$

mživs $P(A) \cap P(B) = P(A \cap B)$.

D`vniY 13 | hw` $A = \{a, b\}$ Ges $B = \{b, c\}$ nq, Zte t` Lvl th, $P(A) \cup P(B) \subset P(A \cap B)$

mgvavb : GLvfb, $P(A) = \{\Phi, \{a\}, \{b\}, \{a, b\}\}$

$$P(B) = \{\Phi, \{b\}, \{c\}, \{b, c\}\}$$

$$\therefore P(A) \cup P(B) = \{\Phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$$

Aevi, $A \cup B = \{a, b, c\}$

$$\therefore P(A \cup B) = \{\Phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

mZivs, $P(A) \cup P(B) \subset P(A \cap B)$.

KvR :

1 | hw` $A = \{1, 2, 3\}$, $B = \{1, 2\}$, $C = \{2, 3\}$ Ges $D = \{1, 3\}$ nq, Zte t` Lvl th,

$$P(A) = \{A, B, C, D, \{1\}, \{2\}, \{3\}, \Phi\}$$

2 | hw` $A = \{1, 2\}$ Ges $B = \{2, 5\}$ nq, Zte t` Lvl th,

$$P(A) = \{A, B, C, D, \{1\}, \{2\}, \{3\}, \Phi\}.$$

$$(i) P(A) \cap P(B) = P(A \cap B)$$

$$(ii) P(A) \cup P(B) \neq P(A \cup B).$$

ev` e mgm`v mgvavfb tmU :

ev` e mgm`v mgvavfb t`fbwP` e`envi Kiv nq | GLvfb D`j L` th, cZtmtUi Dcv`vb msL`v t`fbwP` t`j Lv nte, Zv KtqKwU D`vni t`Yi gva`tg t` Lvfbv ntj v |

D`vniY 14 | 50 Rb tj v`Ki gta` 35 Rb Bsti`R 25 Rb Bsti`R | evsj v ej tZ cvti Ges cZ`tKB `BwU fvlvi AŠZ GKwU ej tZ cvti | evsj v ej tZ cvti KZ Rb ? tKej gv` evsj v ej tZ cvti KZ Rb ?

mgvavb : gtb Kw, mKj tj v`Ki tmU S Ges Zv` i gta` hviv Bsti`R ej tZ cvti Zv` i tmU E, hviv evsj v ej tZ cvti Zv` i tmU B |

Zvntj cKubvnti, $n(S) = 50$, $n(E) = 35$, $n(E \cap B) = 25$ Ges

$$S = E \cup B$$

gtb Kw, $n(B) = x$

Zvntj, $n(S) = n(E \cup B) = n(E) + n(B) - n(E \cap B)$ t` tK cvB,

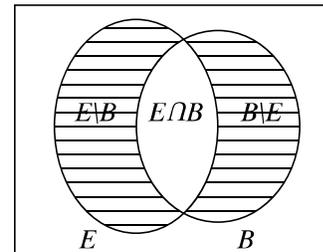
$$50 = 35 + x - 25$$

ev, $x = 50 - 35 + 25 = 40$

A`f, $n(B) = 40$

\therefore evsj v ej tZ cvti 40 Rb |

GLb, hviv tKej evsj v ej tZ cvti, Zv` i tmU nt`Q ($B \setminus E$) |



gfb Kwī, $n(B \setminus E) = y$ thfnZl $E \cap B$ Ges $B \setminus E$ wbtñ` Ges $B = (E \cap B) \cup (B \setminus E)$ [tfbwPĪ`
`be`]

mZivs $n(B) = n(E \cap B) + n(B \setminus E)$

$\therefore 40 = 25 + y$

ev, $y = 40 - 25 = 15$

A_ŕ, $n(B \setminus E) = 15$

\therefore tKej evsj v ej tZ cvti 15 Rb | AZGe, evsj v ej tZ cvti 40 Rb Ges tKej gvĪ evsj v ej tZ
cvti 15 Rb |

D`vniY 15 | f#Mvj I BwZnm wel tğ covi bv Ki tğ Ggb QvĪ t` i tmU h_vµtğ $G \cap H$ ntj wbtğe cĀkē
DĒi `vl (D tğ L`, tm tUi m`m` wbt` R Ki tğ x e`envi Kiv ntğ tğ |)

(a) (i) f#Mvj I BwZnm Dfğ wel tğ covi bv Kti tğ Ggb QvĪ t` i msL`v

(ii) i agvĪ BwZnm covi bv Ki tğ Ggb QvĪ t` i msL`v

tfbwPĪ Mvp Kti t` Lvl |

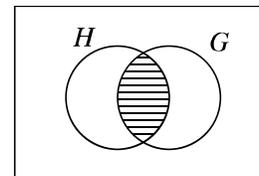
(b) tKv tğ Kv tmi 32 Rb QvĪ i gta` cĀZ`K QvĪ AšZ f#Mvj ev BwZnm wel tğ covi bv Ki tğ | Zv t` i
gta` 22 Rb f#Mvj Ges 15 Rb BwZnm | KZRb QvĪ BwZnm I f#Mvj Dfğ wel tğ c tğ Zv tfbwPĪ
t` Lvl |

mgvavb : (a) (i) $x \in H$ Ges $x \in G$

i.e. $x \in H \cap G$

(ii) $x \in H$ Ges $x \notin G$

i.e. $x \in H \setminus G$

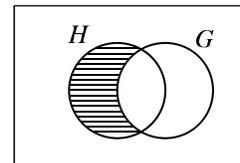


(b) awi, BwZnm wel tğ c tğ Ggb QvĪ t` i tmU H

f#Mvj wel tğ c tğ Ggb QvĪ t` i tmU G

Zv ntj $H \cap G$ f#Mvj I BwZnm wel tğ c tğ Ggb QvĪ t` i tmU

awi, $n(H \cap G) = x$



thfnZl GK wel tğ AšZ cĀZ`K c tğ $H \cup G = U$

$n(H \cup G) = n(U)$

i.e. $(22 - x) + x + (15 - x) = 32$

$\Rightarrow 37 - x = 32$

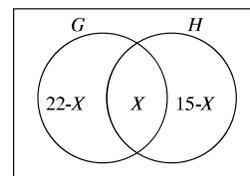
$\therefore x = 5$

mZivs 5 Rb QvĪ BwZnm I f#Mvj Dfğ wel tğ c tğ |

D`vniY 16 | GKwU tkŕYi 35 Rb evj Kvi cĀZ`K t` šo, muZvi I bv tPi th tKv tğ GKwU Z AskMŕY

Kti | Zv t` i gta` 15 Rb t` šo, 4 Rb muZvi I bv P, Rb i ay t` šo, 7 Rb muZv tĪ AskMŕY Kti wKš`

bv tP bq |



$$n(B \cup V) = (18 - x) + x + (12 - x) = 30 - x$$

$$\therefore n(B \cup V) = (18 - x) + x + (12 - x) = 30 - x$$

(b) $n(B \cap V)$ ¶i Zg hLb $B \cup V = U$ ZLb, $n(B \cup V) = n(U) = 30 - x = 24$ ev $x = 6$

\therefore mæte" ¶i Zg gvb $x = 6$

(c) $n(B \cap V)$ epËg hLb $V \subseteq B = U$ ZLb, $n(B \cap V) = n(V) = x = 12$

\therefore mæte" epËg gvb $x = 12$

KvR :

- 1| tKvfbv tkñyi 30Rb QvT i 20Rb dtej Ges 15Rb wµtKU cQ` Kti | cQZ`K QvT `BwU tLj vi thtKvfbv GKwU tLj v cQ` Kti | KZRb QvT `BwU tLj vB cQ` Kti ?
- 2| wKQzmsL`K tj vtKi gta" 50Rb evsj v, 20Rb Bsti wR Ges 10Rb evsj v I Bsti wR ej Z cvti | `BwU fvlvi AšZ GKwU fvlv KZRb ej tZ cvti ?
- 3| XvKv wekte`vj tqi AvaybK fvlv Bbw÷wUDtUi 100Rb wk¶v_¶i gta" 42Rb tdÂ, 30Rb RvgfB, 28Rb `úwob wbtqtQ | 10Rb wbtqtQ tdÂ I `úwobk, 8Rb wbtqtQ RvgfB I `úwobk, 5Rb wbtqtQ RvgfB I tdÂ, 3Rb wZbwU fvlvB wbtqtQ |
 - (i) KZRb wk¶v_¶H wZbwU fvlvi GKwU t bqmb ?
 - (ii) KZRb wk¶v_¶H wZbwU fvlvi tKej GKwU fvlv wbtqtQ ?
 - (iii) KZRb wk¶v_¶H wZbwU fvlvi tKej `BwU fvlv wbtqtQ |
- 4| tKvfbv `qj i beg tkñyi gvbwEK kvLvi 50Rb wk¶v_¶i gta" 29Rb tcŠi bñwZ, 24Rb f¶Mvj Ges 11Rb tcŠi bñwZ I f¶Mvj Dfq welq wbtqtQ | KZRb wk¶v_¶i tcŠi bñwZ ev f¶Mvj welq `BwU tKvbwUB t bqmb ?

Abkxj bx 1.1

- 1| i. tKvb tmñUi m`m" msL`v $2n$ ntj, Gi DctmñUi msL`v nte 4^n
- ii. mKj gj` msL`vi tmU $Q = \left\{ \frac{p}{q} : p, q \in Z, q \neq 0 \right\}$
- iii. $a, b \in R;]a, b[= \{x : x \in R \text{ Ges } a < x < b\}$

Dcti i Zt`i Avtj vtK wbtPi tKvbwU mñWK ?

- K. i | ii L. ii | iii M. i | iii N. i, ii | iii

Wbtpi Zt_i Avtj vtK (2-4) bs c0k0 DEi `vl :

$$c0Z''K \ n \in \mathbb{N} \ \text{Gi} \ \text{Rb''} \ A_n = \{n, 2n, 3n, \dots\}$$

2| $A_1 \cap A_2$ Gi gvb Wbtpi tKvbwU ?

K. A_1 L. A_2 M. A_3 N. A_4

3| Wbtpi tKvbwU $A_3 \cap A_6$ Gi gvb Wbtpi R Kti ?

K. A_2 L. A_3 M. A_4 N. A_6

4| $A_2 \cap B_3$ Gi cwietZ'Wbtpi tKvbwU tj Lv hvq ?

K. A_3 L. A_4 M. A_5 N. A_6

5| t`l qv AvtQ $U = \{x : 3 \leq x \leq 20, n \in \mathbb{Z}\}$, $A = \{x : x \text{ wefRvo msL'v}\}$ Ges $B = \{x : x \text{ tgS'ij K msL'v}\}$ Wbtg0e tmUti Dcv`vb_s,tj vi Zvwj Kv wj wce x Ki :

(a) A Ges B

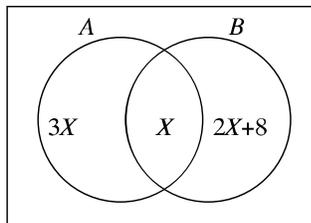
(b) $C = \{x : x \in A \text{ Ges } x \in B\}$ Ges

$D = \{x : x \in A \ \text{A_ev} \ x \in B\}$

tmU C Ges D Gi eY0v `vl

6| t'fbwpt' A Ges B tmUti Dcv`vb_s,tj v t`Lvfbv ntqtQ | hw` $n(A) = n(B)$ nq, Zte WbY0 Ki

(a) x Gi gvb (b) $n(A \cup B)$ Ges $n(A \cap B)$.



7| t'fbwpt' A Ges B tmUti c0Z''Ki Dcv`vb_s,tj v t`Lvfbv ntqtQ | $n(A' \cap B')$ WbY0 Ki |

(a) x Gi gvb (b) $n(A)$ Ges $n(B)$

8| hw` $U = \{x : x \text{ avvZ'K cY'fmsL'v}\}$, $A = \{x : x \geq 5\}$ Ges $B = \{x : x < 12\}$

Zte $n(A \cap B)$ Ges $n(A')$ Gi gvb WbY0 Ki |

9| hw` $U = \{x : x \text{ tRvo cY'fmsL'v}\}$, $A = \{x : 3x \geq 25\}$ Ges $B = \{x : 5x < 12\}$ nq, Zvntj

$n(A \cap B)$ Ges $n(A' \cap B')$ Gi gvb WbY0 Ki |

10| t`Lvl th, (K) $A \setminus A = \Phi$ (L) $A \setminus (A \setminus A) = A$

11| t`Lvl th, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

12| hw` $A \subset B$ Ges $C \subset D$ nq, Zte t`Lvl th, $(A \times C) \subset (B \times D)$

13| t`Lvl th, $A = \{1, 2, 3, \dots, n\}$ Ges $B = \{1, 2, 2^2, \dots, 2^{n-1}\}$ tmU `BwU mgZj |

14| t`LvI th, t`fweK msL`vmg#ni etMP tmU $S = \{1,4,9,25,36,\dots\}$ GKwU AŠ-tmU |

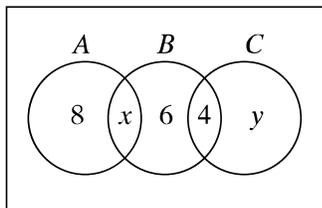
15| c0vY Ki th, $n(A) = p, n(B) = q$ Ges $A \cap B = \Phi$ Ges ntj, $n(A \cup B) = p + q$ |

16| c0vY Ki th, A, B, C mvs-tmU ntj, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$ |

17| hw` $A = \{a, b, x\}$ Ges $B = \{c, y\}$ mwweR tmU $U = \{a, b, c, x, y, z\}$ Gi Dc-tmU ntj, hvPvB Ki th, (a)(i) $A \subset B'$, (ii) $A \cup B' = B'$, (iii) $A' \cap B = B$
 (b) wby0 Ki : $(A \cap B) \cup (A \cap B')$

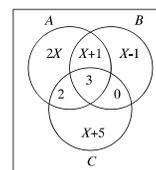
18| tKvfbv tkwYi 30Rb wkqV_0 gta` 19Rb A_0wZ, 17Rb f#Mvj, 11Rb tcSi bwwZ, 12Rb A_0wZ I f#Mvj, 4Rb tcSi bwwZ I f#Mvj, 7 Rb A_0wZ I tcSi bwwZ Ges 5 Rb wZbwU wel qb wbtqtQ | KZRb wkqV_0 wZbwU wel tqi tKvbwUB tbqwb ?

19| tFbwP#t mwweR tmU U Ges Dc-tmU A, B, C Gi m`m` msL`v Dc`vcb Kiv ntqtQ |
 (a) hw` $n(A \cap B) = n(B \cap C)$ nq, Zte x Gi gvb wby0 Ki |
 (b) hw` $n(B \cap C') = n(A' \cap C)$ nq, Zte y Gi gvb wby0 Ki |
 (c) $n(U)$ Gi gvb wby0 Ki |



20| tFbwP#t A, B, C tm#Ui Dcv`vb_#j v Ggbf#te t` I qv AvtQ thb, $U = A \cup B \cup C$

- (a) hw` $n(U) = 50$ nq, Zte x Gi gvb wby0 Ki |
- (b) $n(B \cap C')$ Ges $n(A' \cap B)$ Gi gvb wby0 Ki
- (c) $n(A \cap B \cap C')$ Gi gvb wby0 Ki



21| wZbwU tmU A, B Ges C Ggbf#te t` I qv AvtQ thb, $A \cap B = \Phi, A \cap C = \Phi$ Ges $C \subset B$ tFbwP#t AsKb K#i tmU_#j vi e`vL`v`vl :

22| t` I qv AvtQ $A = \{x : 2 < x \leq 5, x \in R\}$ Ges $B = \{x : 1 \leq x < 3, x \in R\}$

Ges $C = \{2, 4, 5\}$ wbtg#e tmU_#j v Abj#c set notation G cKvk Ki :

- (a) $A \cap B$ (b) $A' \cap B'$ Ges (d) $A' \cup B$

23| t` I qv AvtQ $U = \{x : x < 10, x \in R\}$, $A = \{x : 1 < x \leq 4\}$ Ges $B = \{x : 3 \leq x < 6\}$. wbtPi tmU_#j v Abj#c tmU wP#y#i gva`tg cKvk Ki :

- (a) $A \cap B$ (b) $A' \cap B$ (c) $A \cap B'$ Ges (d) $A' \cap B'$

24| wbtg#e A I B tmU t` I qv AvtQ | c0Zt#t#t $A \cup B$ wby0 Ki Ges hvPvB Ki th $A \subset (A \cup B)$ Ges $B \subset (A \cup B)$

- i. $A = \{-2, -1, 0, 1, 2\}$ Ges $B = \{-3, 0, 3\}$

$$ii. A = \{x : x \in N, x < 10 \text{ Ges } x, 2 \text{ Gi } \text{wYZK}\}$$

$$\text{Ges } B = \{x : x \in N, x < 10 \text{ Ges } x, 3 \text{ Gi } \text{wYZK}\}$$

25 | wbtgic tmlu, tjv e'envi Kti $A \cap B$ wbyq Ki Ges hvPvB Ki th,

$$(A \cap B \subset A \text{ Ges } (A \cap B) \subset B$$

$$(i) A = \{0, 1, 2, 3, 5\}, B = \{-1, 0, 2\}$$

$$(ii) A = \{a, b, c, d\}, B = \{b, x, c, y\}$$

26 | Avtbvqiv gnwe`vj tqi QvTxf`i gta` wevPiv, mUvbx I cePvX cvT Kvi cvVvfvrm m'uvKcwi Pwv Z GK mgxvqvq t`Lv tMj 60% QvTx wevPiv, 50% QvTx mUvbx, 50% QvTx cePvX, 30% QvTx wevPiv I mUvbx, 30% QvTx wevPiv I cePvX, 20% QvTx mUvbx I cePvX Ges 10% QvTx wZbvU cvT KvB cto |

(i) kZKiv KZRb QvTx D^3 cvT Kv wZbvUi tKvbwB cto bv ?

(ii) kZKiv KZRb QvTx D^3 cvT Kv, tjvi gta` tKej `BvU cto ?

$$27 | A = \{x : x \in R \text{ Ges } x^2 - (a + b)x + ab = 0\}$$

$$B = \{1, 2\} \text{ Ges } C = \{2, 4, 5\}$$

K. A tmvUi Dcv`vbmgn wbyq Ki |

L. t`Lvl th, $P(B \cap C) = P(B) \cap P(C)$

M. cgvY Ki th, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

28 | GKwU tkvYi 100 Rb QvTf`i gta` 42 Rb dbej, 46 Rb wvTku Ges 39 Rb nvK tLj | Gf`i gta` 13 Rb dbej I wvTku, 14 Rb wvTku I nvK Ges 12 Rb dbej I nvK tLj tZ cvfi | GQvov 7 Rb tKvfbv tLj vq cvi`kPbq-

K. Dvj wLZ wZbvU tLj vq cvi`kPggb QvTf`i tmlu Ges tKvfbv tLj vq cvi`kPbq Ggb QvTf`i tmlu tfbvPvT t`Lvl -

L. KZRb QvT Dvj wLZ wZbvU tLj vq cvi`kPZv wbyq Ki |

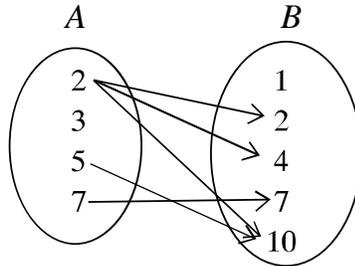
M. KZRb QvT tKej gvT GKwU tLj vq cvi`kP Ges KZRb AšZ `BvU tLj vq cvi`kP?

Ašq (Relation) Ges d'yskb (Function)

3, 2 Gi tPtq eo, 3 Gi eM'9 | G, tjv Aštqi D`vniY | cDg D`vniY j`v` Kvi, ŌGi tPtq eoŌ K_wU ms'vqZ Kiv ntqtQ hv mKj ev`e msL`vi tmlu R Gi AšMZ | Avevi 2q D`vniY m'uvKw ŌGi eM' I ev`e msL`vi tmlu R Gi AšMZ |

DtjL` th mKj ev`e msL`v R tmvU AšMZ wKš' Ašq bq | thgb 3 tgšv K msL`vŌ Ašq bq KviY 3 Ab`tkvfbv msL`vi msM m'uvKZ bq | hv tfbvPvT i gva`tg D`vniY 1-G t`Lvfbv ntjv |

D`vniY 1 | gtb Kwí $A = \{2,3,5,7\}$ Ges $B = \{1,2,4,7,10\}$ | A Gi th th m`m` Øviv B Gi th th m`m` w`fvR` nq Gt` i Aw¼Z Kti wbtP wPtÎ t` Lvfbv ntjv :



Gi fc AwšZ m`m`t`i Øviv MWZ µgtRvotjvi tmU $A = \{(2,2)(2,4)(2,10)(5,10)(7,7)\}$ Øviv GB w`fvR`Zv m`úKw eYØv Kiv hvq | D tmU AšfP µgtRvotjvi cŭg c` A Gi m`m` I wØZxq c` B Gi m`m` thLvfb cŭg c` Øviv wØZxq c` w`fvR` |

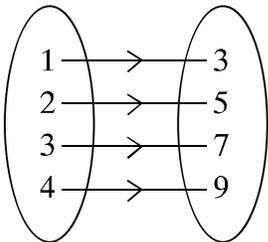
A_{f} , $D \subset A \times B$ Ges $D = \{(x, y) : x \in A, y \in B \text{ Ges } x \text{ Øviv } y \text{ w`fvR}\}$, GLvfb D tmUw A tmU t`tk B tmU GKw Ašq |

D`vniY 2 | ev`e msL`vi µgtRvotjvi tmU $L = \{(x, y) : x \in R, y \in R \text{ Ges } x < y\}$ w`tePbv Kwí th, `Bw ev`e msL`v a, b Gi Rb` $a < b$ hw` I tKej hw` $(a, b) \in L$ nq | mZivs L tmU Øviv ev`e msL`vi tQvU-eo m`úKwY nq |

msÁv : A I B tmU ntj $A \times B$ Gi tKvfbv Akb` DctmUtK A t`tk B G GKw Ašq (*relation*) ejv nq |

msÁv : A GKw tmU ntj $A \times A$ Gi tKvfbv Ai`b` DctmUtK A t`tk A G GKw Ašq ejv nq |

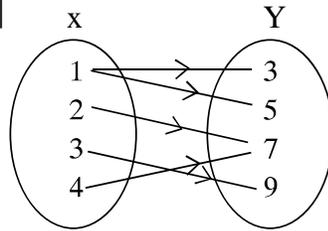
gše` : cŭZ`K Ašq GK ev GKwaK µgtRvotjvi GKw tmU |
 tfbwPtÎ tmU $X = \{1, 2, 3, 4\}$ Gi Dcv`vb,tjv tmU $Y = \{3, 5, 7, 9\}$ Gi Dcv`b,tjvi m½ A`v`iv (*arrow*) wPy Øviv msthM `vcb t` Lvfbv ntqtQ | tmU X I Y Gi gvtS Gi fc msthM `vcbtK X ntZ Y G Ašq (*relation*) ejv nq |



tfbwPtÎ X tmU m`m` 1 Gi mvt_ Y tmU m`m` 3 Gi m`úK 1 → 3 Øviv cKvK Kti Avgiv ejv cŭi w`K m`m` 1 Ges tkl m`m` 3 A_ev 1 g`wcs 3 | Z`c 2 → 5, 3 → 7 Ges 4 → 9.

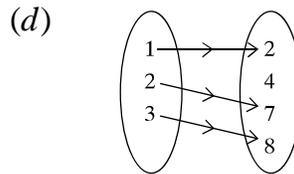
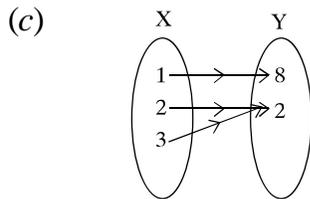
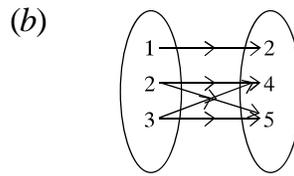
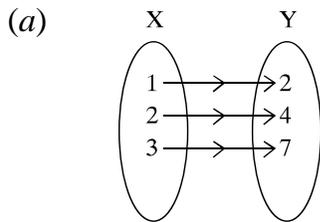
mZivs X tmU cŭZw m`m` x, Y tmU GKw gvt m`m` y Gi mvt_ m`úK | G m`úK dskb ev g`wcs etj |

tfbvPit Avti Kuu m'ukwetePbv Kwi |



wPit t_ik t' Lv hvq, $1 \rightarrow 3, 1 \rightarrow 5, 2 \rightarrow 7, 3 \rightarrow 9$ Ges $4 \rightarrow 7$.

X tm'Ui m'm' 1, Y tm'Ui 'Biu m'm' 3 Gi 5 Gi mvt_g'wics | G m'uk'Kx d'vskb ?
 D'vni Y 3 | wbt'Pi t'Kvb Ašq'w (relation) d'vskb bq ? h'v³ `vl |



m'gravb: (a) Ges (d) m'uk'w d'vskb w'Kš' (b) m'uk'w d'vskb bq Kvi Y $3 \rightarrow 4$ Ges $3 \rightarrow 5$ |
 d'vskbt'K m'vavi YZ Bst'i w'R t'QvU nvt'Zi A'q'i f, g, h BZ'w' Øviv wbt' R Kiv nq |

tm'U $X = \{1, 2, 3, 4\}$ nt'Z tm'U $Y = \{3, 5, 7, 9\}$ d'vskb wbt' R Ki t'Z Avgiv wj wL
 $f : 1 \rightarrow 3, f : 2 \rightarrow 5, f : 3 \rightarrow 7$ Ges $f : 4 \rightarrow 9$

GLv'tb 3 t'K ejv nq 1 Gi B'tgR | Z`'c 5, 7 | 9 h_v'p'tg 2, 3 | 4 Gi B'tgR |

AwaKš'X tm'Ui th't'Kv'tbv m'm' x Ges Y tm'Ui th't'Kv'tbv m'm' y Gi m'uk'w't'K Avgiv $y = 2x + 1$
 Av'Kv'ti c'K'v'k Ki t'Z c'w'i |

AZGe, d'vskbt'K Avgiv wbt'g'e w'boqg Ab'yni Y K'ti c'K'v'k Ki t'Z c'w'i -

$f : x \rightarrow y$ thL'v'tb $y = 2x + 1$

A_ev $f : x \rightarrow 2x + 1$

thL'v'tb Avgiv wj L't'Z c'w'i $f(x) = 2x + 1$

Zvnt'j $f(1) = 3$ nt'j v 1 Gi B'tgR Ges $f(x)$ nt'j v x Gi B'tgR |

Avtj vPZ dsktbi gta" c0g tmU $X = \{1, 2, 3, 4\}$ tK dskbui Avavi (Domain) Ges c0tgv³ tmU ntZ c0E wbtqg c0B wZxq tmfui msL'v,tj vi tmUtK dskbui we'hi (Range) ejv nq|

Ab'frte ejv hvq $y = f(x)$ dsktbi Avavi ntjv x Gi Ggb GKwU tmU thLvfb mg⁻ x Gi Rb" $f(x)$ dsktbi gvb wbyq Kiv mae| Avi dskbui Avavi x Gi Rb" $f(x)$ Gi th mg⁻-gyb cvl qv hvq, Gf` i msM0tK Gi we'hi ev tiA etj |

D`vni Y 4 | $f : x \rightarrow 2x^2 + 1$ Gi Btgr wbyq Ki : thLvfb tWtgb $X = \{1, 2, 3\}$

mgvavb : $f(x) = 2x^2 + 1$

1,2,3 Gi Btgr ntjv : $f(1) = 2(1)^2 + 1 = 3, f(2) = 2(2)^2 + 1 = 9$

Ges $f(3) = 2(3)^2 + 1 = 19$

∴ Btgr tmU $R = \{3, 9, 19\}$.

D`vni Y 5 : cvtki wptf $f : x \rightarrow mx + c$ dsktbi Ask t_tK wbyq Ki -

(a) m Ges c Gi gvb

(b) f Gi Aaxtb 5 Gi Btgr

(c) Btgr 3 ntj m`m` msL'v

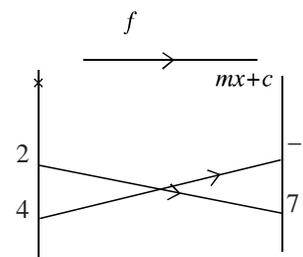
mgvavb : (a) $f(x) = mx + c$

$f : 2 \rightarrow 7 \Rightarrow f(2) = 7$

i.e. $2m + c = 7 \dots\dots\dots(1)$

$f : 4 \rightarrow -1 \Rightarrow f(4) = -1$

i.e. $f(4) = 4m + c \Rightarrow 4m + c = -1 \dots\dots\dots(2)$



(i) | (ii) t_tK cvB $m = -4$ Ges $c = 15$

(b) f Aaxtb 5 Gi Btgr $f(5) = -4 \times 5 + 15 = -5$

(c) awi x wbyq m`m` msL'v hv i Btgr 3 ZLb $f(x) = 3 \Rightarrow -4x + 15 = 3 \Rightarrow x = 3$

D`vni Y 6 | $F = \{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$ Ašqiu Kx dskb ? Gi tWtgb I tiA wbyq Ki |

DĖi : $A = \{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$ Ašqiu GKwU dskb | KviY Gi μgtRvo,tj vi c0g Dcv`vb wfbwfbq

ewYZ dsktb $F(-2) = 4, F(-1) = 1, F(0) = 0, F(1) = 1, F(2) = 4$

∴ tWtgb $A = \{-2, -1, 0, 1, 2\}$ Ges tiA $B = \{0, 1, 4\}$

A Gi wefbwem`tm`i Qwe j`K Ki tj t`Lv hvq th, GLvfb $x \in A$ Gi Rb" $F(x) = x^2$, G dskbwUtK

$F : A \rightarrow B, F(x) = x^2$ wj tL c0Kiv Kiv hvq|

gše" : tKvfbv dvskb F Gi tWtgb Ges tWtgbtbi cZ"K m`m` x Gi Abb` Qwe $F(x)$ wv` Q Kiv ntb B dvskbwU wba"i Z nq| AtbK mgq tWtgb Dn` ivLv nq| Gifc t"t" tWtgb wntmte R Gi H Dc"mU"K M"Y Kiv nq, hvi cZ"K m`m` x Gi Rb` R G $F(x)$ wba"i Z _v"K|

D`vni Y 7| $F(x) = \sqrt{1-x}$ Oviv ewYZ dvsktbi tWtgb wvY" Ki |

$F(-3), F(0), F(\frac{1}{2}), F(1), F(2)$ Gi gfa` th,tj v ms"vqZ tm,tj v wvY" Ki |

mgvavb : $F(x) = \sqrt{1-x} \in R$ hw` l tKej hw` $1-x \geq 0$ ev $1 \geq x$ A`R, $x \leq 1$ mYZivs

tWvg $F = \{x \in R : x \leq 1\}$

GLvfb $F(-3) = \sqrt{1-(-3)} = 4 = 2$

$F(0) = \sqrt{1-0} = \sqrt{1} = 1$

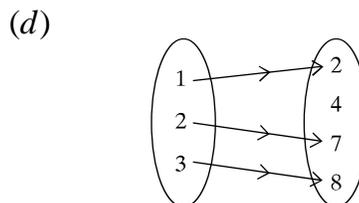
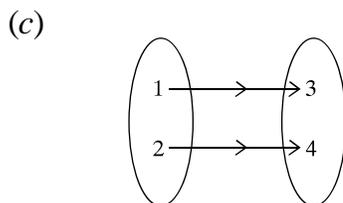
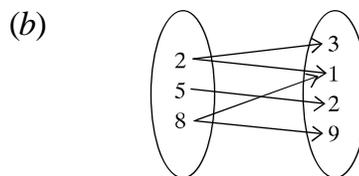
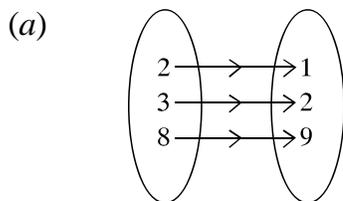
$F(\frac{1}{2}) = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$F(1) = \sqrt{1-1} = 0$

$F(2)$ ms"vqZ bq, tKbbv $2 \notin \text{dom}F$.

KvR :

1| wbtPi tKvb A"q"U dvskb bq ? hv"3 `vl |

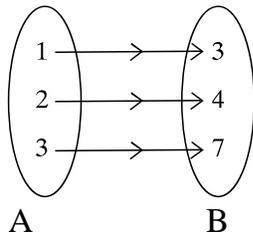


2| $f : x \rightarrow 4x+2$ Oviv ewYZ dvskb hvi tWtgb $D = \{-1, 3, 5\}$ Zvntj dvskbwU BtgR tmU wvY" Ki |

- 3| cÖ Ę S AšqWtK ZWj Kv c×wZtZ eYÖv Ki Ges tKvb,tj v dVskb Zv wbañY Ki | tWwg S I tiÄ S wbyq Ki | thLvtb $A = \{-2, -1, 0, 1, 2\}$
- (K) $S = \{(x, y) : x \in A, y \in A \text{ Ges } x + y = 1\}$
- (L) $S = \{(x, y) : x \in A, y \in A \text{ Ges } x - y = 1\}$
- (M) $S = \{(x, y) : x \in A, y \in A \text{ Ges } y = x^2\}$
- (N) $S = \{(x, y) : x \in A, y \in A \text{ Ges } y^2 = x\}$
- 4| $f(x) = 2x - 1$ Öv iv ewYZ dVsktbi Rb"-
- (K) $F(-2), F(0)$ Ges $F(2)$ wbyq Ki
- (L) $F\left(\frac{a+1}{2}\right)$ wbyq Ki, thLvtb $a \in R$
- (M) $F(x) = 5$ ntj, x wbyq Ki
- (N) $F(x) = y$ ntj x wbyq Ki thLvtb $y \in R$

GK-GK dVskb (One One Function)

tfbwptÄ A Ges B tmtu j ¶i Kwi -



tfbwptÄ f dVsktbi Aaxtb wfbawfbam`tm`i Qwe me©v wfbq

msÁv : hw` tKvb dVsktbi Aaxtb Gi tWtggtbi wfbawfbam`tm`i Qwe me©v wfbæng, Zte dVskbwutK GK-GK (One-One) dskb ej v nq|

msÁv t_tK t`Lv hvq, GKwU dVskb $f : A \rightarrow B$ tK GK-GK dVskb ej v nte, hw`

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \text{ thLvtb } x_1, x_2 \in A.$$

D`vni Y 8| $f(x) = 3x + 5, x \in R$ GKwU GK-GK dVskb|

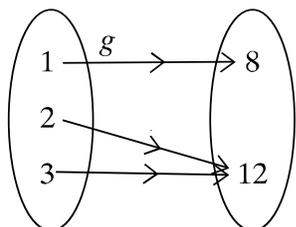
hvpvB : t`l qv AvtQ $f(x) = 3x + 5$

awi, $a, b \in R$ Zv ntj $f(a) = 3a + 5$ Ges $f(b) = 3b + 5$

$$\text{GLb, } f(a) = f(b) \Rightarrow 3a + 5 = 3b + 5 \Rightarrow a = b$$

mYZivs f GK-GK dVskb|

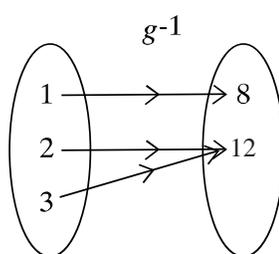
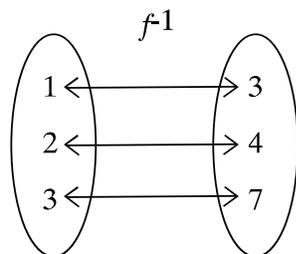
Aevi, g G ewYZ dVskbwU j ¶i Kwi th,



ƒWŵtǵtbi `Bŵŵ ŵFbom` ƒm'i Qŵe GKb $g(2) = 12, g(3) = 12$

mŵZivs g dŵskbŵŵ GK-GK bq|

Dcƒii ŵPĪ `Bŵŵi Zxi ŵPƒyi (arrows) ŵ` K Dƒĕv Ki ƒj ƒ` Lv hvq-



cŴgŵŵ GK-GK dŵskb, GB dŵskbƒK ej v nq f dŵskƒbi ŵecixZ dŵskb|

ŵKŠ' ŵecixZ g' ŵmcs hv g Gi Aaxƒb Kiv nƒqƒQ Zv GK-GK dŵskb bq| ZvB g dŵskƒbi ŵecixZ dŵskb

cvl qv hvte bv| GLvƒb GK-GK dŵskb Ges Bŵvi ŵecixZ f^{-1} Gi ga" mŵúKƒ` Lvƒbv nƒj v :

f Gi Aaxb	f^{-1} Gi Aaxb
$f(1) = 2 \Leftrightarrow$	$f^{-1}(2) = 1$
$f(2) = 4 \Leftrightarrow$	$f^{-1}(4) = 2$
$f(3) = 7 \Leftrightarrow$	$f^{-1}(7) = 3$
$f(a) = b \Leftrightarrow$	$f^{-1}(b) = a$

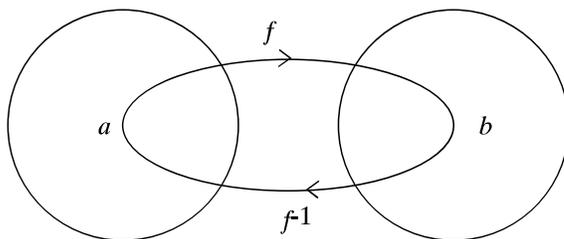
f GK-GK dŵskb hv`

(i) f Gi GKŵŵ ŵecixZ dŵskb f^{-1} nq

(ii) b BƒgR nq $f(a)$ Aaxb $\Leftrightarrow a$ BƒgR nq $b = f(a)$ A_ŵŵ $b = f(a) \Leftrightarrow a = f^{-1}(b)$

GŵŵƒK ŵPƒĪ ƒ` Lvƒbv hvq

$$b = f(a) \Leftrightarrow a = f^{-1}(b)$$



D`vniY 9 | $f: R \rightarrow R, f(x) = x^2$ dıskbıW GK-GK bq |

mgvavb : GLvıb tWvg $F = R, x_1 = -1, x_2 = 1$ wbtq t`wL th, $x_1 \in tWvg F, x_2 \in tWvg F, x_1 \neq x_2$

wKŞ' $f(x_1) = f(-1) = (-1)^2 = 1, f(x_2) = f(1) = (1)^2$

A_w, $f(x_1) = f(x_2), \therefore f$ GK-GK bq |

`be` : tKvb dısktbi wecixZ AŞq dıskb bvl ntZ cvtı |

D`vniY 10 | $f(x) = \frac{x}{x-2}, x \neq 2$ ewYZ dısktbi Rb` wbyq Ki (K) $f(5)$ (L) $f^{-1}(2)$

mgvavb : (K) $f(x) = \frac{x}{x-2}, x \neq 2$

$$f(5) = \frac{5}{5-2} = \frac{5}{3} = 1\frac{2}{3}$$

(L) awi, $a = f^{-1}$ ZLb $f(a) = 2$

$$\frac{a}{a-2} = 2 \Rightarrow a = 2a - 4 \Rightarrow a = 4$$

$$\therefore f^{-1}(2) = 4$$

D`vniY 11 | $f(x) = 3x+1, 0 \leq x \leq 2$

(a) f Gi Mıd AwK Ges tiÄ t`Lvl

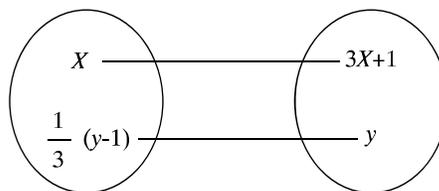
(b) t`Lvl th f GK-GK dıskb

(c) f^{-1} wbyq Ki Ges f^{-1} Gi Mıd Aıwb Ki |

mgvavb : $f(x) = 3x+1, 0 \leq x \leq 2$

ntZ cvB kıl qe`y (0,1) Ges (2,7)

\therefore tiÄ $f: R = \{y: 1 \leq y \leq 7\}$



(b) thınZıçÖZ`K $y \in R$ Gi Rb` GKgvı $x \in R$ Gi Bıgr y t`Lvıv ntqtQ |

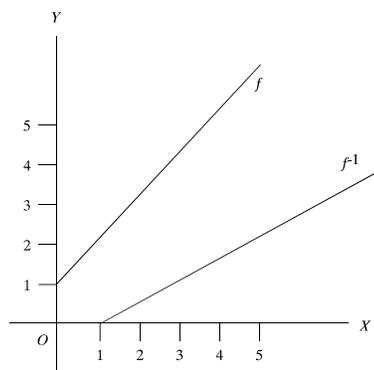
mZıvs f GK-GK dıskb |

(c) awi, $y = f(x), x$ Gi Bıgr

$$Zıntj, y = 3x + 1$$

$$\Rightarrow x = \frac{1}{3}(y - 1)$$

wecixZ dıskb $f^{-1}: y \rightarrow x$ thLvı, $x = \frac{1}{3}(y - 1)$



ev, $f^{-1} : y \rightarrow \frac{1}{3}(y-1)$ hv wPŕĤ t`Lvŕbv nŕqŕQ|

y Gi tŕj x vcb Kŕi cvB, $f^{-1} : x \rightarrow \frac{1}{3}(x-1)$

f^{-1} Gi AwŕZ tŕLv $y = \frac{1}{3}(x-1)$, $1 \leq x \leq 7$ t`Lvŕbv nŕqŕQ|

mweŕ dŕskb A_{ev} Ablŕdŕskb (Onto Function)

tŕbwPŕĤ dŕskb f Gi Aaxŕb tmU $A = \{1, 2, 3\}$ Ges $B = \{5, 7, 9\}$ wetePbv Kwĭ |

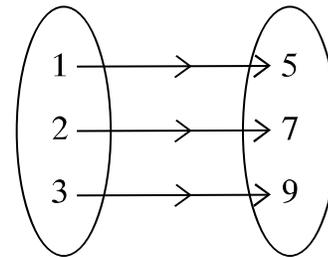
GLŕŕb $f(1) = 5 = 2 \cdot 1 + 3$ tŕLv hvq

Abjŕc $f(2) = 7 = 2 \cdot 2 + 3$

$f(3) = 9 = 2 \cdot 3 + 3$

mvavi Yfŕŕe Avgŕv wj LŕZ cwi $y = f(x) = 2x + b$

AZGe, ej v hvq $f(a) = b$



msÁv : GKŭ dŕskb $f : A \rightarrow B$ tK mweŕ dŕskb A_{ev} Ablŕdŕskb ej v nŕe hw` $f(A) = B$

A_{ev}, GKŭ dŕskb $f : A \rightarrow B$ tK mweŕ dŕskb A_{ev} Ablŕdŕskb ej v nŕe hw` cŕZ`K $b \in B$ Gi Rb` GKŭ $a \in A$ cvl qv hvq thb $f(a) = b$ nq| Avŕj vP` dŕskb $f : A = \{1, 2, 3\} \rightarrow B = \{5, 7, 9\}$ tK $y = f(x) = 2x + 3$ Ővi v msÁwŕZ Kiv nq| Zŕe f GKŭ mweŕ dŕskb |

wecixZ dŕskb (Inverse function)

gŕb Kwĭ $f : A \rightarrow B$ GKŭ GK-GK Ges Ablŕdŕskb, Zv nŕj GKŭ dŕskb $f^{-1} : B \rightarrow A$ we`gvb AvŕQ thLvŕb cŕZ`K $b \in B$ Gi Rb` GKŭ Abb` $f^{-1}(b) \in A$ we`gvb| Zŕe f^{-1} tK f Gi wecixZ dŕskb ej v nq|

gŕb Kwĭ, $f : A \rightarrow B$ Ges $g : B \rightarrow A$ DfŕqB GK-GK Ges Ablŕdŕskb | Zv nŕj g tK f Gi wecixZ dŕskb ej v nŕe hw` $f(g(x)) = g(f(x)) = x$ nq thLvŕb $f(x) \in B$ Ges $g(x) \in A$. GLŕŕb $g = f^{-1}$

D`vniY 12 | hw` $f : R \rightarrow R$ Ges $g : R \rightarrow R$ dŕskb `Bŭ $f(x) = x + 5$ Ges $g(x) = x - 5$ Ővi v msÁwŕZ nq, Zŕe t`Lvl th, f Gi wecixZ dŕskb g |

mgvavb : t`l qv AvŕQ $f(x) = x + 5 \dots \dots (1)$ Ges

$g(x) = x - 5 \dots \dots (2)$

GLb $f(g(x)) = f(x - 5)$: [(2) bs Ővi v]

$= (x - 5) + 5$ [(1) bs Ővi v]

$= x$

$$\begin{aligned} \text{Ges } g(f(x)) &= f(x) - 5 \quad [(2) \text{ bs } \emptyset \text{vi v}] \\ &= (x+5) - 5 \quad [(1) \text{ bs } \emptyset \text{vi v}] \\ &= x \end{aligned}$$

$$\text{th\~{t}nZl } f(g(x)) = g(f(x)) = x$$

m\~{y}Zivs f Gi w ecixZ d\~{v}skb g .

D`vniY 13 | h\~{w} $f : \mathbb{R} \rightarrow \mathbb{R}$ d\~{v}skbuU $f(x) = x^3 - 5$ \emptyset vi v ms\~{A}v\~{w}qZ nq, Z\~{t}e $y = f^{-1}(x)$ w\~{b}Y\~{q} Ki |
mgvavb : ms\~{A}vb\~{m}v\~{t}i , $f(f^{-1}(x)) = x$

$$f(y) = x \dots \dots (1) \quad [\text{th\~{t}nZl } f^{-1}(x) = y]$$

$$\text{t`I qv Av\~{t}Q } f(x) = x^3 - 5$$

$$\Rightarrow f(y) = y^3 - 5$$

$$\Rightarrow x = y^3 - 5 \quad [(1) \emptyset \text{vi v}]$$

$$\Rightarrow x + 5 = y^3 \Rightarrow y = (x + 5)^{\frac{1}{3}}$$

$$\therefore f^{-1}(x) = (x + 5)^{\frac{1}{3}}.$$

KvR :

1 | w\~{b}t\~{g}ie c\~{o}ZwU GK-GK d\~{v}sk\~{t}bi ms\~{w}k\~{o} f^{-1} w\~{b}Y\~{q} Ki |

$$(K) \quad y = (x + 5)^{\frac{1}{3}} \qquad (L) \quad f(x) = \frac{3}{x-1}, x \neq 1$$

$$(M) \quad f(x) = \frac{2x}{x-2}, x \neq 2 \qquad (N) \quad f : x \rightarrow \frac{2x+3}{2x-1}, x \neq \frac{1}{2}$$

2 | ew\~{Y}Z d\~{v}skb $f(x) = \frac{4x-9}{x-2}, x \neq 2$ Gi t\~{q}\~{t}\~{t}

$$(K) \quad f^{-1}(-1) \text{ Ges } f^{-1}(1) \text{ w\~{b}Y\~{q} Ki |}$$

$$(L) \quad x \text{ Gi gvb w\~{b}Y\~{q} Ki thb } 4f^{-1}(x) = x$$

3 | ew\~{Y}Z d\~{v}skb $f(x) = \frac{2x+2}{x-1}, x \neq 1$ Gi Rb`

$$(K) \quad f^{-1}(3) \text{ w\~{b}Y\~{q} Ki |}$$

$$(L) \quad \text{t`I qv Av\~{t}Q } f^{-1}(p) = kp, p \text{ Gi m\~{v}t\~{c}\~{t}\~{q} k tK c\~{K}v\~{k} Ki |}$$

4 | w\~{b}t\~{g}w\~{e}3 c\~{o}Z`K t\~{q}\~{t}\~{t} c\~{o}E m\~{a}u\~{K}©F GKwU d\~{v}skb w\~{K}bv Zv w\~{b}Y\~{q} Ki | F d\~{v}skb n\~{t}j Dnvi
t\~{W}t\~{g}b Ges ti\~{A} w\~{b}Y\~{q} Ki , Dnv GK-GK w\~{K}bv ZvI w\~{b}a\~{f}i Y Ki :

$$(K) \quad F\{(x, y) \in \mathbb{R}^2 : y = x\}$$

$$(L) \quad F\{(x, y) \in \mathbb{R}^2 : y = x^2\}$$

(M) $F\{(x, y) \in R^2 : y^2 = x\}$

(N) $F\{(x, y) \in R^2 : y = \sqrt{x}\}$

5 | (a) h \ddot{u} $f : \{-2, -1, 0, 1, 2\} \rightarrow \{-8, -1, 0, 1, 8\}$ d \ddot{u} skb \ddot{u} $f(x) = x^3$ \emptyset iv ms \ddot{A} wiqZ, t \ddot{u} Lvl th, f GK-GK Ges Ab \ddot{u} z |

(b) $f : \{1, 2, 3, 4\} \rightarrow R$ GK \ddot{u} d \ddot{u} skb hv $f(x) = 2x + 1$ \emptyset iv ms \ddot{A} wiqZ, t \ddot{u} Lvl th, f GK-GK d \ddot{u} skb w \ddot{K} Š' Ab \ddot{u} z d \ddot{u} skb bq |

Ašq (Relation) I d \ddot{u} skt \ddot{u} bi tj L \ddot{u} P \ddot{I}

tj L \ddot{u} P \ddot{I} nt \ddot{u} v d \ddot{u} skt \ddot{u} bi R \ddot{u} wg \ddot{u} ZK Dc \ddot{u} vcb | $y = f(x)$ tj L \ddot{u} P \ddot{I} AsKt \ddot{u} bi Rb \ddot{u} 'O' w \ddot{e} š \ddot{u} žZ ci \ddot{u} i t \ddot{O} x j \ddot{u} š \ddot{u} š \ddot{u} B \ddot{u} W mij t \ddot{u} i Lv XOX' Ges YOY' j l qv nq | O t \ddot{K} gj w \ddot{e} š \ddot{u} y XOX' t \ddot{K} x A \ddot{u} | Ges YOY' t \ddot{K} y A \ddot{u} | ej v nq |

$y = f(x)$ d \ddot{u} skt \ddot{u} bi tj L \ddot{u} P \ddot{I} AsKt \ddot{u} bi Rb \ddot{u} $a \leq x \leq b$ e \ddot{u} ewat \ddot{u} Z \ddot{u} axb Pj K x Ges Aaxb Pj K y Gi gvb \ddot{u} tj vi Zvwj Kv c \ddot{O} ž Ki t \ddot{u} Z nq | AZ:ci Zvwj Kvi m \ddot{u} wgZ msL \ddot{u} K w \ddot{e} š \ddot{u} y \ddot{u} t \ddot{u} v t \ddot{K} xy mgZt \ddot{u} \ddot{u} vcb Ki t \ddot{u} Z nq | c \ddot{O} š w \ddot{e} š \ddot{u} y \ddot{u} t \ddot{u} v t \ddot{K} mij t \ddot{u} i Lv A \ddot{u} ev e \ddot{u} ti Lv \emptyset iv h \ddot{u} š Ki t \ddot{u} Z $y = f(x)$ d \ddot{u} skt \ddot{u} bi tj L \ddot{u} P \ddot{I} cvl qv hvq | gva \ddot{u} wgK exRM \ddot{u} YžZ tj L \ddot{u} P \ddot{I} m \ddot{u} š \ddot{u} t \ddot{K} c \ddot{O} š \ddot{u} wgK avi Yv c \ddot{O} v \ddot{u} Kiv nt \ddot{u} t \ddot{O} | GLv t \ddot{u} b, mij % \ddot{u} LK (Linear) d \ddot{u} skb, w \ddot{O} NvZ (Quadratic) d \ddot{u} skb Ges e \ddot{u} š \ddot{u} i tj L \ddot{u} P \ddot{I} A $\frac{1}{4}$ b m \ddot{u} š \ddot{u} t \ddot{K} Av t \ddot{u} j v Pbv Kiv nt \ddot{u} t \ddot{O} |

mij % \ddot{u} LK d \ddot{u} skb

mij \hat{u} i \ddot{u} LK d \ddot{u} skb Gi m \ddot{u} vavi Y ifc nt \ddot{u} v $f(x) = mx + b$

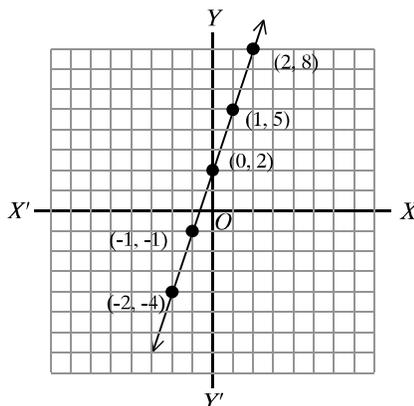
thL \ddot{u} v t \ddot{u} b, m Ges b ev \ddot{u} e msL \ddot{u} v | Gi tj L \ddot{u} P \ddot{I} GK \ddot{u} ti Lv hvi Xvj nt \ddot{u} v m Ges y A \ddot{u} | i t \ddot{O} K b |

GLv t \ddot{u} b, avi $m = 3$ Ges $b = 2$ Zvnt \ddot{u} Z d \ddot{u} skb \ddot{u} \ddot{u} wovq $f(x) = 3x + 2$

ew \ddot{u} ž d \ddot{u} skb nt \ddot{u} Z x | y Gi w \ddot{u} g \ddot{u} š \ddot{u} m \ddot{u} š \ddot{u} k \ddot{u} gvb cvl qv hvq :

x	-2	-1	0	1	2
y	-4	-1	2	5	8

$\therefore f(x) = 3x + 2$ Gi tj L w \ddot{u} t \ddot{u} g \ddot{u} š \ddot{u} L \ddot{u} v t \ddot{u} bv nt \ddot{u} v :



WNVZ dskb (Quadratic function)

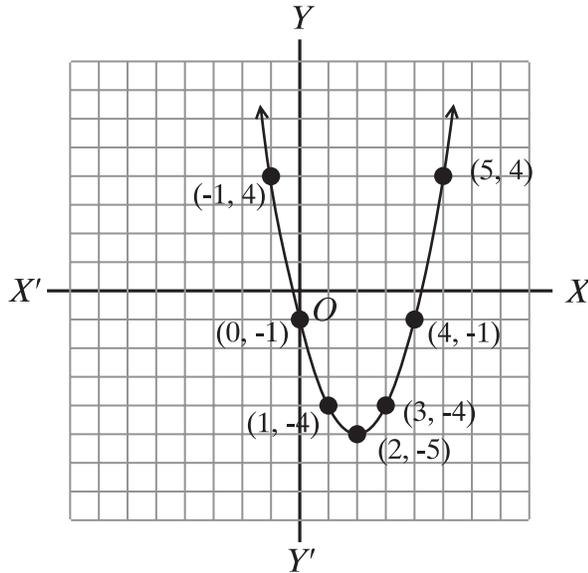
WNVZ dskb ntj v GKW dskb hv $y = ax^2 + bx + c$ mgxKiY Øviv eWZ thLvfb a, b Ges c ev e mSLv Ges $a \neq 0$.

cØ E dskfb awi $a = 1, b = -4, c = -1$

Zvntj $y = ax^2 + bx + c$ tK tj Lv hvq $y = x^2 - 4x - 1$

eWZ dskb ntj x | y Gi msukó gv b cvl qv hvq |

x	$x^2 - 4x - 1$	y
-1	$(-1)^2 - 4(-1) - 1$	4
0	$0^2 - 4(0) - 1$	-1
1	$1^2 - 4(1) - 1$	-4
2	$2^2 - 4(2) - 1$	-5
3	$3^2 - 4(3) - 1$	-4
4	$4^2 - 4(4) - 1$	-1
5	$5^2 - 4(5) - 1$	4



Bnv wbtYØ WNVZ dskb-Gi tj LwPÎ |

GB WNVZ dskb Gi wKQzmvavi Y eukó j ¶i Kwi |

- (i) civeE AvKvi
- (ii) y A¶i mgvšivj ti Lv ev y A¶i eivei cØZmvg we` ycvl qv hvte |
- (iii) GKW we` fZ dskbWj gv b ¶i Zg ev epEg nte |

e¶Ei tj LwPÎ

Dtj L` th p, q | r a`eK Ges $r \neq 0$ ntj $R \text{ G } S = \{(x, y) : (x - p)^2 + (y - q)^2 = r^2\}$

Aštqi tj L GKW eE hvi tK`^a p, q Ges e`vma[®] r (gva`wgK exRMWZ cy`K`be`)|

OK KvM¶R p, q we` ycvZb Kti H we` fK tK`^aKti r e`vma[®]btq eE A¼b Kti tj LwPÎW cvl qv hvq | gše` : th Aštqi mgvavb tmU Amxg, Gi tj LwPÎ A¼tbi `KZ c×WZ ntj v hťó mSL`K mgvavtbi cØZifcx we` yOK KvM¶R cvZb Kti mvej xj fvte (.....) H me we` ythvM Kiv, hvfZ AšqWj tj LwPÎ i aiY Ø`_¶xbfvte tevSv hvq |

wKš` th Aštqi tj LwPÎ eE, Gi Rb` K`úm e`envi Kij KvR mnR I my`i nq weavq tktlv³ cšv Aej ¶b Kiv ntj v |

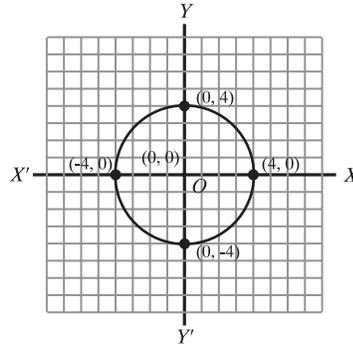
D`vniY 14 |

$S = \{(x, y) : x^2 + y^2 = 16\}$

ev, $x^2 + y^2 = 4^2$

mZivS S Gi tj LwPÎ GKW eE hvi tK`^aC (0,0) Ges e`vma[®] $r = 4$.

S Gi tj LwPÎ wbtgaf` Lvfbv ntj v :



KvR :

1| wbtgaf` dvsktbi mvavi Y ifc (Standard Form) wj L :

(K) $y - 2 = 3(x - 5)$ (L) $y - 2 = \frac{1}{2}(x + 3)$

(M) $y - (5) = -2(x + 1)$ (N) $y - 5 = \frac{4}{3}(x - 3)$

2| tj LwPÎ A¼b Ki :

(K) $y = 3x - 1$ (L) $x + y = 3$

(M) $x^2 + y^2 = 9$ (N) $y = \frac{1}{3}x + 1.$

Abkxj bx 1.2

1| $\{(2, 2), (4, 2), (2, 10), (7, 7)\}$ Aštqi tWwtgb tKvbwU ?

- (K) $\{2, 4, 7\}$ (L) $\{2, 2, 10, 7\}$
 (M) $\{2, 2, 10, 7\}$ (N) $\{2, 4, 2, 5, 7\}$

2| $S = \{(x, y) : x \in A, y \in A \text{ Ges } y = x^2\}$ Ges $A = \{-2, -1, 0, 1, 2\}$ wbtPi tKvbwU S Aštqi m`m` ?

- (K) $(2, 4)$ (L) $(-2, 4)$
 (M) $(-1, 1)$ (N) $(1, -1)$

3| hw` $S = \{(1, 4), (2, 1), (3, 0), (4, 1), (5, 4)\}$ nq Zte,

- (i) S Aštqi tiÄ $S = \{4, 1, 0, 4\}$
 (ii) S Aštqi weci xZ Aštq, $S^{-1} = \{(4, 1), (1, 2), (0, 3), (1, 4), (4, 5)\}$
 (iii) S Aštqwu GKwU dvskb

Dcti i Zt`i Avtj vtK wbtPi tKvbwU mwVK ?

- (K) i | ii (L) ii | iii (M) i | iii (N) i, ii | iii

wbtPi Zt`i Avtj vtK wbtPi 4-6 bs cOkkE DEi `vl :

hw` $F(x) = \sqrt{x-1}$

- 4| $F(10) = KZ$?
 (K) 9 (L) 3 (M) -3 (N) $\sqrt{10}$
- 5| $f(x) = 5$ ntj x Gi gv b KZ ?
 (K) 5 (L) 24 (M) 25 (N) 26
- 6| d v s k b m U i t W t g b m b t P i t K v b m U ?
 (K) t W g $F = \{x \in R : x \neq 1\}$ (L) t W g $F = \{x \in R : x \geq 1\}$
 (M) t W g $F = \{x \in R : x \leq 1\}$ (N) t W g $F = \{x \in R : x > 1\}$
- 7| (a) c 0 E S A s t q i t W t g b , t i A I m e c i x Z A s t q m b Y q K i |
 (b) S A _ e v S ^{-1} d v s k b m K b v Z v m b a f l Y K i |
 (c) d v s k b _ t j v G K - G K m K b v ?
 (K) $S = \{(1, 5), (2, 10), (3, 15), (4, 20)\}$
 (L) $S = \{(-3, 8), (-2, 3), (-1, 0), (0, -1), (1, 0), (2, 3), (3, 8)\}$
 (M) $S = \left\{ \left(\frac{1}{2}, 0 \right), (1, 1), (1, -1), \left(\frac{5}{2}, 2 \right), \left(\frac{5}{2}, -2 \right) \right\}$
 (N) $S = \{(-3, -3), (-1, -1), (0, 0), (1, 1), (3, 3)\}$
 (O) $S = \{(2, 1), (2, 2), (2, 3)\}$
- 8| $F(x) = \sqrt{x-1}$ 0 v i v e m Y Z d v s k t b i R b " -
 (K) $F(1), F(5),$ Ges $F(10)$ m b Y q K i (L) $F(a^2 + 1)$ m b Y q K i t h L v t b $a \in R$
 (M) $F(x) = 5$ ntj , x m b Y q K i (N) $F(x) = y$ ntj , x m b Y q K i t h L v t b $y \geq 0$.
- 9| $F : R \rightarrow R, F(x) = x^2$ d v s k t b i R b " -
 (K) t W g F Ges t i A F m b Y q K i (L) t ^ L v l t h , F G K - G K d v s k b b q |
- 10| (K) $f : R \rightarrow R$ G K m U d v s k b h v $f(x) = ax + b; a, b \in R$ 0 v i v m s A m q Z ntj , t ^ L v l t h , f G K - G K Ges A b U z |
 (L) $f : [0, 1] \rightarrow [0, 1]$ d v s k b m U $F(x) = \sqrt{1-x^2}$ 0 v i v m s A m q Z , Z t e t ^ L v l t h , f G K - G K Ges A b U z |
- 11| (K) h w ^ $f : R \rightarrow R$ Ges $g : R \rightarrow R$ d v s k b 0 q $f(x) = x^3 + 5$ Ges $g(x) = (x-5)^{\frac{1}{3}}$ 0 v i v m s A m q Z n q , Z t e t ^ L v l t h , $g = f^{-1}$

HAKER

M

Wozxq Aa'vq exRMmYwZK i vnk

(Algebraic Expression)

weirfbocKvfi i exRMmYwZK i vnk i mt½ Avgiv cwi uPZ | GK ev GKwaK msL'v I msL'v vbt` RK cZxKtK +, -, ×, ÷ NvZ ev gj` wPtýi thtKvfbv GKwU A_ev GKwaKtKi mrvth` A_@nfvte mshy³ Kijtj th bZb msL'v vbt` RK cZxKtKi mjo nq, GtK exRMmYwZK i vnk (Algebraic expression) ev mst`|tc i vnk ej v nq | thgb, $2x, 2x + 3ay, 6x + 4y^2 + a + \sqrt{z}$ BZ'w` cZ'fKB GK GKwU exRMmYwZK i vnk | Aa'vq tktl vkw`v`fv -

- euc`xi avi Yv e'vL'v Kitz cvi te |
- D`vni tYi mrvth` GK Pj Kwekó euc`x e'vL'v Kitz cvi te |
- euc`xi ,Y I fvm e'vL'v Kitz cvi te |
- fMtkl Dccv` I Drcv`K Dccv` e'vL'v Ges Zv cZ'qM Kti euc`xi Drcv`K wtklY Kitz cvi te |
- mggwî K i vnk, cZmg i vnk Ges Pµ-µwgK i vnk e'vL'v Kitz cvi te |
- mggwî K i vnk, cZmg i vnk Ges Pµ-µwgK i vnk i Drcv`K wbyq Kitz cvi te |
- gj` fMsktK AvskK fMsk cKvk Kitz cvi te |

2.1 Avtj vP` Aa'vq msL'v ej tZ Avgiv ev`e msL'vB eSe | A, B, C BZ'w` i vnk ,tj vi tKvfbwUB hw` GKwaK i vnk i thvMdj ev wetsvMdj bv nq, Zte Gt` i cZ'KwUfK $A + B + C + \dots$ AvKvfi i vnk i GK GKwU c` ej v nq | thgb, $5x + 3y^2 - 2b + \sqrt{2}$ i vnkUfZ $5x, 3y^2, -2b, \sqrt{2}$ GK GKwU c` | tKvfbv Avtj vPbvq msL'v vbt` RK GKwU A`|i cZxK ev Pj K (Variable) ev a'eK (Constant) ntZ cvi | hw` Gifc GKwU cZxK GKwaK m`m'wekó tKvfbv msL'v tmUti thtKvfbv Amba'ni Z m`m' vbt` R Kti, Zte cZxKwUfK Pj K ej v nq Ges tmUwUfK Gi tWtgb ej v nq | hw` cZxKwU GKwU wbw` @ msL'v vbt` R Kti, Zte GtK a'eK ej v nq | tKvfbv Avtj vPbvq GKwU Pj K Gi tWtgb t`fK thtKvfbv gvb MthY Kitz cvi | wKs' GKwU a'etKi gvb tKvfbv Avtj vPbvq wbw` @ _vfk |

euc`x :

euc`x wtkl ai tbi exRMmYwZK i vnk | Gifc i vnk tZ GK ev GKwaK c` _vfk Ges c` ,tj v GK ev GKwaK Pj tKi i aygvî AFYvZK cYmvsL`K NvZ I a'etKi ,Ydj nq |

GK Pj tKi euc`x :

gtb Kwi, x GKwU Pj K| Zvntj, (1) a , (2) $ax+b$ (3) ax^2+bx+c (4) ax^3+bx^2+cx+d
BZ`w` AvKvti i i vktK x Pj tKi euc`x ejv nq, thLvfb, a,b,c,d BZ`w` wv` e msL`v (a^eK) |

mvavi Yfite, x Pj tKi euc`xi c`mgf Cx^p AvKvti nq, thLvfb C GKwU (x -ewRZ) wv` e msL`v (hv
kb`I ntZ cvti) Ges p GKwU AFYvZK cYfmsL`v | p kb` ntj c`wU i ayC nq Ges C kb` ntj
c`wU euc`xtZ Abtj, L_vtK | Cx^p cf` C tK x^p Gi mnM (Coefficient) Ges p tK GB cf` i
gvIv ev NvZ (degree) ejv nq |

tKvfbv euc`xtZ Dvj wLZ c`mgfni Mwi o gvIvtK euc`xwU gvIv ejv nq | euc`xtZ Mwi o gvIvhj
c`wU tK gL`c` I gL`cf` i mnM tK gL`mnM Ges 0 gvIvhj A_{f} , Pj K-ewRZ c`wU tK $a^ec`$ ejv nq |
thgb, $2x^6-3x^5-x^4+2x-5$, x Pj tKi GKwU euc`x, hvi gvIv 6, gL`c` $2x^6$, gL`mnM 2 Ges
 $a^ec` -5$ |

$a \neq 0$ ntj, ctef (1) euc`xi gvIv 0, (2) euc`xi gvIv 1, (3) euc`xi gvIv 2 Ges (4) euc`xi
gvIv 3 | thtKvfbv Akb` a^eK ($a \neq 0$) Pj tKi 0 gvIvi euc`x ($a = ax^0$ weteP) | 0 msL`wU tK kb`
euc`x wetePbv Kiv nq Ges kb` euc`xi gvIv AmsAwqZ aiv nq |

x Pj tKi euc`xtK mvaviYZ x Gi NvtZi Aatmtg (A_{f} , gL`c` t_tK i i" Kti mtg mtg a^e c`
chS) eYv Kiv nq | Gi fc eYv tK euc`xwU Av` kcf (Standard form) ejv nq |

e`envti i mjeavt` x Pj tKi euc`xtK $P(x), Q(x)$ BZ`w` cZxK vvi mPZ Kiv ntj v |

thgb, $P(x) = 2x^2 + 7x + 5$

Gi fc $P(x)$ cZxtK x Gi Dci euc`xwU gvIvi wbfPZv wv` R Kti | $P(x)$ euc`xtZ x Pj tKi
cwi etZ q Kvtbv wv` e msL`v a emvtj euc`xwU th gvb cvl qv hvq, GtK $P(a)$ vvi mPZ Kiv nq |

D`vniY 1 | hw` $P(x) = 3x^3 + 2x^2 - 7x + 8$ nq, Zte $P(0), P(1), P(-2), P\left(\frac{1}{2}\right), P(2)$ Ges $P(a)$

wvYq Ki |

mgvavb : cD E euc`xtZ x Gi cwi etZ $^0, 1, -2, \frac{1}{2}, 2, a$ emvtq cvB,

$$P(0) = 3(0)^3 + 2(0)^2 - 7(0) + 8 = 8$$

$$P(1) = 3(1)^3 + 2(1)^2 - 7(1) + 8 = 6$$

$$P(-2) = 3(-2)^3 + 2(-2)^2 - 7(-2) + 8 = 6$$

$$P\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) + 8 = \frac{43}{8}$$

$$P(a) = 3a^3 + 2a^2 - 7a + 8$$

β Pj tKi euc`x

$$2x + 3y - 1$$

$$x^2 - 4xy + y^2 - 5x + 7y + 1$$

$$8x^3 + y^3 + 10x^2y + 6xy^2 - 6x + 2$$

G,tjv x l y Pj tKi euc`x | mvariYfvte Gifc euc`xi c` ,tjv $Cx^p x^q$ AvKvti i nq thLvfb C GKwU wbu` msL`v (a`eK) Ges p l q AFYvZK cYmL`v | $Cx^p x^q$ ct` C nt`Q $x^p x^q$ Gi mnM Ges $p+q$ nt`Q GB ct` i gvIv | Gifc euc`xtK Dwj,mLZ c`mgfni Mwi o gvIv tK euc`xwU gvIv ejv nq | Gifc euc`xtK $p(x, y)$ AvKvti i cZxK Øviv mPZ Kiv nq | thgb, $p(x, y) = 8x^3 + y^3 - 4x^2 + 7xy + 2y - 5$ euc`xi gvIv 3 Ges $P(1, 0) = 8 - 4 - 5 = -1$.

wZb Pj tKi euc`x

x, y l z Pj tKi euc`xi c` ,tjv $Cx^p x^q z^r$ AvKvti i nq | thLvfb C (a`eK) c`wU mnM Ges p, q, r AFYvZK cYmsL`v | $(p+q+r)$ tK GB ct` i gvIv Ges euc`xtK Dwj,mLZ c`mgfni Mwi o gvIv tK euc`xwU gvIv ejv nq | Gifc euc`xtK $P(x, y, z)$ AvKvti i cZxK Øviv mPZ Kiv nq | thgb, $P(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ euc`xi gvIv 3 Ges $P(1, 1, -2) = 1 + 1 - 8 + 6 = 0$ |

gse` : βwU euc`xi thvMdj , wftqvMdj Ges ,Ydj memgq euc`x nq | wKŠ` euc`xi fvMdj euc`x ntZI cvti A_ev bvl ntZ cvti |

KvR :

1 | wbtPi tKvbwU euc`x wBY@ Ki :

(K) $2x^3$	(L) $7 - 3a^2$	(M) $x^3 + x^{-2}$
(N) $\frac{a^2 + a}{a^3 - a}$	(O) $5x^2 - 2xy + 3y^2$	(P) $6a + 3b$
(Q) $C^2 + \frac{2}{0} - 3$	(R) $3\sqrt{n-4}$	(S) $2x(x^2 + 3y)$
(T) $3x - (2y + 4z)$	(U) $\frac{6}{x} + 2y$	(V) $\frac{3}{4}x - 2y$

2 | cZct` i msL`v Abjvqx euc`x wPwYZ Ki :

(K) $x^2 + 10x + 5$	(L) $3a + 2b$	(M) $4xyz$
(N) $2m^2n - mn^2$	(O) $7a + b - 2$	(P) $6a^2b^2c^2$

3 | wbtPi euc`x,tjvi cZKwU (i) x Pj tKi euc`xi Av`k`ifc Zv eYØv Ki Ges x Pj tKi euc`x ifc Gi gvIv, gL` mnM l a`e c` wBY@ Ki | (ii) y Pj tKi euc`xi Av`k`AvKvti eYØv Ki Ges y Pj tKi euc`xi ifc Gi gvIv, gL` mnM l a`e c` wBY@ Ki |

(K) $3x^2 - y^2 + x - 3$	(L) $x^2 - x^6 + x^4 + 3$	(M) $5x^2y - 4x^4y^4 - 2$
(N) $x + 2x^2 + 3x^3 + 6$	(O) $3x^3y + 2xyz - x^4$	

4 | hw` $P(x) = 2x^2 + 3$ nq, Zte $P(5)$, $P(6)$, $P\left(\frac{1}{2}\right)$ Gi gvb wbyq Ki |

fVM m \hat{t} :

hw` $D(x) \mid N(x)$ DfqB x Pj tKi euc`x nq Ges $D(x)$ Gi gv \hat{I} v) $\leq (N(x)$ Gi gv \hat{I} v) nq, Zte mvavi Y wbyq $D(x)$ \emptyset vi $N(x)$ tK fVM Kti fVMdj $Q(x)$ fVMtkl $R(x)$ cvl qv hvq | thLv \hat{t} b,

- (1) $Q(x) \mid R(x)$ DfqB x Pj tKi euc`x
- (2) $Q(x)$ Gi gv \hat{I} v) = $N(x)$ Gi gv \hat{I} v) - $D(x)$ Gi gv \hat{I} v)
- (3) $R(x) = 0$ A_ev $R(x)$ Gi gv \hat{I} v) < ($D(x)$ Gi gv \hat{I} v)
- (4) mKj x Gi Rb` $N(x) = D(x)Q(x) + R(x)$

g \hat{s} e` : (4) bs wbyq tK fVR` = fVRK \times fVMdj + fVMtkl wntmte D \hat{t} j, L Kiv nq |

mgZv m \hat{t} :

- (1) hw` mKj x Gi Rb` $ax + b = px + q$ nq, Zte $x = 0 \mid x = 1$ ewmtq cvB, $b = q$ Ges $a + b = p + q$ hv t \hat{t} K t` Lv hvq th, $a = p, b = q$
- (2) hw` mKj x Gi Rb` $ax^2 + bx + c = px^2 + qx + r$ nq, Zte $x = 0, x = 1 \mid x = -1$ ewmtq cvB, $c = r, a + b + c = p + q + r$ Ges $a - b + c = p - q + r$ hv t \hat{t} K t` Lv hvq th, $a = p, b = q, c = r$.
- (3) mvavi Yfvte t` Lv hvq th, hw` mKj x Gi Rb` $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = p_0x^n + p_1x^{n-1} + \dots + p_{n-1}x + p_n$ nq, Zte, $a_0 = p_0, a_1 = p_1, \dots, a_{n-1} = p_{n-1}, a_n = p_n$ A \hat{r} , mgZv w \hat{t} y \hat{i} Dfq c \hat{t} | x Gi GKB NvZhy \hat{s} mnM \hat{t} q ci \hat{u} i mgvb |

g \hat{e} ` : x Pj tKi n gv \hat{I} vi euc`xi eY \hat{v} i mnM \hat{t} j v \hat{t} K a_0 (a mve-wR \hat{t} iv), a_1 (a mve-l qvb) BZ \hat{w} t \hat{b} l qv m \hat{y} earRbK |

\hat{B} u euc`x $P(x) \mid Q(x)$ mKj x Gi Rb` mgvb ntj, G \hat{t} i mgZv \hat{t} K A \hat{t} f` ej v nq Ges Zv tevS \hat{t} Z A \hat{t} bK mgq $P(x) \equiv Q(x)$ t \hat{t} Lv nq | G \hat{t} | t \hat{t} $P(x) \mid Q(x)$ euc`x \hat{B} u Awfbenq | \equiv w \hat{p} y \hat{t} K A \hat{t} f` w \hat{p} y ej v nq | mvavi Yfvte, \hat{B} u exRMwYZxq iwki mgZv \hat{t} K A \hat{t} f` (*identity*) ej v nq, hw` iwki \hat{B} u t \hat{z} t \hat{K} v \hat{t} bv GKw Pj tKi t \hat{w} t \hat{g} b GKB nq Ges Pj Kmg \hat{t} ni t \hat{w} t \hat{g} bf \hat{s} gv \hat{t} bi Rb` iwki \hat{B} u gvb mgvb nq | thgb, $x(y + z) = xy + xz$ GKw A \hat{t} f` |

2.2 fVMtkl I Drcv` K Dccv`

GB Ab \hat{t} "Q \hat{t} i ay x Pj tKi euc`x wetePbv Kiv nte | c \hat{t} tg \hat{B} u D \hat{v} ni Y wetePbv Kwi |

D`vniY 1 | hW` $P(x) = x^2 - 5x + 6$ nq, Zte $P(x)$ tK $(x-4)$ Øviv fVM Ki Ges t`Lvl th, fVMtKl $P(4)$ Gi mgvb |

mgvavb : $P(x)$ tK $x-4$ Øviv fVM Kwi ,

$$\begin{array}{r} x-4 \) \ x^2 - 5x + 6 \\ \underline{x^2 - 4x} \\ -x + 6 \\ \underline{-x + 4} \\ 2 \end{array}$$

GLvfb fVMtKl 2,

thtnZl $P(4) = 4^2 - 5(4) + 6 = 2.$

mZi vs, fVMtKl $P(4)$ Gi mgvb |

D`vniY 2 | hW` $P(x) = ax^3 + bx + c$ nq, Zte $P(x)$ tK $x-m$ Øviv fVM Kti t`Lvl th, fVMtKl $P(m)$ Gi mgvb |

mgvavb : $P(x)$ tK $x-m$ Øviv fVM Kwi ,

$$\begin{array}{r} x-m \) \ ax^3 + bx + c \\ \underline{ax^3 - amx^2} \\ amx^2 + bx + c \\ \underline{amx^2 - am^2x} \\ (am^2 + b)x + c \\ \underline{(am^2 + b)x - (am^2 + b)m} \\ am^3 + bm + c \end{array}$$

GLvfb fVMtKl = $am^3 + bm + c$

Aevi , $P(m) = am^3 + bm + c$, mZi vs fVMtKl $P(m)$ Gi mgvb |

GB D`vniY `BwU t`tK wbtgde cÅZÁwU mæútk®avi Yv cvl qv hvq |

fVMtKl DCCV`" (Remainder Theorem)

cÅZÁv 1 | hW` $P(x)$ abvZtK gvIvi euc`x nq Ges a tKvfbv wbu`Ø msL`v nq, Zte $P(x)$ tK $x-a$ Øviv fVM Ki tJ fVMtKl $P(a)$ nte |

cØvY : $P(x)$ tK $x-a$ Øviv fVM Ki tJ fVMtKl nq 0 A_ev Akb` a`eK nte |

gtb Kwi, fVMtKl R Ges fVMdj $Q(x)$ ZvntJ , fvtMi wbtg, mKj x Gi Rb`-

$$P(x) = (x-a)Q(x) + R.....(1)$$

(1) bs G $x = a$ ewmtq cvB, $P(a) = 0 \cdot Q(a) + R = R.$

mZi vs, $P(x)$ tK $x-a$ Øviv fVM Ki tJ fVMtKl $P(a)$ nte |

D`vniY 3 | $P(x) = x^3 - 8x^2 + 6x + 60$ euc`xK $x+2$ Øviv fVM KiTj, fVMtKl KZ nte ?

mgvavb : thtnZl $x-a = x+2$, $\therefore x+2 = x-(-2) \Rightarrow a = -2$

mYZivs, fVMtKl $P(-2) = (-2)^3 - 8(-2)^2 + 6(-2) + 60 = 8$

cØZÁv 1 Gi AbKItY cØvY Kiv hvq th,

cØZÁv 2 | hw` $P(x)$ avZK gvIvi euc`x nq Ges $a \neq 0$ nq, Zte $P(x)$ tK $ax+b$ Øviv fVM KiTj

fVMtKl $P\left(\frac{-b}{a}\right)$ nte |

D`vniY 4 | euc`x $P(x) = 36x^2 - 8x + 5$ tK $(2x-1)$ Øviv fVM KiTj fVMtKl KZ nte ?

mgvavb : wbtYq fVMtKl $P\left(\frac{1}{2}\right) = 36\left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 5 = 9 - 4 + 5 = 10$

D`vniY 5 | hw` $P(x) = 5x^3 + 6x^2 - ax + 6$ tK $x-2$ Øviv fVM KiTj fVMtKl 6 nq, Zte a Gi gvb wbtYq Ki |

mgvavb : $P(x)$ tK $x-2$ Øviv fVM KiTj fVMtKl nte,

$$P(2) = 5(2)^3 + 6(2)^2 - a(2) + 6$$

$$= 40 + 24 - 2a + 6$$

$$= 70 - 2a$$

kZØvnti, $70 - 2a = 6$

$$\text{ev } 2a = 70 - 6 = 64 \Rightarrow a = 32$$

D`vniY 6 | hw` $P(x) = x^3 + 5x^2 + 6x + 8$ nq Ges $P(x)$ tK $x-a$ Ges $x-b$ Øviv fVM KiTj GKB

fVMtKl vtK thLvtb $a \neq b$, Zte t`Lvl th, $a^2 + b^2 + ab + 5a + 5b + 6 = 0$.

mgvavb : $P(x)$ tK $x-a$ Øviv fVM KiTj fVMtKl nte $P(a) = a^3 + 5a^2 + 6a + 8$

Ges $P(x)$ tK $x-b$ Øviv fVM KiTj fVMtKl nte $P(b) = b^3 + 5b^2 + 6b + 8$

kZØvnti, $a^3 + 5a^2 + 6a + 8 = b^3 + 5b^2 + 6b + 8$

$$\text{ev, } a^3 - b^3 + 5(a^2 - b^2) + 6(a - b) = 0$$

$$\text{ev, } (a-b)(a^2 + b^2 + ab + 5a + 5b + 6) = 0$$

$\therefore a^2 + b^2 + ab + 5a + 5b + 6 = 0$, thtnZl $(a-b) \neq 0$ i.e $a \neq b$.

Drcv`K Dccv`" (Factor theorem)

c0ZÁv 3 | hw` $P(x)$ abvZK gvÍvi eüc`x nq Ges $P(a) = 0$ nq, Zte $P(x)$ Gi GKwU Drcv`K $x - a$ nte |

c0yvY : $P(x)$ eüc`x $x - a$ 0viv fvM Ki t j fvM t k l = $P(a)$ [fvM t k l Dccv`" Abhvqx]
= 0 [c0 E kZ@_tK]

A_# $P(x)$ eüc`x $x - a$ 0viv wfvR |

∴ $x - a$ n t "Q $P(x)$ Gi GKwU Drcv`K |

Drcv`K Dccv`" i weci xZ Dccv`"

c0ZÁv 4 | hw` $P(x)$ eüc`xi $x - a$ GKwU Drcv`K nq, Zte t`Lvl th, $P(a) = 0$

c0yvY : th t nZl $P(x)$ eüc`xi $x - a$ GKwU Drcv`K, mZivs Av t i KwU eüc`x $Q(x)$ cvl qv hvq thb,
 $P(x) = (x - a)Q(x)$

GLv t b $x = a$ eim t q t`Lv hvq th, $P(a) = (a - a)Q(a) = 0 \cdot Q(a) = 0$.

D`vniY 7 | t`Lvl th, $P(x) = ax^3 + bx^2 + cx + d$ eüc`xi $x - 1$ GKwU Drcv`K nte hw` I tKej
hw` $a + b + c + d = 0$ nq |

mgvavb : g t b Kwí, $a + b + c + d = 0$

Zvnt j, $P(1) = a + b + c + d = 0$ [kZ@ymv t i]

mZivs, $x - 1$, $P(x)$ Gi GKwU Drcv`K [Drcv`K Dccv`" i mrvnt h"]

Gevi g t b Kwí $P(x)$ Gi GKwU Drcv`K $x - 1$

Zte, Drcv` t Ki weci xZ Dccv`" i mrvnt h" cvB, $P(1) = 0$ A_# $a + b + c + d = 0$.

gše` : abvZK gvÍvi th t Kv t bv eüc`xi $x - 1$ GKwU Drcv`K nte hw` I tKej hw` eüc`xwU
mnMmg t ni mgwó 0 nq |

D`vniY 8 | g t b Kwí, $P(x) = ax^3 + bx^2 + cx + d$ eüc`xi mnM t j v cYmsL`v, $a \neq 0, d \neq 0$ Ges
 $x - r$ eüc`xwU GKwU Drcv`K | t`Lvl th, (K) hw` r cYmsL`v nq, Zte r, d Gi Drcv`K nte | (L)

hw` $r = \frac{p}{q}$ j wó AvKv t i cKw kZ gj` msL`v nq, Zte p, d Gi Drcv`K I q, a Gi Drcv`K nte |

mgvavb : (K) Drcv`K Dccv`" t_ t K t`Lv hvq th, $P(r) = ar^3 + br^2 + cr + d = 0$

$$\text{ev, } (ar^2 + br + c)r = -d$$

th t nZl $(ar^2 + br + c)$, r I d c0Z t KB cYmsL`v, mZivs, r, d Gi GKwU Drcv`K |

(L) Drcv`K Dccv`" t_ t K t`Lv hvq th, $P(r) = ar^3 + br^2 + cr + d = 0$

$$\text{ev, } P\left(\frac{p}{q}\right) = a\left(\frac{p}{q}\right)^3 + b\left(\frac{p}{q}\right)^2 + c\left(\frac{p}{q}\right) + d = 0$$

$$\text{ev, } ap^3 + bp^2q + cpq^2 + dq^3 = 0 \dots (1)$$

$$(1) \text{ t_tK cvl qv hvq } (ap^2 + bpq + cq^2)p = -dq^3 \dots (2)$$

$$\text{Ges } (bp^2 + cqp + dq^2)q = -ap^3 \dots (3)$$

GLb, $ap^2 + bpq + cq^2, bp^2 + cpq + dq^2, p, q, d, a$ c0Z`tKB cYmsL`v|

mZivs (2) t_tK cvl qv hvq, p, dq^3 Gi GKwJ Drcv`K Ges (3) t_tK cvl qv hvq, q, ap^3 Gi GKwJ Drcv`K | wKŠ'p | q Gi ±1 Qvov tKvfbv maviY Drcv`K tbB | mZivs p, d Gi GKwJ Drcv`K Ges q, a Gi GKwJ Drcv`K |

`be` : Dctii D`vniY t_tK Avgiv j¶ | Kw th, Drcv`K Dccv`i mrvvth` cYmsL`K mnMwewkó euc`x $P(x)$ Gi Drcv`K wby¶qi Rb` c0tg $P(r)$ Ges cti $P\left(\frac{r}{s}\right)$ cix¶v Kiv thtZ cvti, thLvfb, r euc`xwJi a`e cti wewfbæDrcv`K ($r = \pm 1$ mn) Ges s euc`xwJi gL` mnMi wewfbæDrcv`K ($s = \pm 1$ mn) |

D`vniY 9 | $P(x) = x^3 - 6x^2 + 11x - 6$ Drcv`tK wctkiY Ki |

mgvavb : c0E euc`xi mnMmgñ cYmsL`v Ges a`e c` = -6, gL` mnM = 1

GLb r hw` cYmsL`v nq Ges $P(x)$ Gi hw` $x - r$ AvKv¶i i tKvfbv Drcv`K _vtK, Zte r Aek`B -6 Gi Drcv`K A_¶, ±1, ±2, ±3, ±6 Gi tKvbwJ nte | GLb r Gi Gifc wewfbægv¶bi Rb` $P(x)$ cix¶v Kw |

$$p(1) = 1 - 6 + 11 - 6 = 0 \therefore x - 1, p(x) \text{ Gi GKwJ Drcv`K}$$

$$p(-1) = -1 - 6 - 11 - 6 \neq 0 \therefore x + 1, p(x) \text{ Gi Drcv`K bq}$$

$$p(2) = 8 - 24 + 22 - 6 = 0 \therefore x - 2, p(x) \text{ Gi GKwJ Drcv`K}$$

$$p(-2) = -8 - 24 - 22 - 6 \neq 0 \therefore x + 2, p(x) \text{ Gi Drcv`K bq}$$

$$p(3) = 27 - 54 + 33 - 6 = 0 \therefore x - 3, p(x) \text{ Gi GKwJ Drcv`K}$$

thtnZi, $P(x)$ Gi gv¶v 3 Ges wZbwJ 1 gv¶vi Drcv`K cvl qv tMt0, mZivs $P(x)$ Gi Ab` tKvb Drcv`K hw` _vtK Zte Zv a`eK nte |

$$\therefore P(x) = k(x - 1)(x - 2)(x - 3), \text{ thLvfb } k \text{ a`eK |}$$

Dfqc¶¶ x Gi mte¶P NvtZi mnM wetePbv Kti t`Lv hvq th, $k = 1$

$$\text{mZivs, } P(x) = (x - 1)(x - 2)(x - 3)$$

“be” : tKvfbv euc`x $P(x)$ tK Drcv` tK wefkIY Kivi Rb` c0tg $(x-r)$ AvKvfi i GKwU Drcv` K wbyq Kfi $P(x)$ tK mivmwi $(x-r)$ Øviv fvM Kfi $A_{ev} P(x)$ Gi c` mgntK cbweØ`vm Kfi $P(x)$ tK $P(x) = (x-r)Q(x)$ AvKvfi tj Lv hvq| tmLvfb $Q(x)$ euc`xi gvIv $P(x)$ Gi gvIv t` tK 1 Kg| AZ:ci $Q(x)$ Gi Drcv` K wbyq Kfi AMñi ntZ nq|

D`vniY 10| Drcv` tK wefkIY Ki :

$$18x^3 + 15x^2 - x - 2$$

mgvavb : gtb KwI , $P(x) = 18x^3 + 15x^2 - x - 2$

$P(x)$ Gi a`e c` -2 Gi Drcv` Kmgñi tmU $F_1 = \{1, -1, 2, -2\}$.

$P(x)$ Gi gL` mnM 18 Gi Drcv` Kmgñi tmU $F_2 = \{1, -1, 2, -2, 3, -3, 6, -6, 9, -9, 18, -18, \}$

GLb $P(a)$ wefepbv KwI , thLvfb, $a = \frac{r}{s}$ Ges $r \in F_1, s \in F_2$

$$a = 1 \text{ ntj , } P(1) = 18 + 15 - 1 - 2 \neq 0$$

$$a = -1 \text{ ntj , } P(-1) = 18 + 15 - 1 - 2 \neq 0$$

$$\begin{aligned} a = -\frac{1}{2} \text{ ntj , } P\left(-\frac{1}{2}\right) &= -18\left(\frac{1}{8}\right) + 15\left(\frac{1}{4}\right) + \frac{1}{2} - 2 \\ &= \frac{-9}{4} + \frac{15}{4} + \frac{1}{2} - 2 \\ &= \frac{17}{4} - \frac{17}{4} = 0 \end{aligned}$$

mZivs $x + \frac{1}{2} = \frac{1}{2}(2x+1)$ A`fr, $(2x+1), P(x)$ Gi GKwU Drcv` K|

$$\begin{aligned} \text{GLb, } 18x^3 + 15x^2 - x - 2 &= 18x^3 + 9x^2 + 6x^2 + 3x - 4x - 2 \\ &= 9x^2(2x+1) + 3x(2x+1) - 2(2x+1) \\ &= (2x+1)(9x^2 + 3x - 2) \end{aligned}$$

$$\begin{aligned} \text{Ges } 9x^2 + 3x - 2 &= 9x^2 + 6x - 3x - 2 \\ &= 3x(3x+2) - 1(3x+2) \\ &= (3x+2)(3x-1) \end{aligned}$$

$$\therefore P(x) = (2x+1)(3x+2)(3x-1)$$

KvR :

1| hw` $P(x) = 2x^4 - 6x^3 + 5x - 2$ nq, Zte $P(x)$ tK wbgvj wLZ euc`x Øviv fvM Kfi fvMtkI wbyq Ki |

- (i) $x-1$ (ii) $x-2$ (iii) $x+2$ (iv) $x+3$ (v) $2x-1$ (vi) $2x+1$

2| fvMtkI Dccv`“i mrvh”- fvMtkI wbyq Ki |

- (i) fvR` : $4x^3 - 7x + 10$, fvRK : $x - 2$

- (ii) fVR : $5x^3 - 11x^2 - 3x + 4$, fVRK : $x + 1$
 (iii) fVR : $2y^3 - y^2 - y - 4$, fVRK : $y + 3$
 (iv) fVR : $2x^3 + x^2 - 18x + 10$, fVRK : $2x + 1$

- 3| t`Lvl th, $3x^3 - 4x^2 + 4x - 3$ Gi GKwU Drcv`K $(x - 1)$
 4| hw` $2x^3 + x^2 + ax - 9$, euc`xi GKwU Drcv`K $x + 3$ ntj a Gi gvb wbyq Ki |
 5| t`Lvl th, $x^3 - 4x^2 + 4x - 3$ euc`xi GKwU Drcv`K $x - 3$ |
 6| hw` $P(x) = 2x^3 - 5x^2 + 7x - 8$ nq, Zte $P(x)$ tK $x - 2$ Øviv fVM Ki tj th fVMtkl _vtK GtK fVMtkl Dccv`i mnvth` wbyq Ki |
 7| t`Lvl th, $4x^3 - 5x^2 + 5x - 2$ euc`xi $x + 1$ Ges $x - 1$ mvavi Y Drcv`K
 8| Drcv`tK wtkl Y Ki :
 (i) $x^3 + 2x^2 - 5x - 6$ (ii) $x^3 + 4x^2 + x - 6$
 (iii) $a^3 - a^2 - 10a - 8$ (iv) $x^4 + 3x^3 + 5x^2 + 8x + 5$

mggwit K, cZmg I Pμ-μvgK i wtk

mggwit K euc`x : tKvfbv euc`xi cZ`K ct`i gviv GKB ntj, GtK mggwit K euc`x (Homogeneous Polynomial) ejv nq| $x^2 + 2xy + 5y^2$ i wtkwU x, y Pj tKi `B gvivi GKwU mggwit K euc`x (GLvfb cZ`K ct`i gviv 2)|

$ax^2 + 2hxy + by^2$ i wtkwU x, y Pj tKi GKwU `B gvivi mggwit K i wtk, thlvfb, a, h, b wv` Ø msl`v| x, y, a, h, b cZ`K tK Pj K wetePbv Kiv ntj GuU GB Pj Kmgfni wZb gvivi mggwit K euc`x nq| $2x^2y + y^2z + 9z^2x - 5xyz$ i wtkwU x, y, z Pj tKi wZb gvivi mggwit K euc`x| (GLvfb cZ`K ct`i gviv 3)|

cZmg i wtk (Symmetric)

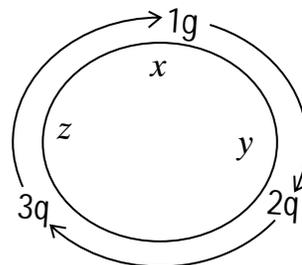
GKwaK Pj K avi YKvi x tKvfbv exRMwYwZK i wtki thtKvfbv `BwU Pj K `vb wvbgtq hw` i wtkwU Acwi ewZ` _vtK, Zte i wtkwU tK H Pj Kmgfni cZmg (Symmetric) i wtk ejv nq| $a + b + c$ i wtkwU a, b, c Pj tKi cZmg i wtk| Kvi Y, a, b, c Pj K wZbwi thtKvfbv `Bwi `vb wvbgtq i wtkwU Acwi ewZ` _vtK| GKbfvte, $ab + bc + ca$ i wtkwU a, b, c Pj tKi Ges $x^2 + y^2 + z^2 + xy + yz + zx$ i wtkwU x, y, z Pj tKi cZmg i wtk|

wKŠ' $2x^2 + 5xy + 6y^2$ i wtkwU x | y Pj tKi cZmg bq Kvi Y i wtkwU tZ x | y Gi ci`ui `vb wvbgtq $2y^2 + 5xy + 6x^2$ i wtk tZ cwi ewZ` nq hv cteP i wtk t`tK wfbq

Pμ-μwgK i vnk (Cyclic)

wZbwU Pj K m=0j Z tKvfbv exRMvWvWZK i vnk tZ c0_g Pj K w0Zxq Pj tKi, w0Zxq Pj K ZZxq Pj tKi Ges ZZxq Pj tKi t_j c0_g Pj K emvtj i vnk w h^w cwi ewZ z bv nq, Zte i vnk w tK H wZb Pj tKi Dwj. wLZ μtg GKwU Pμ-μwgK i vnk ev Pμ c0Zmg i vnk ev (Cyclically symmetric expression) ej v nq | Pj K, t_j vi t_vb cwi eZ z cvtki wP t i gZ PμvKvfi Kiv nq etj B Gifc i vnk tK Pμ-μwgK i vnk ej v nq | _v tK |

$x^2 + y^2 + z^2 + xy + yz + zx$ i vnk wU x, y, z Pj tKi GKwU Pμ-μwgK i vnk, Kvi Y G tZ PμvKvfi x Gi cwi e tZ y, y Gi cwi e tZ z Ges z Gi cwi e tZ x emvtj i vnk wU GKB _v tK | GKBfvte $x^2y + y^2z + z^2x$ i vnk wU x, y, z Pj tKi GKwU Pμ-μwgK i vnk |



$x^2 - y^2 + z^2$ i vnk wU Pμ-μwgK i vnk bq, Kvi Y G tZ x Gi t_j y, y Gi t_j z Ges z Gi t_j x emvtj i vnk wU $y^2 - z^2 + x^2$ i vnk tZ cwi ewZ z nq hv c t e P i vnk t t_K wfb q

wZbwU Pj tKi c0Z K c0Zmg i vnk Pμ-μwgK | wK s' c0Z K Pμ-μwgK i vnk c0Zmg bq | thgb, $x^2(y - z) + y^2(z - x) + z^2(x - y)$ i vnk wU Pμ-μwgK, wK s' c0Zmg bq | Kvi Y, i vnk wU tZ x Ges y t_vb wvbgq Ki t_j $y^2(x - z) + x^2(z - y) + z^2(y - x)$ i vnk cvl qv hvq hv c t e P i vnk wU t t_K wfb q

`be` : eY v i m ye a t _ x, y Pj tKi i vnk tK $F(x, y)$ AvKvfi i Ges x, y, z Pj tKi i vnk tK $F(x, y, z)$ AvKvfi i c0Z xK 0v i v m PZ Kiv nq |

D`vni Y 1 | t`Lv l th, $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ i vnk wU c0Zmg bq wK s' PμμwgK |

$$\left[F(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x} \right] \text{ a t i w b t R K i}$$

Pμ-μwgK euc`xi Drcv` tK w e t K I Y

Gifc euc`x tK Drcv` tK w e t K I Y Kivi tKvfbv aiv-ewav wbgq t b | m v a v i Y Z, i vnk wU i c` t_j v t K c p w e v m K t i Drcv` K t e i Kiv nq | A t b k m g q i vnk wU t K t K v f b v G K w U P j t K i e u c` x a t i Drcv` K D c c v t` i m v n v h` G K e v G K w a K Drcv` K w b Y q Kiv nq Ges i vnk wU i P μ - μ w g K I m g g w i t K ` e w k o` w e t e P b v K t i A c i v c i Drcv` K w b Y q Kiv nq |

G c h t t 1/2 D t j, L` th, a, b, c Pj tKi

(K) tKvfbv Pμ-μwgK euc`xi $(a - b)$ GKwU Drcv` K n t j, $(b - c)$ Ges $(c - a)$ i vnk wU i Drcv` K n t e |

(L) GK g v i v i I ` β g v i v i m g g w i t K P μ - μ w g K e u c` x h _ v μ t g $k(a + b + c) | k(a^2 + b^2 + c^2) + m(ab + bc + ca)$ t h L v t b $k | m$ a` e K |

(M) `Bw euc`x hw` Ggb nq th, Pj K,tj vi mKj gvftbi Rb` Gt` i gvb mgvb nq, Zte euc`x `Bw Abjfc c` `Bw mnM ci`ui mgvb nte|

D`vni Y 2| $bc(b - c) + ca(c - a) + ab(a - b)$ tK Drcv` tK wetkIY Ki |

mgvavb : c0g c×wZ-

$$bc(b - c) + ca(c - a) + ab(a - b)$$

$$= bc(b - c) + c^2a - ca^2 + a^2b - ab^2$$

$$= bc(b - c) + a^2b - ca^2 - ab^2 + c^2a$$

$$= bc(b - c) + a^2(b - c) - a(b^2 - c^2)$$

$$= (b - c)\{bc + a^2 - a(b + c)\}$$

$$= (b - c)\{bc + a^2 - ab - ac\}$$

$$= (b - c)\{bc - ab - ac + a^2\}$$

$$= (b - c)\{b(c - a) - a(c - a)\}$$

$$= (b - c)(c - a)(b - a)$$

$$= -(a - b)(b - c)(c - a)$$

w0Zxq c×wZ : c0E iwkwJtK a Gi euc`x $P(a)$ ati ZvtZ a Gi cwietZ^ob ewmtq t`wL th, $P(b) = bc(b - c) + cb(b - b) + b^2(a - b) = 0$ mZivs, Drcv`K Dccv` Abhvqx $(a - b)$ c0E iwki GKwJ Drcv`K GLb thtnZic0E iwkwJ Pμ-μwgK iwkwJ tmtnZi $(b - c)$ Ges $(c - a)$ Dftq c0E iwkwJi Drcv`K| c0E iwkwJ wZb gvIvi mggwI K Ges Gi wZbwJ GK gvIvi Drcv`K cvl qv tmq| mZivs Ab` Drcv`K hw` _vtK Zv a`eK nte|

A_ŕ, $bc(b - c) + ca(c - a) + ab(a - b) = k(a - b)(c - a)$(1) tmLvfb k GKwJ a`eK| a, b, c

Gi mKj gvftbi Rb` (1) mZ`| (1) bs G $a = 0, b = 1, c = 2$ ewmtq cvB, $2(-1) = k(-1)(-1)(2)$

$$\Rightarrow k = -1$$

$$\therefore bc(b - c) + ca(c - a) + ab(a - b) = -(a - b)(b - c)(c - a).$$

D`vni Y 3| $a^3(b - c) + b^3(c - a) + c^3(a - b)$ tK Drcv` tK wetkIY Ki |

mgvavb : c0E iwkwJtK a Gi euc`x $P(a)$ wetepbv Kti ZvtZ a Gi cwietZ^ob ewmtq cvB,

$$P(b) = b^3(b - c) + b^3(c - b) + c^3(b - b) = 0$$

mZivs Drcv`K Dccv` Abhvqx $(a - b)$ c0E iwki GKwJ Drcv`K| GLb thtnZic0E iwkwJ Pμ-μwgK iwkwJ tmtnZi $(b - c)$ Ges $(c - a)$ Dftq c0E iwkwJi Drcv`K| Avevi c0E iwkwJ Pvi gvIvi mggwI K iwkwJ Ges $(a - b)(b - c)(c - a)$ wZb gvIvi mggwI K iwkwJ| mZivs c0E iwki Aci Drcv`KwJ Aek`B Pμ-μwgK Ges GK gvIvi mggwI K iwkwJ nte| A_ŕ, Zv $k(a + b + c)$ nte, thLvfb k GKwJ a`eK|

$$\therefore a^3(b-c) + b^3(c-a) + c^3(a-b) = k(a-b)(b-c)(c-a)(a+b+c) \dots (1)$$

a, b, c Gi mKj gv̄bi Rb̄ (1) mZ̄ |

m̄Z̄is (1) bs G $a = 0, b = 1, c = 2$ eim̄tq cv̄B, $2 + 8(-1) = k(-1)(-1)(2)(3)$ ev $k = -1$

(1) G $k = -1$ eim̄tq cv̄B,

$$a^3(b-c) + b^3(c-a) + c^3(a-b) = -(a-b)(b-0)(c-a)(a+b+c).$$

D`vniY 4 | $(b+c)(c+a)(a+b) + abc$ tK Drcv`tK w̄tKlY Ki |

mgvavb : i v̄k̄ūt̄K a Gi eūc`x $P(a)$ w̄t̄ePbv K̄ti Zv̄tZ a Gi cwīēt̄Z[⊗] $-b-c$ eim̄tq cv̄B, $P\{-b+c\} = (b+c)(c-b-c)(-b-c+b) + (-b-c)bc = bc(b+c) - bc(b+c) = 0$.

m̄Z̄ivs Drcv`K Dccv` Ab̄hvqx $(a+b+c)$ c̄l̄Ē i v̄k̄i GK̄ū Drcv`K | c̄l̄Ē i v̄k̄ū w̄Zb gv̄Īvi m̄ggw̄ĪK P̄μ-μ̄wgK eūc`x Ges Gi GK̄ gv̄Īvi GK̄ū Drcv`K cv̄l̄qv t̄M̄t̄Q | m̄Z̄ivs Ac̄i Drcv`K `B̄ gv̄Īvi m̄ggw̄ĪK P̄μ-μ̄wgK eūc`x n̄te, A_{f} , $k(a^2 + b^2 + c^2) + m(bc + ca + ab)$ Av̄Kv̄ti i n̄te, t̄hLv̄t̄b $k \mid m$ a`eK |

$$\therefore (b+c)(c+a)(a+b) + abc = (a+b+c)\{k(a^2 + b^2 + c^2) + m(bc + ca + ab)\} \dots (1)$$

a, b, c Gi mKj gv̄bi Rb̄ (1) mZ̄ |

(1) G c̄l̄t̄g $a = 0, b = 0, c = 1$ Ges c̄ti $a = 1, b = 1, c = 0$ eim̄tq h_v̄μ̄t̄g cv̄B, $0 = k$ Ges $2 = 2(k \times 2 + m)$

$$\therefore k = 0, m = 1.$$

GLb $k \mid m$ Gi gv̄b eim̄tq cv̄B, $(b+c)(c+a)(a+b) + abc = (a+b+c)(bc + ca + ab)$

ḡš̄e` : D`vniY 2 Gi mgvav̄bi c̄l̄g c̄x̄w̄Zi Ab̄jfc c̄x̄w̄Zi D`vniY 3 Ges D`vniY 4 G ew̄ȲZ i v̄k̄ `B̄w̄t̄K Drcv`tK w̄t̄KlY Kiv hv̄te |

GK̄ū w̄t̄Kl̄ exRM̄w̄Ȳw̄ZK m̄t̄ : $a, b \mid c$ Gi mKj gv̄bi Rb̄ w̄b̄t̄gm̄t̄w̄i `B̄w̄ c̄ḡv̄Y t̄` l̄qv n̄t̄j v:

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

c̄l̄g c̄ḡv̄Y (mi v̄m̄wi m̄nR exRM̄w̄Ȳw̄ZK c̄l̄μ̄qv c̄l̄qv̄M K̄ti) :

$$\begin{aligned} & a^3 + b^3 + c^3 - 3abc \\ &= (a+b)^3 - 3ab(a+b) + c^3 - 3abc \\ &= (a+b)^3 + c^3 - 3ab(a+b+c) \\ &= (a+b+c)\{(a+b)^2 - (a+b)c + c^2\} - 3ab(a+b+c) \\ &= (a+b+c)(a^2 + 2ab + b^2 - ac - bc + c^2) - 3ab(a+b+c) \\ &= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \end{aligned}$$

w̄Z̄xq c̄ḡv̄Y (m̄ggw̄ĪK P̄μ-μ̄wgK eūc`xi avi Yv e`envi K̄ti) :

$$a^3 + b^3 + c^3 - 3abc \text{ i v̄k̄ūt̄K } a \text{ Pj t̄Ki eūc`x } P(a) \text{ āti Zv̄tZ } a = -(b+c) \text{ eim̄tq cv̄B,}$$

$$p\{-(b+c)\} = -(b+c)^3 + b^3 + c^3 + 3(b+c)bc$$

$$= -(b+c)^3 + (b+c)^3 = 0$$

mȳZivs $a+b+c$ weŧePbvaxb i vnkWUj GKWU Drcv`K | thŧnZl $a^3+b^3+c^3-3abc$ wZb gvŧvi mggwŧK Pμ-μmgK eŧc`x, mȳZivs i vnkWUj Aci Drcv`K $k(a^2+b^2+c^2)+m(ab+bc+ca)$ AvKŧii nŧe, thLvŧb $k | m$ a*eK | AZGe, mKj $a, b | c$ Gi Rb`

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)\{k(a^2+b^2+c^2) + m(ab+bc+ca)\}$$

GLvŧb cŧŧg $a=1, b=0, c=0$ Ges cŧi $a=1, b=1, c=0$ ewŧŧq cvB, $k=1$ Ges

$$2 = 2(k \times 2 + m) \Rightarrow k = 1 \text{ Ges } 1 = 2 + m \Rightarrow m = -1$$

$$\therefore k = 1 \text{ Ges } m = -1.$$

$$\therefore a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{Abȳm} \times \text{vŧŧ-1} | a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$$

cŧŧvY : thŧnZl $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$$

$$= \frac{1}{2}\{(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)\}$$

$$= \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$$

$$\therefore a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$$

Abȳm}vŧŧ-2 | hw` $a+b+c=0$ nq, Zŧe $a^3+b^3+c^3=3abc$.

Abȳm}vŧŧ-3 | hw` $a^3+b^3+c^3=3abc$ nq, Zŧe $a+b+c=0$ A_ev $a=b=c$.

D`vniY 5 | $(a-b)^3 + (b-c)^3 + (c-a)^3$ ŧK Drcv`ŧK weŧkIY Ki |

mgvavb : awi $A = a-b, B = b-c$ Ges $C = c-a$. Zvŧŧj ,

$$A + B + C = a - b + b - c + c - a = 0$$

mȳZivs, $A^3 + B^3 + C^3 = 3ABC$

$$A_{\text{ŧ}}, (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a).$$

KvR : Drcv`ŧK weŧkIY Ki

$$1 | \text{ (K) } a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$$

$$\text{ (L) } a^2(b-c) + b^2(c-a) + c^2(a-b)$$

- (M) $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$
 (N) $bc(b^2 - c^2) + ca(c^2 - a^2) + ab(a^2 - b^2)$
 (O) $a^4(b-c) + b^4(c-a) + c^4(a-b)$
 (P) $a^2(b-c)^3 + b^2(c-a)^3 + c^2(a-b)^3$
 (Q) $x^4(y^2 - z^2) + y^4(z^2 - x^2) + z^4(x^2 - y^2)$
 (R) $a^3(b-c) + b^3(c-a) + c^3(a-b)$

$$2 | \text{hw} \quad \frac{x^2 - yz}{a} = \frac{y^2 - zx}{b} = \frac{z^2 - xy}{c} \neq 0 \text{ nq,}$$

$$\text{Zte f`Lvl th, } (a+b+c)(x+y+z) = ax+by+cz.$$

$$3 | \text{hw} \quad (a+b+c)(ab+bc+ca) = abc \text{ nq, Zte f`Lvl th, } (a+b+c)^3 = a^3 + b^3 + c^3.$$

gj` fMusk (Rational Fractions)

GKwU euc`xtK ni Ges GKwU euc`xtK je wbtq MwYZ fMuskfK gj` fMusk ejv nq| thgb

$$\frac{x}{(x-a)(x-b)} \text{ Ges } \frac{a^2+a+1}{(a-b)(a-c)} \text{ gj` fMusk |}$$

$$\text{D`vniY 1 | mij Ki : } \frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)}$$

$$\begin{aligned} \text{mgvavb : c0 E i vnk } & \frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)} \\ & = \frac{a}{-(a-b)(c-a)} + \frac{b}{-(b-c)(a-b)} + \frac{c}{-(c-a)(b-c)} \\ & = \frac{a(b-c) + b(c-a) + c(a-b)}{-(a-b)(b-c)(c-a)} \\ & = \frac{ab - ca + bc - ab + ca - bc}{-(a-b)(b-c)(c-a)} \\ & = \frac{0}{(a-b)(b-c)(c-a)} \\ & = 0 \end{aligned}$$

$$\text{D`vniY 2 | mij Ki : } \frac{a^2 - (b-c)^2}{(a+c)^2 - b^2} + \frac{b^2 - (a-c)^2}{(a+b)^2 - c^2} + \frac{c^2 - (a-b)^2}{(b+c)^2 - a^2}$$

$$\text{mgvavb : c0_g fMusk = } \frac{(a+b-c)(a-b+c)}{(a+b+c)(a-b+c)} = \frac{a+b-c}{a+b+c}$$

$$\text{w0Zxq fMusk = } \frac{(a+b-c)(b-a+c)}{(a+b+c)(a+b-c)} = \frac{b-a+c}{a+b+c}$$

$$\text{ZZxq fMusk} = \frac{(c+a-b)(c-a+b)}{(b+c+a)(b+c-a)} = \frac{c+a-b}{a+b+c}$$

$$\begin{aligned} \therefore \text{c0 \u00c7 i vk} &= \frac{a+b-c}{a+b+c} + \frac{b-a+c}{a+b+c} + \frac{c+a-b}{a+b+c} \\ &= \frac{a+b-c+b-a+c+c+a-b}{a+b+c} \\ &= \frac{a+b+c}{a+b+c} = 1 \end{aligned}$$

$$\text{D`vni Y 3 | mij Ki : } \frac{(ax+1)^2}{(x-y)(z-x)} + \frac{(ay+1)^2}{(x-y)(y-z)} + \frac{(az+1)^2}{(y-z)(z-x)}$$

$$\text{mgvab : c0 \u00c7 i vk} = \frac{(ax+1)^2(y-z) + (ay+1)^2(z-x) + (az+1)^2(x-y)}{(x-y)(y-z)(z-x)} \dots\dots\dots(1)$$

$$\begin{aligned} (1) \text{ Gi je} &= (a^2x^2 + 2ax + 1)(y-z) + (a^2y^2 + 2ay + 1)(z-x) + (a^2z^2 + 2az + 1)(x-y) \\ &= a^2\{x^2(y-z) + y^2(z-x) + z^2(x-y)\} + 2a\{x(y-z) + y(z-x) + z(x-y)\} \\ &\quad + \{(y-z) + (z-x) + (x-y)\} \end{aligned}$$

$$\text{\u00d0S' } x^2(y-z) + y^2(z-x) + z^2(x-y) = -(x-y)(y-z)(z-x)$$

$$\text{Z`pwi , } x(y-z) + y(z-x) + z(x-y) = 0 \text{ Ges } (y-z) + (z-x) + (x-y) = 0$$

$$\therefore (1) \text{ Gi je} = -a^2(x-y)(y-z)(z-x)$$

$$\text{m\u00d0i vs c0 \u00c7 i vk} = \frac{-a^2(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)} = -a^2$$

$$\text{D`vni Y 4 | mij Ki : } \frac{1}{x+a} + \frac{2x}{x^2+a^2} + \frac{4x^3}{x^4+a^4} + \frac{8x^7}{a^8-x^8}$$

$$\text{mgvab : c0 \u00c7 i vk i ZZxq I PZL`c\` i thvMdj} = \frac{4x^3}{x^4+a^4} + \frac{8x^7}{a^8-x^8} = \frac{4x^3}{x^4+a^4} \left(1 + \frac{2x^4}{a^4-x^4} \right)$$

$$= \frac{4x^3}{x^4+a^4} \times \frac{a^4-x^4+2x^4}{a^4-x^4} = \frac{4x^3}{x^4+a^4} \times \frac{a^4+x^4}{a^4-x^4} = \frac{4x^3}{a^4-x^4}$$

$$\therefore \text{\u00d0ZZxq, ZZxq Ges PZL`c\` i thvMdj} = \frac{2x}{x^2+a^2} + \frac{4x^3}{a^4-x^4} = \frac{2x}{x^2+a^2} \left[1 + \frac{2x^2}{a^2-x^2} \right]$$

$$= \frac{2x}{x^2+a^2} \times \frac{a^2-x^2+2x^2}{a^2-x^2} = \frac{2x}{x^2+a^2} \times \frac{a^2+x^2}{a^2-x^2} = \frac{2x}{a^2-x^2}$$

$$\therefore \text{c0 \u00c7 i vk} = \frac{1}{x+a} + \frac{2x}{a^2-x^2} = \frac{a-x+2x}{a^2-x^2} = \frac{a+x}{a^2-x^2} = \frac{1}{a-x}$$

KivR :
 mij Ki :

$$1) \frac{b+c}{(a-b)(a-c)} + \frac{c+a}{(b-c)(b-a)} + \frac{a+b}{(c-a)(c-b)}$$

$$2) \frac{a^3-1}{(a-b)(a-c)} + \frac{b^3-1}{(b-c)(b-a)} + \frac{c^3-1}{(c-a)(c-b)}$$

$$3) \frac{bc(a+d)}{(a-b)(a-c)} + \frac{ca(b+d)}{(b-c)(b-a)} + \frac{ab(c+d)}{(c-a)(c-b)}$$

$$4) \frac{a^3+a^2+1}{(a-b)(a-c)} + \frac{b^3+b^2+1}{(b-c)(b-a)} + \frac{c^3+c^2+1}{(c-a)(c-b)}$$

$$5) \frac{a^2+bc}{(a-b)(a-c)} + \frac{b^2+ca}{(b-c)(b-a)} + \frac{c^2+ab}{(c-a)(c-b)}$$

AvsukK fMusK (Partial Fraction)

hw` tKvfbv fMusktK GKwaK fMusiki thvMdj ifc cKvk Kiv nq, Zte tktlv³ fMusktjvi cÖZ`KwUtK cÖtgv³ fMusiki AvsukK fMusK ejv nq| aiv hvK, GKwU fMusK $\frac{3x-8}{x^2-5x+8}$ GtK tj Lv

hvq, $\frac{3x-8}{x^2-5x+8} = \frac{2(x-3) + (x-2)}{(x-3)(x-2)} = \frac{2}{x-2} + \frac{1}{x-3}$

GLvfb cÖE fMuskuUtK `BwU fMusiki thvMdj ifc cKvk Kiv ntqtQ| A_@, fMuskuUtK `BwU AvsukK fMuskt weF³ Kiv ntqtQ|

hw` $N(x) | D(x)$ DfqB x Pj tKi euc`x Ges je $N(x)$ Gi gvIv ni $D(x)$ Gi gvIv Atc¶v tQvU nq, Zvntj fMuskuU cKZ fMusK (Proper Fraction)| hw` je $N(x)$ Gi gvIv ni $D(x)$ Gi gvIvi mgvb A_ev Zv Atc¶v eo ntj fMuskuUtK AcKZ fMusK (Improper Fraction) ejv nq|

thgb, $\frac{x^2+1}{(x+1)(x+2)(x-3)}$ GKwU cKZ fMusK|

Ges $\frac{2x^4}{x+1} | \frac{x^3+3x^2+2}{x+2}$ DfqB AcKZ fMusK|

DtjL` th, AcKZ fMusiki j etK ni Øviv mvariY wqqtg fM Kti A_ev j tei c` ,tj vtK mjeavRbKfvte c¶veØ`vm Kti fMuskuUtK GKwU euc`x (fMdj) Ges GKwU cKZ fMusiki thvMdj ifc cKvk Kiv hvq|

thgb, $\frac{x^3+3x^2+2}{x+2} = (x^2+x-2) + \frac{6}{x+2}$

cKZ gj` fMusktK Kxfite AvsukK fMuskt cwiYZ Kiv nq, Zv vbtgawefbocxwZtZ t` Lvfbv ntj v|

c0g c×wZ : hLb nti ev`e | GKvZvewkó Drcv`K _vfk wKŠ` tKvb Drcv`KB cpivevE nq bv|

D`vniY 1| $\frac{5x-7}{(x-1)(x-2)}$ tK AvsukK fMuskt cKvk Ki |

mgvavb : awi , $\frac{5x-7}{(x-1)(x-2)} \equiv \frac{A}{x-1} + \frac{B}{x-2}$ (1)

(1) Gi Dfqc¶tK $(x-1)(x-2)$ Øviv ,Y Ki tñ cvB, $5x-7 = A(x-2) + B(x-1)$(2)

hv x Gi mKj gvftbi Rb` mZ` | GLb (2) Gi Dfqc¶t¶ $x=1$ eimtq cvB, $5-7 = A(1-2) + B(1-1)$
ev, $-2 = -A \quad \therefore A = 2$

Avevi , (2) Gi Dfqc¶t¶ $x=2$ eimtq cvB, $10-7 = A(2-2) + B$

ev, $3 = B \quad \therefore B = 3$

GLb A Ges B Gi gvb (1) G eimtq cvB, $\frac{5x-7}{(x-1)(x-2)} = \frac{2}{x-1} + \frac{3}{x-2}$; GtZB c0 È fMuskuW

AvsukK fMuskt wef³ ntj v|

gŠe` : c0 È fMuskuWi AvsukK fMuskt cKvk th h_vh_ ntqtQ Zv cix¶v Kti t` Lv thtZ cvti |

Wwbc¶ = $\frac{2}{x-1} + \frac{3}{x-2} = \frac{2(x-2) + 3(x-1)}{(x-1)(x-2)} = \frac{5x-7}{(x-1)(x-2)} = \text{evgc¶}$

D`vniY 2| $\frac{x+5}{(x-1)(x-2)(x-3)}$ tK AvsukK fMuskt cKvk Ki |

mgvavb : awi , $\frac{x+5}{(x-1)(x-2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ (1)

(1) Gi Dfqc¶tK $(x-1)(x-2)(x-3)$ Øviv ,Y Kti cvB,

$x+5 \equiv A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$(2)

(2) Gi Dfqc¶t¶ x Gi mKj gvftbi Rb` mZ` |

(2) Gi Dfqc¶t¶ $x=1$ eimtq cvB, $1+5 = A(-1)(-2) \Rightarrow 6 = 2A \Rightarrow A = 3$

Avevi (2) Gi Dfqc¶t¶ $x=2$ eimtq cvB, $2+5 = B(1)(-1) \Rightarrow 7 = -B$

$\therefore B = -7$

Ges (2) Gi Dfqc¶t¶ $x=3$ eimtq cvB, $3+5 = C(2)(1)$ ev $8 = 2C$ ev $C = 4$

GLb, A, B Ges C Gi gvb . (1) G eimtq cvB, $\frac{x+5}{(x-1)(x-2)(x-3)} = \frac{3}{x-1} - \frac{7}{x-2} + \frac{4}{x-3}$

GvUB c0 È fMuskti AvsukK fMuskt cKvk |

wØZxq c×wZ : hLb j tēi NvZ nti i NvZ Atc¶v epEi ev mgvb nq, ZLb j etK ni Øviv fVM Kti j tēi NvZ Atc¶v ¶i Zi Kitz nq|

D`vni Y 3 | $\frac{(x-1)(x+5)}{(x-2)(x-4)}$ tK AvsukK fVMusk cKvk Ki |

mgvavb : awi , $\frac{(x-1)(x+5)}{(x-2)(x-4)} \equiv 1 + \frac{A}{x-2} + \frac{2}{2-4} \dots\dots(1)$

(1) Dfqc¶tK $(x-2)(x-4)$ Øviv , Y Kti cvB,
 $(x-1)(x-5) \equiv (x-2)(x-4) + A(x-4) + B(x-2) \dots\dots(2)$

(2) Gi Dfqc¶tK ch¶qµtg $x = 2, 4$ ewmtq cvB, $(2-1)(2-5) = A(2-4)$ ev, $A = \frac{3}{2}$

Ges $(4-1)(4-5) = B(4-2)$ ev, $B = \frac{-3}{2}$

GLb A Ges B Gi gvb (1) G ewmtq cvB, $\frac{(x-1)(x+5)}{(x-2)(x-4)} = 1 + \frac{3}{2(x-2)} - \frac{3}{2(x-4)}$

hv vbtY¶ AvsukK fVMusk |

D`vni Y 4 | $\frac{x^3}{(x-1)(x-2)(x-3)}$ tK AvsukK fVMusk cKvk Ki |

mgvavb : awi , $\frac{x^3}{(x-1)(x-2)(x-3)} \equiv 1 + \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} \dots\dots(1)$

(1) Dfqc¶¶ $(x-1)(x-2)(x-3)$ Øviv , Y Kti cvB,
 $x^3 \equiv (x-1)(x-2)(x-3) + A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \dots\dots(2)$

(2) Gi Dfqc¶¶ ch¶qµtg $x = 1, 2, 3$ ewmtq cvB,

$1 = A(-1)(-2)$ ev, $A = \frac{1}{2}$

$8 = B(1)(-1)$ ev, $B = -8$

Ges $27 = C(2)(1)$ ev, $C = \frac{27}{2}$

GLb A, B Ges C Gi gvb (1) G ewmtq cvB,

$\frac{x^3}{(x-1)(x-2)(x-3)} = 1 + \frac{1}{2(x-1)} - \frac{8}{x-2} + \frac{27}{2(x-3)}$

hv vbtY¶ AvsukK fVMusk |

ZZxq c×wZ : hLb nti ev`e I GKNvZ wewkó Drcv`K _vtK Ges Dnv` i gta` KtqKwU cpi vevE nq|

D`vni Y 5 | $\frac{x}{(x-1)^2(x-2)}$ tK AvsukK fMusk cKvk Ki |

mgvavb : awi $\frac{x}{(x-1)^2(x-2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \dots\dots\dots(1)$

(1) Gi Dfqc¶tK $(x-1)^2(x-2)$ Øviv ,Y Kti cvB,

$x = A(x-1)(x-2) + B(x-2) + C(x-1)^2 \dots\dots\dots(2)$

(2) Gi Dfqc¶t¶ $x=1, 2$ emvBqv cvB, $1 = B(1-2)$ ev, $B = -1$

Ges $2 = C(2-1)^2$ ev, $2 = C \Rightarrow C = 2$

Averi, (2) G x^2 Gi mnM mgxKZ Kti cvB, $0 = A + C$ ev, $A = -C = -2$

GLb A, B Ges C Gi gvb (1) G emtq cvB, $\frac{x}{(x-1)^2(x-2)} = \frac{-2}{x-1} + \frac{-1}{(x-1)^2} + \frac{2}{x-2}$

hv vbtY¶ AvsukK fMusk |

PZL`C×vZ : hLb nti ev`e I vØNvZ weukó Drcv`K _vtK vKŠ' tKvbtv cpi vevE nq bv |

D`vni Y 6 | $\frac{x}{(x-1)(x^2+4)}$ tK AvsukK fMusk cKvk Ki |

mgvavb : awi, $\frac{x}{(x-1)(x^2+4)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+4} \dots\dots\dots(1)$

Dfqc¶tK $(x-1)(x^2+4)$ Øviv ,Y Kti cvB, $x = A(x^2+4) + (Bx+C)(x-1) \dots\dots\dots(2)$

(2) G $x=1$ emvBqv cvB, $1 = A(5) \Rightarrow A = \frac{1}{5}$

x^2 I x Gi mnM mgxKZ Kti cvB, $A + B = 0 \dots\dots\dots (3)$ Ges $C - B = 1 \dots\dots\dots (4)$

(3) bs G $A = \frac{1}{5}$ emvBqv cvB, $B = -\frac{1}{5}$

(4) bs G $B = -\frac{1}{5}$ emvBqv cvB $C = \frac{4}{5}$

GLb, A, B I C Gi gvb (1) bs G emtq cvB,

$$\frac{x}{(x-1)(x^2+4)} = \frac{\frac{1}{5}}{x-1} + \frac{-\frac{x}{5} + \frac{4}{5}}{x^2+4} = \frac{1}{5(x-1)} - \frac{x-4}{5(x^2+4)}, \text{ hv vbtY¶ AvsukK fMusk |}$$

cÅg c×vZ : hLb nti e`e I vØNvZ weukó Drcv`K _vtK Ges Dnv` i gta` KtqKvU cpi vevE NtU |

D`vniY 7| $\frac{1}{x^2(x^2+1)^2}$ tK Avsıkk fMusk cKvk Ki |

mgvavb : awi , $\frac{1}{x^2(x^2+1)^2} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$

(1) Gi Dfqct¶| $x^2(x^2+1)^2$ Øviv , Y Kti cvB,

$1 = Ax(x^2+1)^2 + B(x^2+1)^2 + (Cx+D)x^2(x^2+1) + (Ex+F)x^2 \dots\dots\dots(1)$

(1) Gi x^5, x^4, x^3, x^2, x Gi mnM Ges a'eK c` mgxKZ Kwi qv cvB,

$A + C = 0,$

$B + D = 0$

$2A + C + E = 0$

$2B + D + F = 0$

$A = 0$

$B = 1$

$A + C = 0$ tZ $A = 0$ emtq cvB, $C = 0$

$B + D = 0$ tZ $B = 1$ emtq cvB, $D = -1$

$2A + C + E = 0$ tZ $A = 0, C = 0$ emtq cvB, $E = 0$

Avevi , $2B + D + F = 0$ tZ $D = -1, B = 1$ emtq cvB, $2 - 1 + F = 0 \Rightarrow F = -1$

$\therefore A = 0, B = 1, C = 0, D = -1, E = 0, F = -1$

(4) Gi emtq cvB, $\frac{x}{x^2(x^2+1)^2} = \frac{1}{x^2} + \frac{0-1}{x^2+1} + \frac{0-1}{(x^2+1)^2} = \frac{1}{x^2} - \frac{1}{x^2+1} - \frac{1}{(x^2+1)^2}$

hv wbtYq Avsıkk fMusk |

KvR :

Avsıkk fMusk cKvk Ki :

1 $\frac{x^2 + x - 1}{x^3 + x^2 - 6x}$	2 $\frac{x^2}{x^4 + x^2 - 2}$	3 $\frac{x^3}{x^4 + 3x^2 + 2}$
4 $\frac{x^2}{(x-1)^3(x-2)}$	5 $\frac{1}{1-x^3}$	6 $\frac{2x}{(x+1)(x^2+1)^2}$

Abjxj bx 2

1| wbtPi tKvttbv i wkwU cZmg?

- (K) $a + b + c$ (L) $xy + yz + zx$ (M) $x^2 - y^2 + z^2$ (N) $2a^2 - 5bc - c^2$

2| (i) hñ $a + b + c = 0$ nq, Zte $a^3 + b^3 + c^3 = 3abc$

(ii) $P(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ i wkwj Ppμwgk

(iii) $\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{x^4-1}$ Gi mij gvb $\frac{1}{x-1}$

Dcti i Zt_ i Avtj vtK wbtPi tkvbiU mivK ?

(K) i | ii (L) ii | iii (M) i | iii (N) i, ii | ii

eüc`x $x^3 + px^2 - x - 7$ Gi GKwJ Drcv`K $x + 7$ | GB Zt_ i Avtj vtK wbtPi 3 Ges 4 bs c0kè DËi `vl |

3| P Gi gvb KZ ?

(K) -7 (L) 7 (M) $\frac{54}{7}$ (N) 477

4| eüc`xwji Aci Drcv`K, tj vi , Ydj KZ ?

(K) $(x-1)(x-1)$ (L) $(x+1)(x-2)$ (M) $(x-1)(x+3)$ (N) $(x+1)(x-1)$

5| $x^4 - 5x^3 + 7x^2 - a$ eüc`xi GKwJ Drcv`K $x - 2$ ntj , t`Lvl th, $a = 4$

6| gtb Ki , $P(x) = x^n - a^n$, thLvfb n abvZK cYmsL`v Ges a GKwJ a`eK

(K) t`Lvl th, $(x-a)$ eüc`xwji GKwJ Drcv`K Ges Ggb $Q(x)$ wbyq Ki thb $P(x) = (x-a)Q(x)$ nq|

(L) n tRvo msL`v ntj t`Lvl th, $(x+a)$ eüc`xwji GKwJ Drcv`K Ges Ggb $Q(x)$ wbyq Ki thb $P(x) = (x+a)Q(x)$ nq|

7| gtb Ki , $P(x) = x^n + a^n$, thLvfb n abvZK cYmsL`v Ges a GKwJ a`eK | n wvRvo msL`v ntj t`Lvl th, $(x+a)$ eüc`xwji GKwJ Drcv`K Ges Ggb $Q(x)$ wbyq Ki thb, $P(x) = (x+a)Q(x)$ nq|

8| gtb Ki , $P(x) = ax^5 + bx^4 + cx^3 + cx^2 + bx + a$ thLvfb a, b, c a`eK Ges $a \neq 0$, t`Lvl th, $(x-r)$ hñ $P(x)$ Gi GKwJ Drcv`K nq, Zte $(rx-1)$ | $P(x)$ Gi GKwJ Drcv`K |

9| Drcv`K wvkiY Ki :

(i) $x^4 + 7x^3 + 17x^2 + 17x + 6$

(ii) $4a^4 + 12a^3 + 7a^2 - 3a - 2$

(iii) $x^3 + 2x^2 + 2x + 1$

(iv) $x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 3xyz$

(v) $(x+1)^2(y-z) + (y+1)^2(z-x) + (z+1)^2(x-y)$

(vi) $b^2c^2(b^2 - c^2) + c^2a^2(c^2 - a^2) + a^2b^2(a^2 - b^2)$

10| hw` $\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} = \frac{3}{abc}$ nq, Zte f`Lvl th, $bc + ca + ab = 0$ A_ev, $a = b = c$

11| hw` $x = b + c - a$, $y = c + a - b$ Ges $z = a + b - c$ nq, Zte f`Lvl th,
 $x^3 + y^3 + z^3 = 4(a^3 + b^3 + c^3 - 3abc)$

12| mij Ki

(a) $\frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-c)(b-a)} + \frac{c^2}{(c-a)(c-b)}$

(b) $\frac{a}{(a-b)(a-c)(x-a)} + \frac{b}{(b-a)(b-c)(x-b)} + \frac{c}{(c-a)(c-b)(x-c)}$

(c) $\frac{(a+b)^2 - ab}{(b-c)(a-c)} + \frac{(b+c)^2 - bc}{(c-a)(b-a)} + \frac{(c+a)^2 - ca}{(a-b)(c-b)}$

(d) $\frac{1}{(1+x)} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \frac{8}{1+x^8} + \frac{16}{x^{16}-1}$

13| Avsikk fMstik cKvk Ki :

(a) $\frac{5x+4}{x(x+2)}$

(b) $\frac{x+2}{x^2-7x+12}$

(c) $\frac{x^2-9x-6}{x(x-2)(x+3)}$

(d) $\frac{x^2-4x-7}{(x+1)(x^2+4)}$

(e) $\frac{x^2}{(2x+1)(x+3)^2}$

14| Pj K x Gi GKwU euc`x $P(x) = 7x^2 - 3x + 4x^4 - a + 12x^3$

(K) euc`xi Av`kPfcwU tj L Ges GKwU ZZxq gvIvi D`ev euc`xi D`vni Y`vl |

(L) $P(x)$ euc`wU Gi GKwU Drcv`K $(x+2)$ ntj a Gi gvb wYq Ki |(M) hw` $Q(x) = 6x^3 - x^2 + 9x + 2$ Gi f`f`f` $Q\left(\frac{1}{2}\right) = 0$ nq, Zte $P(x)$ Ges $Q(x)$ Gi

mvavi Y Drcv`K `BwU wYq Ki |

15| x, y, z Gi GKwU euc`x ntj v, $F(x, y, z) = x^3 + y^3 + z^3 - 2xyz$

(K) f`Lvl th, $F(x, y, z)$ ntj v GKwU Pμ-μwgK i vnk |(L) $F(x, y, z)$ tK Drcv`tK w`tklY Ki Ges hw` $F(x, y, z) = 0$, $x + y + z \neq 0$ nq, Ztef`Lvl th, $(x^2 + y^2 + z^2) = (xy + yz + zx)$

(M) hwi` $x = b + c - a$, $y = c + a - b$ Ges $z = a + b - c$ nq, Zte t`Lvl th,
 $F(a, b, c) : F(x, y, z) = 1 : 4$

16| Pj K x Gi PviW i vnk n t j v, $(x+x), (x^2-9), (x^3+27)$ Ges (x^4-81)

(K) Dcwi D³ i vnk , t j vi n t Z GKwJ c K Z gj` f M usk Ges GKwJ Ac K Z gj` f M usk `Zwi Ki |

(L) $\frac{x^3+27}{x^2-9}$ t K m a e` Av s i k K f M us t ki m g u o i f c D c ` v c b Ki |

(M) D c t i i c l g, w Z x q Ges P Z L i v n k m g t n i c o Z t k i , Y v Z k w e c i x Z i v n k i m g u o t k m i j i f c c k v k Ki |

17| $(x+1)^3 y + (y+1)^2$ i v n k w t k

(K) x P j t k i e u c ` x i A v ` k A v K v t i e Y b v Ki Ges x P j t k i e u c ` x i f c Z v i g v t v, g L ` m n M I a e c ` w b Y q Ki |

(L) y P j t k i e u c ` x i A v ` k A v K v t i e Y b v Ki Ges y P j t k i e u c ` x i f c Z v i g v t v, g L ` I m n M I a e c ` w b Y q Ki |

(M) x I y P j t k i e u c ` x i f c w e t e P b v K t i Z v i g v t v w b Y q Ki |

ZZxq Aa'vq R'wgvZ

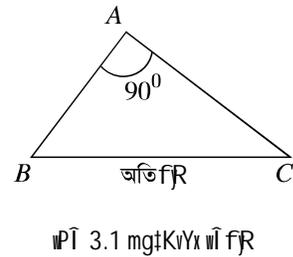
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Aa'vq tktl wk'v_Ūv -

- j x^Awf't'q|tci aviYv e'vL'v Ki tZ cvi te |
- cx_vtMvi v'tmi Dccv'' i Dci wfw'É Kti cŪ É Dccv'' ,tj v cŪvY I cŪqvM Ki tZ cvi te |
- w'f't'ri cwi tK>' , fi tK>' I j x^e'ym'útk'Z Dccv'' ,tj v cŪvY I cŪqvM Ki tZ cvi te |
- e'pv_ t'Bi Dccv'' cŪvY I cŪqvM Ki tZ cvi te |
- Utj wgi Dccv'' cŪvY I cŪqvM Ki tZ cvi te |

3 (K) cx_vtMvi vm m'útk'Z Avtj vPbv

w'f'oi R'tb'i cŪq 600 eQi AvtM weL'vZ M'K cwŪZ cx_vtMvi vm mg'tKivYx w'f'f'ri t'q|t' Gkiw AZ'š- i "ZcY'© Dccv'' (Theorem) eY'v Kti b | GB Dccv'' wŪ Zvi bvgv'v'v'ti cx_vtMvi v'tmi Dccv'' etj cwi wPZ | Rv'v hvq Zvi I cŪq 1000 eQi AvtM w'gkixq f'v'g Rwi cKvi xM't'Yi GB Dccv'' wŪ mg'tŪ aviYv wŪj | cx_vtMvi v'tmi Dccv'' we'f'v'v'te cŪvY Kiv hvq | wbgvav'gK ch'q Gi `BwŪ cŪvY t' I qv AvtQ | ZvB GLv'tb t'Kiv'tv cŪvY t' I qv n'te bv | wk'v_Ūv Gi cŪvY Aek'B wbgvav'gK R'wgvZtZ Kite | GLv'tb i agv'Ū Gi eY'v I wKQzAvtj vPbv _vK'te |



Dccv'' 3-1

cx_vtMvi v'tmi Dccv'' :

Gkiw mg'tKivYx w'f'f'ri Aw'f'f'ri Dci Aw'k'Z eM'q|t'i t'q|t' dj Aci `B ev'ui Dci Aw'k'Z eM'q|t' Ūtqi t'q|t' d'tj i mg'v'oi mg'v'b |

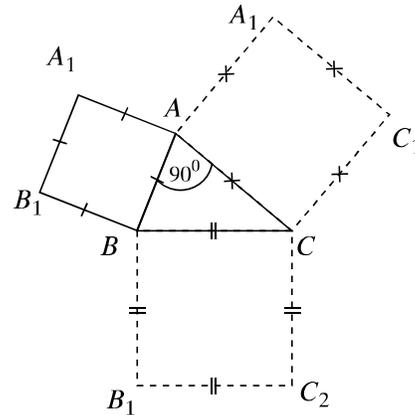
WPT 3.2 Gi $\triangle ABC$ wif fRwU GKwU mgtkvYx wif fR | $\angle BAC$ mgtkvY Ges BC AwZfR | BC AwZfRi Dci tkvfbv eMfRfTi AsKb Kitj Zvi th fRfTdj nte mgtkvY msj Mæevú0q $AB \perp AC$ Gi Dci eMfRfTi AwZfRi Kitj Zvfi fRfT dj i thvMdj Zvi mgvb nte |

A_w $BC^2 = AB^2 + AC^2$

GLvfb $BC^2 = BB_1C_2C$ eMfRfTfTi fRfTdj |

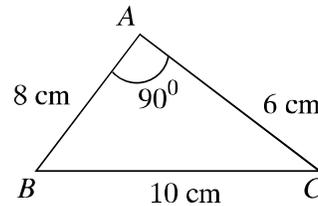
$AB^2 = AA_1B_1B$ 0 0

$AC^2 = AA_1C_1C$ 0 0



WPT : 3.2

D`vniY fjc, GKwU mgtkvYx wif fRi (WPT : 3.3) mgtkvY msj Mæevú0tqi N° h_vmtg 8 tm.wg | 6 tm.wg. ntj cx_vfMvifmi Dccvfi gva`g mnfRB ejv hvq Gi AwZfRi N° 10 tm.wg nte | Abjfcfvte, thtkvfbv `B evui N° gva`tg ZZxq evui N° Rvbn mæe | wbtgæ Dccvfi wU cx_vfMvifmi Dccvfi wecixZ c0ZÁv wnmvte cwipZ |

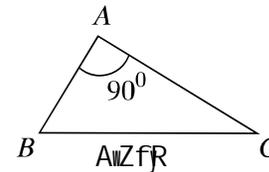


WPT : 3.3

Dccvfi 3.2

tkvfbv wif fRi GKwU evui Dci AwZfR eMfRfTfTi fRfTdj Aci `B evui Dci AwZfR eMfRfTfTfTqi fRfTdj i mgw0i mgvb ntj tkflv³ evn0tqi AšfB tkvYwU mgtkvY nte | (WPT : 3.4) j R Ki |

$\triangle ABC$ Gi BC evü AwZfR Ges Aci `B evü h_vmtg $AB \perp AC$.



WPT : 3.4

BC evui Dci AwZfR eMfRfTfTi fRfTdj Aci `B evü h_vmtg $AB \perp AC$ evui Dci AwZfR eMfRfTfTfTqi fRfTdj i mgw0i mgvb |

A_w, $BC^2 = AB^2 + AC^2$

myZivs, $\angle BAC$ GKwU mgtkvY |

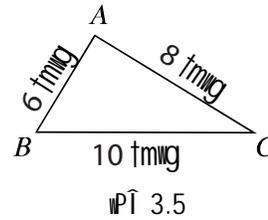
D`vniY fjc Avgiv ejfZ cwii $\triangle ABC$ Gi $AB, BC \perp CA$ evui N° h_vmtg 6 tm.wg 10 tm.wg | 8 tm.wg nq Zvntj $\angle BAC$ Aek`B mgtkvY nte | thtnZi $AB^2 = 6^2$ e. tm. wg. = 36 e. tm. wg.

$$BC^2 = 10^2 \text{ e. tm. wg.} = 100 \text{ e. tm. wg.}$$

$$AC^2 = 8^2 \text{ e. tm. wg.} = 64 \text{ e. tm. wg.}$$

$$\therefore BC^2 = 100 = 36 + 64 = AB^2 + AC^2.$$

$$\therefore \angle BAC = 90^\circ = \text{mg}\ddot{\text{t}}\text{KvY}$$

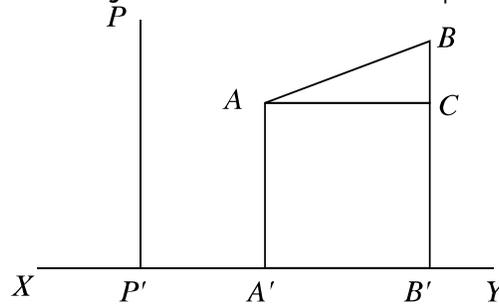


3 (L) j α Awf\ddot{t}q|c (Orthogonal Projection)

wex`y j α Awf\ddot{t}q|c : tKv\ddot{t}bv wbow`0 mij ti Lvi Dci tKv\ddot{t}bv wex`y j α Awf\ddot{t}q|c ej tZ tmB wex`yt_tK D³ wbow`0 ti Lvi Dci Aw\ddot{t}Z j t\ddot{t} cv` wex` tK e\ddot{t}vq|

gtbKwi, XY GKwU wbow`0 mij ti Lv Ges P th\ddot{t}Kv\ddot{t}bv wex`y (wPÎ 3.6) | P wex`yt_tK XY ti Lvi Dci Aw\ddot{t}Z j α PP' Ges j α PP' Gi cv` wex`y P' |

mZivs, P' wex`y XY ti Lvi Dci P wex`y j α Awf\ddot{t}q|c | A_w tKv\ddot{t}bv wbow`0 ti Lvi Dci tKv\ddot{t}bv wex`y j α Awf\ddot{t}q|c GKwU wex`y Avgiv G avibv t_tK ej tZ cwii tKv\ddot{t}bv mij ti Lvi Dci j α th\ddot{t}Kvb mij ti Lvi j α Awf\ddot{t}q|c GKwU wex`y tm t\ddot{t}tÎ D³ j α Awf\ddot{t}q|tci ``N⁰nte kb`|



wPÎ : 3-6 wbow`0 ti Lv XY Gi Dci tKv\ddot{t}bv wex`y P Ges ti Lvsk AB Gi j α Awf\ddot{t}q|c|

ti Lvst\ddot{t}ki j α Awf\ddot{t}q|c :

awi, AB ti Lvst\ddot{t}ki c0s`wex`0q A | B (wPÎ : 3.6) | GLb A | B wex`yt_tK XY ti Lvi Dci Aw\ddot{t}Z j α h_vK\ddot{t}g AA' | BB' | AA' j t\ddot{t} cv` wex`y A' Ges BB' j t\ddot{t} cv` wex`y B' | GB A'B' ti LvskB nt`Q XY ti Lvi Dci AB ti Lvst\ddot{t}ki j α Awf\ddot{t}q|c |

mZivs, t` Lv hv\ddot{t}Q j α A\ddot{t}tbi gva\ddot{t}g Awf\ddot{t}q|c wbyq Kiv nq| Ziv A'B' ti Lvst\ddot{t}k\ddot{t}K XY ti Lvi Dci AB ti Lvst\ddot{t}ki j α Awf\ddot{t}q|c (Orthogonal Projection) ej v nq|

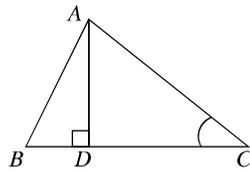
j q|Yxq :

1 | tKv\ddot{t}bv ti Lvi Dci tKvb wex`yt_tK Aw\ddot{t}Z j t\ddot{t} cv` wex`0 H wex`y j α Awf\ddot{t}q|c |

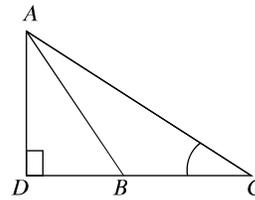
2 | tKv\ddot{t}bv ti Lvi Dci j α ti Lvi j α Awf\ddot{t}q|c GKwU wex`y dtj j α Awf\ddot{t}q|tci ``N⁰kb`|

3 | tKv\ddot{t}bv wbow`0 ti Lvi mgv\ddot{t}vj ti Lvst\ddot{t}ki j α Awf\ddot{t}q|c H ti Lvst\ddot{t}ki mgvb nte|

wPÎ 3.6 G AB ti Lvsk XY Gi mgv\ddot{t}vj ntj AB = A'B' nte|



ŵPĪ : 3·8 (K)



ŵPĪ : 3·8 (L)

cġvY : $\triangle ABD$ Gi $\angle ADB$ mgġKvY |

$$\therefore AB^2 = AD^2 + BD^2 \text{ [cx_vġMvi vġmi Dccv`"} \dots\dots (1)$$

cġg ŵPĪ $BD = BC - DC$

ŵZxq ŵPĪ $BD = DC - BC$

$$\begin{aligned} \therefore \text{Dfqt} \text{ġĪ} \quad BD^2 &= (BC - DC)^2 = (DC - BC)^2 \\ &= BC^2 + DC^2 - 2BC \cdot DC \\ &= BC^2 + CD^2 - 2BC \cdot CD \quad [\because CD = DC] \end{aligned}$$

$$\therefore BD^2 = BC^2 + CD^2 - 2BC \cdot CD \dots\dots(2)$$

GLb mgxKiY (1) | (2) nġZ cvl qv hvq

$$AB^2 = AD^2 + BC^2 + CD^2 - 2BC \cdot CD$$

$$\text{ev, } AB^2 = AD^2 + CD^2 + BC^2 - 2BC \cdot CD \dots\dots(3)$$

Avevi $\triangle ADC$ mgġKvYx ŵĪ fġR Ges $\angle D$ mgġKvY

$$\therefore AC^2 = AD^2 + CD^2 \text{ [cx_vġMvi vġmi Dccv`"} \dots\dots (4)$$

mgxKiY (3) | (4) nġZ cvB,

$$AB^2 = AC^2 + BC^2 - 2BC \cdot CD. \quad [cġvYZ]$$

ŵe. `ġ : C ŵe> yġ_ġK AB Gi Dci j α Aġġbi gvaġġg GKB fvġe Dccv`"ŵ cġvY Kiv hvq |

cx_vġMvi vġmi Dccv`", j α Awfġġc Ges cx_vġMvi vġmi Dccv`"i Dci wfvĒ Kġi ewYZ Dccv`"mgġ nġZ j ġYxq ŵel q Ges ŵm×vŠ-mgġ |

j ġYxq :

1 | mgġKvYx ŵĪ fġRi ġġġġ mgġKvġYi mġbunZ evġġq ci ŵi j α ŵeavq Zvġi cġZ"Kŵi j α Awfġġc kb" | mġZivs $BC \cdot CD = 0$.

2 | Dccv`" 3·3 | Dccv`" 3·4, Dccv`" 3·1 Gi wfvĒi Dci cġZŵóZ | ZvB Dccv`" 3·3 | Dccv`" 3·4 ġK Dccv`" 3·1 A_ġ cx_vġMvi vġmi Dccv`"i Abyġm×vŠ-ej v hvq |

Dctiv³ Avġj Pbv mġctġġ MġnZ ŵm×vŠmgġ :

$\triangle ABC$ Gi ġġġġ,

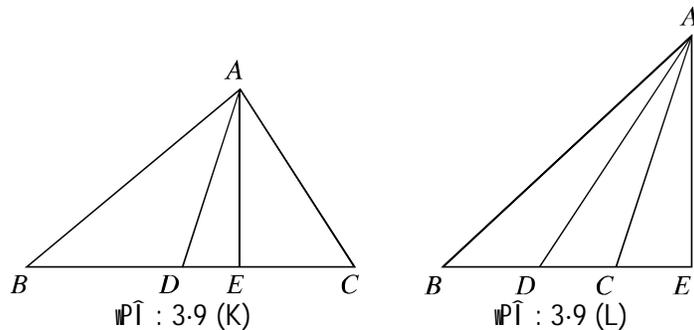
1 | $\angle C$ ġġġKvY nġj ,

$$AB^2 > AC^2 + BC^2 \quad [Dccv`" 3·3]$$

- 2| $\angle C$ mg#KvY ntj ,
 $AB^2 = AC^2 + BC^2$ [Dccv`" 3-1]
- 3| $\angle C$ m#KvY ntj ,
 $AB^2 < AC^2 + BC^2$ [Dccv`" 3-4]

wbtgpe³ Dccv`"wU cx_v#Mvivotmi Dccv#`"i we`wi A_# Dccv`" 3.3 I Dccv`" 3.4 Gi Dci wfwE Kti c#Zw#Z | GB Dccv`"wU G`vtctj vmbqvm KZ# evYZ etj GuU G`vtctj vmbqv#mi Dccv`" bvtg cwi wPZ |

Dccv`" 3-5 (G`vtctj vmbqv#mi Dccv`")
 w# f#Ri th#Kv#v# `B ev#i Dci Aw#Z eM#q#i#tqi t#q#dtj i mgw#, ZZxq ev#i A#f#Ki I ci Aw#Z eM#q#i#t i t#q#dj Ges H ev#i mgw#LE#K ga`gvi I ci Aw#Z eM#q#i#t i t#q#dtj i mgw#i w#_Y |
 we#kl wePb : $\triangle ABC$ Gi AD ga`gv BC ev#K mgw#Lw#Z Kti#Q |
 c#yY Ki#Z n#e th, $AB^2 + AC^2 = 2(AD^2 + BD^2)$



A#b : BC ev#i Dci (wP# : 3-9 (K)) Ges BC ev#i ewaZv#tki (wP# 3-9 (L)) AE j #A#b Kwi |

c#yY : $\triangle ABD$ Gi $\angle ADB$ #KvY Ges BD tiLvi ewaZv#tki Dci AD tiLvi j #Awf#q#c DE [Df#q wP#i#]

\therefore #Kv#Yi t#q#i# cx_v#Mvivotmi Dccv#`"i we`wZ Abymv#i [Dccv`" 3-3]

Avgiv cvB, $AB^2 = AD^2 + BD^2 + 2 BD \cdot DE$(1)

Aevi, $\triangle ACD$ Gi $\angle ADC$ m#KvY Ges DC tiLvi (wP# 3.9 (K)) Ges DC tiLvi ewaZv#tki (wP# 3.9 (L)) Dci AD tiLvi j #Awf#q#c DE .

\therefore m#Kv#Yi t#q#i# cx_v#Mvivotmi Dccv#`"i we`wZ Abymv#i (Dccv`" 3.4) cvB,

$AC^2 = AD^2 + CD^2 - 2CD \cdot DE$(2)

GLb mgxKiY (1) I (2) thwM Kti cvB,

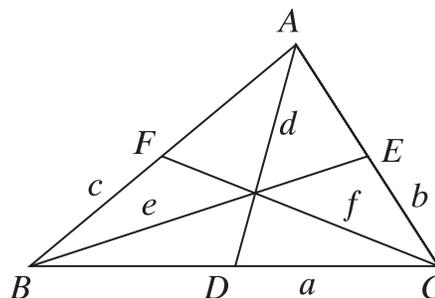
$$\begin{aligned}
 AB^2 + AC^2 &= 2AD^2 + BD^2 + CD^2 + 2BD \cdot DE - 2CD \cdot DE \\
 &= 2AD^2 + BD^2 + BD^2 + 2BD \cdot DE - 2BD \cdot DE; \quad [\because BD = CD] \\
 &= 2AD^2 + 2BD^2 \\
 &= 2(AD^2 + BD^2). \quad [c\hat{o}wYZ]
 \end{aligned}$$

wm×vŠ-: G'v'cv'tj wbcv'tmi Dccv'`i gva'tg wĭ f'f'ri evû l ga'gvi m'p'úK'q'byĕ |
 g'tb Kwĭ, ΔABC Gi BC, CA | AB evûi %N'©h_v'μ'tg a, b | c | BC, CA | AB evûi Dci
 Aw'4Z ga'g'v AD, BE | CF Gi ^N'©h_v'K'tg d, e | f.
 Zvntj, G'v'cv'tj wbcv'tmi Dccv'` n'tZ cvB,

$$\begin{aligned}
 AB^2 + AC^2 &= 2(AD^2 + BD^2) \\
 \text{ev, } c^2 + b^2 &= 2\left(d^2 + \left(\frac{1}{2}a\right)^2\right) \quad \left[\because BD = \frac{1}{2}a\right] \\
 \text{ev, } b^2 + c^2 &= 2d^2 + 2 \cdot \frac{1}{4}a^2 \\
 \text{ev, } b^2 + c^2 &= 2d^2 + \frac{a^2}{2} \\
 \text{ev, } d^2 &= \frac{2(b^2 + c^2) - a^2}{4}
 \end{aligned}$$

Abj'fc'v'te cvl qv hvq,

$$\begin{aligned}
 e^2 &= \frac{2(c^2 + a^2) - b^2}{4} \\
 \text{Ges } f^2 &= \frac{2(a^2 + b^2) - c^2}{4}
 \end{aligned}$$



∴ t'Kv'tbv wĭ f'f'ri evûi ^N'©R'v'v_v'K'tj ga'g'v'mg'f'ni ^N'©R'v'v hvq |
 Avevi,

$$\begin{aligned}
 d^2 + c^2 + f^2 &= \frac{2(b^2 + c^2) - a^2}{4} + \frac{2(c^2 + a^2) - b^2}{4} + \frac{2(a^2 + b^2) - c^2}{4} \\
 &= \frac{3}{4}(a^2 + b^2 + c^2)
 \end{aligned}$$

$$\therefore 3(a^2 + b^2 + c^2) = 4(d^2 + c^2 + f^2).$$

m'Z'ivs ej v hvq t'Kv'tbv wĭ f'f'ri wZ'bwĭ evûi Dci Aw'4Z eM'f'f'ĭ mg'f'ni t'f'ĭ d'tj i mg'w'oi wZ'b_Y D³
 wĭ f'f'ri ga'g'v ĩt'qi Dci Aw'4Z eM'f'f'ĭ mg'f'ni t'f'ĭ d'tj i mg'w'oi Pvi _t'Yi mg'v |

wĭ f'f'rwĭ mg't'Kv'Yx A_ŕ ∠C = mg't'Kv'Y Ges AB Aw'Z'f'R n'tj

$$c^2 = a^2 + b^2$$

$$\therefore a^2 + b^2 + c^2 = 2c^2$$

$$\text{ev, } \frac{4}{3}(d^2 + c^2 + f^2) = 2c^2$$

$$\text{ev, } 2(d^2 + c^2 + f^2) = 3c^2.$$

mYivs, ejv hvq mgtKvYx wlfRi ga'gvltqi Dci Aw4Z eMfRi mgfni tRi dtji mgwoi w0_Y AwZfRi Dci Aw4Z eMfRi i tRi dtji wZY_tYi mgvb |

Abkxj bx 3.1

- 1) $\triangle ABC$ Gi $\angle B = 60^\circ$ ntj cgvY Ki th, $AC^2 = AB^2 + BC^2 - AB \cdot BC$
- 2) $\triangle ABC$ Gi $\angle B = 120^\circ$ ntj cgvY Ki th, $AC^2 = AB^2 + BC^2 + AB \cdot BC$
- 3) $\triangle ABC$ Gi $\angle B = 90^\circ$ Ges BC Gi ga'we`y D | cgvY Ki th, $AB^2 = AD^2 + 3BD^2$
- 4) $\triangle ABC$ G AD, BC evui Dci j a^Ges BE, AC Gi Dci j a^ | t`Lvl th, $BC \cdot CD = AC \cdot CE$
- 5) $\triangle ABC$ Gi BC evu $P | Q$ we`fZ wZbw mgvb Astk wef³ ntqtQ | cgvY Ki th, $AB^2 + AC^2 = AP^2 + AQ^2 + 4PQ^2$.
[mstKZ : $BP = PQ = QC$; $\triangle ABQ$ Gi ga'gv AP .
 $AB^2 + AQ^2 = 2 \cdot (BP^2 + AP^2) = 2PQ^2 + 2AP^2$
 $\triangle APC$ Gi ga'gv AQ ,
 $AP^2 + AC^2 = 2PQ^2 + 2AQ^2$]
- 6) $\triangle ABC$ Gi $AB = AC$ | fwg BC Gi Dci P thtKvfbv we`y | cgvY Ki th, $AB^2 - AP^2 = BP \cdot PC$.
[mstKZ : BC Gi Dci AD j a^AwK Zvntj $AB^2 = BD^2 + AD^2$ Ges $AP^2 = PD^2 + AD^2$]
- 7) $\triangle ABC$ Gi ga'gvltq G we`fZ wgwj Z ntj cgvY Ki th, $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$
[mstKZ : G'vtvtj vbcvtmi Dccv` i Avtj vtK MpxZ wmxvS-mgn t` LtZ nte A_@, wlfRi evui N^I ga'gvi m^uK^ LtZ nte]

3 (M) wlfRi I eE welqK Dccv`

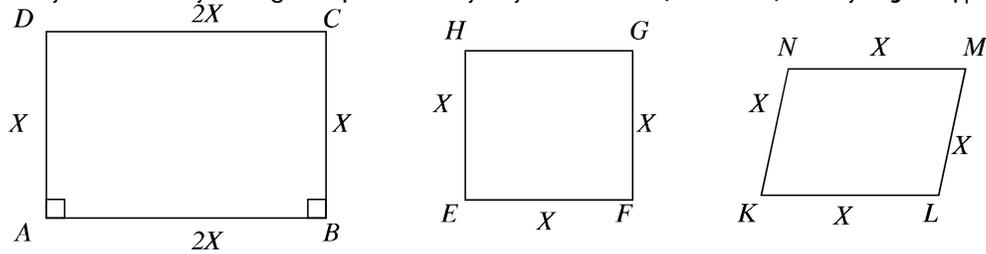
GB Astk wlfRi I eE welqK KtqKw jZcyDccv` i hv³gj K cgvY Dc`vcb Kiv nte | Dccv` mgn cgvYi Rb` Bw wlfRi m`kZv m^uK ceAvb_vKv Avek`K | gra'wgK R'wgvZtZ wlfRi m`kZv m^uK we`wv Z Avtj vPbv Kiv ntqtQ | GB Dccv` t`jv cgvYi cteK_R'v wlfRi m`kZv m^uK tRb wote | wkv` i m^eav` wlfRi m`kZv m^uK mstRtC Avtj vPbv Kiv ntjv |

tKvYi t¶¶t m`kZv : mgvb msL`K evuwnkó `Bw eufRi GKwI tKvY,tjv hw` avivwnKfvte AciwI tKvY,tjvi mgvb nh, Zte eufR `BwtK m`kKvYx eufR ejv nq|

evüi AbjfcZi t¶¶t m`kZv : mgvb msL`K evuwnkó `Bw eufRi GKwI kxle`y,tjvtK hw` avivwnKfvte AciwI kxle`y,tjvi mt½ Ggbfvte wj Kiv hvq th, eufR `Bwi -

(1) Abjfc tKvY,tjv mgvb nq Ges

(2) Abjfc `Bw evüi AbjfcZ mgvb nq, Zte eufR `BwtK m`k (Similar) eufR ejv nq|



w¶ 3.10

Dctii w¶t j ¶¶ Ki tj t`Le th,

(1) AvqZ ABCD | eM°EFGH m`k bq hw` | Zviv m`kKvYx|

(2) eM°EFGH | i°m KLMN m`k bq hw` | Zvt`i kxle`y,tjvi thtKvfbv avivwnK wj Ki tYi dtj Abjfc evü `Bwi AbjfcZ,tjv mgvb nq|

`Bw w¶fvRi tejvq Aek` G iKg nq bv| `Bw w¶fvRi kxle`y,tjvi tKvY wj Ki tYi dtj hw` m°v,vi msAvq DttwLZ kZ° Bwi GKwI mZ` nq, Zte AciwI mZ` nq Ges w¶fvR `Bw m`k nq|

G cñt½ DttwL` th,

(1) `Bw w¶fvR m`kKvYx ntj mgvb tKvY `BwtK Abjfc tKvY Ges Abjfc tKvYi wccixZ evü `BwtK Abjfc evü aiv nh|

(2) `Bw w¶fvRi GKwI wZb evü AciwI wZb evüi mgvbcwZK ntj, AvbcwZK evü `BwtK Abjfc evü Ges Abjfc evüi wccixZ tKvY `BwtK Abjfc tKvY aiv nq|

(3) Dfvtt¶¶t Abjfc tKvY,tjvi kxle`y wj Kti w¶fvR `Bw eY°v Kiv nq| thgb, $\Delta ABC | \Delta DEF$ Gi Abjfc tKvY,tjv nt°Q $\angle A | \angle D, \angle B | \angle E, \angle C | \angle F$ Ges Abjfc evü,tjv nt°Q $AB | DE, AC | DF, BC | EF.$ |

`Bw w¶fvRi m`kZv m°vKZ KtqKw Dccvt`i msw¶B eY°v t`l qv ntj v|

Dccv` 3-6

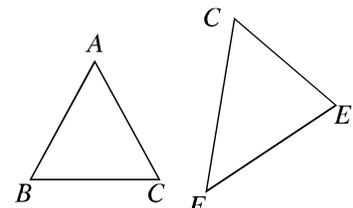
`Bw w¶fvR m`kKvYx ntj Zvt`i Abjfc evü,tjv mgvbcwZK nte|

cvtk¶ w¶t $\Delta ABC | \Delta DEF$ m`kKvYx w¶fvR|

A_¶, $\angle A = \angle D, \angle B = \angle E$ Ges $\angle C = \angle F.$ nI qvq

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

nte| A_¶ Abjfc evü,tjv mgvbcwZK nte|



w¶ : 3-11

Abjmxvš: `Bw wî fR m`kKvYx ntj , Zviv m`k nq|
 gše` : `Bw wî fRi GKwI `B tKvY AciwI `B tKvYi mgvb ntj wî fR `Bw m`kKvYx Ges Gi dtj
 G,tjv m`k nq| KviY thtKvYi wî fRi wZb tKvYi mgwó `B mgKvY|

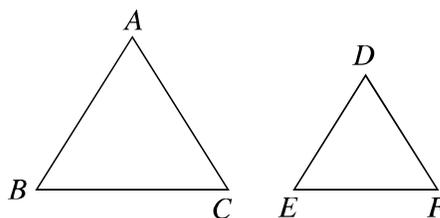
Dccv` 3.7

`Bw wî fRi evú,tjvi mgvbcwZK ntj Abjfc evúi weciXZ
 tKvY,tjv ci`úi mgvb nq|

cvtkP wPÎ $\triangle ABC \parallel \triangle DEF$ Gi evú,tjv mgvbcwZK A_ŕ,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \text{ nI qvq wî fR} \theta \text{tqi tKvY,tjv ci`úi}$$

mgvb| A_ŕ, $\angle A = \angle D, \angle B = \angle E$ Ges $\angle C = \angle F.$ |



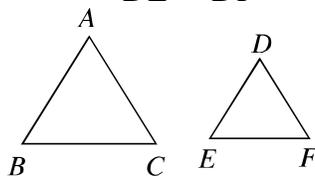
wPÎ 3.12

Dccv` 3.7 tK Dccv` 3.6 Gi weciXZ wmwteI ejv thtZ cvti |

Dccv` 3.8

`Bw wî fRi GKwI GK tKvY AciwI GK tKvYi mgvb Ges mgvb tKvY msj Mæevú,tjv mgvbcwZK
 ntj wî fR `Bw m`k nte|

cvtkP wPÎi (wPÎ : 3.13) $\triangle ABC \parallel \triangle DEF$ Gi $\angle A = \angle D$ Ges mgvb tKvY msj Mæ evú0q
 AB, AC Ges $DE \parallel EF$ mgvbcwZK| A_ŕ, $\frac{AB}{DE} = \frac{AC}{DF}$ nI qvq $\triangle ABC \parallel \triangle DEF$ m`k|



wPÎ : 3.13

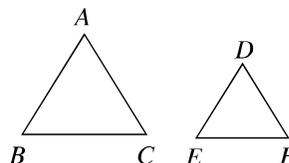
Dccv` 3.9

`Bw m`k wî fRtŕtŕi tŕŕI dtj 0tqi AbcvZ Zvt`i thtKvYi `B Abjfc evúi I ci AwZ eMŕtŕi
 tŕŕI dtj 0tqi AbcvZi mgvb|

cvtkP wPÎi $\triangle ABC \parallel \triangle DEF$ wî fR0q m`k| wî fR `BwI Abjfc evú $BC \parallel EF$ | GB Ae`vq
 wî fR0tqi tŕŕI dtj AbcvZ $BC \parallel EF$ evú0tqi I ci AwZ eMŕtŕi tŕŕI dtj 0tqi AbcvZi mgvb|

$$A_ŕ \frac{\triangle ABC}{\triangle DEF} = \frac{BC^2}{EF^2} |$$

wî fRi tŕŕI Dctiv³ Avtj vPbv I Dccv` mgŕtK wfvE Kti w0tgv³
 Dccv` mgŕni hv³ gj K c0vY Dc`vcb Kiv ntj v|

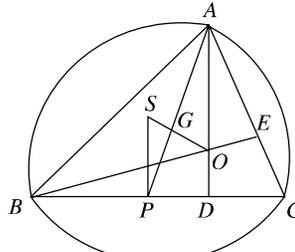


wPÎ : 3.14

Dccv` 3.10

wl ffrRi cwitk>` ; fitk>` i j r^we` ymgfLl

wetkl wePb : gtb KwI, ΔABC Gi j r^we` y O cwitk>` a S Ges AP GKwU ga`gv | j r^we` y O Ges cwitk>` a S Gi msthM tiLv AP ga`gvK G we` tZ tQ` KtitQ | S, P thvM Kitj SP tiLv BC Gi Dci j r^f Zvntj , G we` yU ΔABC Gi fitk>` a GwU cgvY KivB ht_ó nte |



wPÍ 3.15

cgvY : Avgiv Rvnb, tKvibv wl ffrRi j r^we` y t_tK kxtlP` i Z; wl ffrRi cwitk>` a t_tK H kxtlP` wecixZ evüi w_ Y | ΔABC Gi j r^we` y O t_tK A kxtlP` i Z; OA Ges cwitk>` a S t_tK A kxtlP` wecixZ evü BC Gi i Z; SP |

∴ OA = 2SP (1)

GLb thtnZl AD I SP DfqB BC Gi lci j r^atmtnZl AD II SP.

GLb AD II SP Ges AP Gf` i tQ` K |

∴ ∠PAD = ∠APS [GKvšt tKvY]

A_w, ∠OAG = ∠SPG

GLb ΔAGO Ges ΔPGS Gi gta`

∠AGO = ∠PGS [wecZxc tKvY]

∠OAG = ∠SPG [GKvšt tKvY]

∴ Aewkó ∠AOG = Aewkó ∠PSG

∴ ΔAGO Ges ΔPGS m` k tKvYx |

mZi vs, $\frac{AG}{GP} = \frac{OA}{SP}$

ev, $\frac{AG}{GP} = \frac{OA}{SP}$

ev, $\frac{AG}{GP} = \frac{2SP}{SP}$ [(1) bs mgxKi Y nřZ]

ev, $\frac{AG}{GP} = \frac{2}{1}$

$\therefore AG : GP = 2 : 1$

A_ŕ, G we`y AP ga`gvK 2:1 AbvcZ wef³ Kti tQ|

$\therefore G$ we`y ΔABC Gi fi tK`q (cŕmYZ)

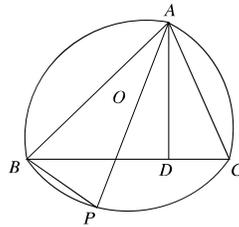
`be` : (1) bewe`pE (Nine Point Circle) : tKv tbn wĭ fĕRi evũ,tj vi ga`we`jĭq, kxl we`y,tj v t_tK wecixZ evũtqi Dci AwZ j`ftqi cv` we`jĭq Ges kxl we`y l j`we`j msthvRK ti Lvĭtqi ga`we`jĭq, meŕgvU GB bqvU we`yGKB eĕEi Dci Ae`vb Kti | GB eĕEKB bewe`pE etj |

(2) wĭ fĕRi j`we`y l cwi tK`a msthvRb Kti Drcbæmng mij ti Lvi ga`we`p bewe`yetEi tK`q

(3) bewe`yetEi e`vmaŕwĭ fĕRi cwi e`vmtaŕ A taŕKi mgvb |

Dccv` 3.11 (eŕv ,tBi Dccv`)

tKv tbn wĭ fĕR th tKv tbn `p evũi AšMZ AvqZ tĕtĭi tĕtĭdj wĭ fĕRi cwi eĕEi e`vm Ges H evũtqi mvavi Y we`y t_tK fvgi Dci AwZ j`ft AšMZ AvqZ tĕtĭi tĕtĭdj i mgvb |



wĕT : 3.16

wetkl wepB : gtb Kwi, ABC wĭ fĕRi cwi tK`a O Ges AP cwi eĕEi GKwU e`vm | ΔABC Gi kxl A t_tK wecixZ evũ BC Gi Dci AD j`ft

cŕmY Ki tZ nte th, $AB \cdot AC = AP \cdot AD$.

A/b : B, P thvM Kwi |

cŕmY : GKB Pvc AB Gi Rb` $\angle APB \mid \angle ACD$ eĕvskw`Z tKvY | AP eĕEi e`vm etj $\angle ABP$ Aaŕeĕ` tKvY Ges BC evũi Dci AD j`ft n l qvq $\angle ADC$ mg tKvY |

GLb $\Delta APB \mid \Delta ADC$ Gi gta` $\angle APB = \angle ACD$ [GKB eĕvskw`Z tKvY mgvb |]

$\angle ABP = Aaŕeĕ` tKvY = GK mg tKvY = \angle ADC$.

\therefore Aenkó $\angle BAP = Aenkó \angle CAD$.

$\therefore \Delta ABP \mid \Delta ADC$ m`k tKvYx |

$\therefore \frac{AB}{AD} = \frac{AP}{AC}$

A_ŕ, $AB \cdot AC = AP \cdot AD$. [cŕvYZ]

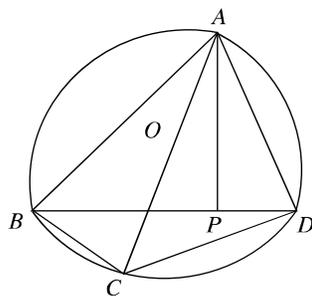
j ŕYxq : $\triangle ABC$ Gi cwieŕĒi e'vma[©]R ntj , $R = \frac{1}{2} AP$

A_ŕ, $AP = 2R$.

∴ Dcŕi Dccv` t_ŕK cvl qv hvq $AB \cdot AC = 2R \cdot AD$.

Dccv` 3.12 (Uŕj wgi Dccv`)

eŕĒ Aŕj ŕLZ tKvŕbv PZŕŕi KYŕqi AŕMZ AvqZŕŕĪ H PZŕŕi wecixZ evŕŕtqi AŕMZ AvqZŕŕĪ i mgvŕi mgvb |



ŕPĪ : 3.17

wetkl wbePb : gŕb KwĪ eŕĒ Aŕj ŕLZ ABCD PZŕŕi wecixZ evŕŕtqv h_vŕŕtg $AB \perp CD$ Ges $BC \perp AD \perp AC$ Ges BD PZŕŕi ŕBŕW KYŕ cŕvY KiŕZ nŕe th,

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$

A¼b : $\angle BAC$ tK $\angle DAC$ Gi tŕvU aŕi wŕtq A we` ŕZ AD ti Lvŕŕki mŕŕ_ $\angle BAC$ Gi mgvb Kŕi $\angle DAP$ Awk thb AP ti Lv BD KYŕK P we` ŕZ tŕ` Kŕi |

cŕvY : A¼b Abŕvŕŕi $\angle BAC = \angle DAP$

Dfŕcŕŕŕ $\angle CAP$ thvM Kŕi cvB,

$$\angle BAC + \angle CAP = \angle DAP + \angle CAP$$

A_ŕ, $\angle BAP = \angle CAD$

GLb $\triangle ABP$ | $\triangle ACD$ Gi gŕa`

$$\angle ADP = \angle ACD$$

$$\angle ABD = \angle ACD \text{ [GKB eĒvskw`Z tKvY mgvb eŕj]}$$

Ges Awkŕ $\angle APB = \text{Awkŕ } \angle ADC$

∴ $\triangle ABP \sim \triangle ACD$ m` kŕKvYx |

$$\therefore \frac{BP}{CD} = \frac{AB}{AC}$$

$$A_ŕ, AC \cdot BP = AB \cdot CD \dots\dots\dots (1)$$

Avevi, $\triangle ABC \parallel \triangle APD$ Gi gta"

$\angle BAC = \angle PAD$ [A/b Abymt'i]

$\angle ADP = \angle ACB$ [GKw e'vskw'Z tKvY mgvb etj]

Ges Aewkó $\angle ABC =$ Aewkó $\angle APD$

$\therefore \triangle ABC \parallel \triangle APD$ m`k'tKvYx|

$\therefore \frac{AD}{AC} = \frac{PD}{BC}$

A_# , $AC \cdot PD = BC \cdot AD$ (2)

GLb mgxKiY (1) I (2) thvM K'ti cvB,

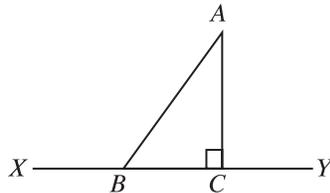
$AC \cdot BP + AC \cdot PD = AB \cdot CD + B \cdot AD$

ev, $AC(BP + PD) = AB \cdot CD + BC \cdot AD$

A_# , $AC \cdot BD = AB \cdot CD + BC \cdot AD$ [th'tnZl $BP + PD = BD$] [c'gvwYZ]

Abkxj bx 3.2

1|

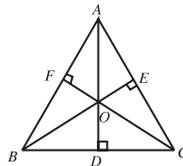


XY ti Lustk AB Gi j a^Awf't'c wbt'Pi tKvbwU?

- K. AB
- M. AC

- L. BC
- N. XY

2|



I c'ti i wP't' tKvbwU j a^we`?

- K. D
- M. F

- L. E
- N. O

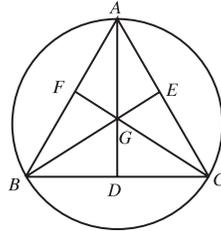
3| i w'f'f'ri ga'gv't'qi tQ` we`'K fi tK`'etj |

ii fi tK`'th'tKv't'bv ga'gv't'K 3:1 Abj'vt'Z wef`³ K'ti |

iii m`k tKvYx w'f'f'ri Abj'fc evu`'tj v mgv'bcwZK wbt'Pi tKvbwU mwVK ?

- K. i I ii
- M. i I iii

- L. ii I iii
- N. i, ii I iii



- D, E, F h_vμtḡ BC, AC l AB Gi ga'we`yntj l cti i wPti i Avtj vtK 4-6 bs cḡkē DĒi `vl :
- 4| G we`j bvg wk?
- K. j^we`y L. Aš:tk>`^a
- M. fi tk>`^a N. cwi tk>`^a
- 5| $\triangle ABC$ Gi kxl qe`yw tq Aw/Z eĒi bvg wk?
- K. cwi eĒ L. Aš:eĒ
- M. ein:eĒ N. bewe`yeĒ
- 6| $\triangle ABC$ Gi tqtĪ wbtPi tKvbW G'vctvj wbcvtmi Dccv`tk mg_ḡ Kti?
- K. $AB^2 + AC^2 = BC^2$
- L. $AB^2 + AC^2 = 2(AD^2 + BD^2)$
- M. $AB^2 + AC^2 = 2(AG^2 + GD^2)$
- N. $AB^2 + AC^2 = 2(BD^2 + CD^2)$
- 7| ABC wĪ ftRi cwi eĒ` th tKvbtv P we`yt_tk BC l CA Gi Dci PD l PE j^A/b Kiv n tq tq | hw` ED ti Lvsk AB tk O we`fZ tḡ` Kti, Zte cḡvY Ki th, PO ti Lv AB Gi Dci j^A PO \perp AB.
- 8| $\triangle ABC$ Gi $\angle C$ mgtkvY | C t_tk Aw/Z ftRi Dci Aw/Z j^A CD ntj, cḡvY Ki th, $CD^2 = AD \cdot BD$.
- 9| $\triangle ABC$ Gi kxl q t_tk wecxZ evu_tjvi lci j^A AD, BE l CF ti Lv tq O we`fZ tḡ` Kti | cḡvY Ki th, $AO \cdot OD = BO \cdot OE = CO \cdot OF$. [mstKZ : $\triangle BOF$ Ges $\triangle COE$ m`k | $\therefore BO : CO = OF : OE$]
- 10| AB evtmi lci Aw/Z AaĒĒi `Bw Rv AC l BD ci`ui P we`fZ tḡ` Kti | cḡvY Ki th, $AB^2 = AC \cdot AP + BD \cdot BP$
- 11| tKvbtv mgevú wĪ ftRi cwi eĒi e`vmva^{3.0} tm.wg. ntj H wĪ ftRi evú `N`wbYq Ki |

12| ABC mgwv evu wif firi kxle y A nZ fig BC Gi lci AwZ j AD Ges wif firi cwie vma^oR ntj cgvY Ki th, $AB^2 = 2R \cdot AD$. [e²v₃ tBi Dccv^t " $AB = AC$]

13| ABC wif firi $\angle A$ Gi mgwv LUK BC tK D we^t Z Ges ABC cwie tE tK E we^t Z tQ^t Kt itQ | t^t Lvl th, $AD^2 = AB \cdot AC - BD \cdot DC$.

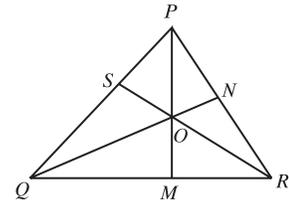
14| ABC wif firi $AC \perp AB$ evu lci h_vvtg $BE \perp CF$ j AF t^t Lvl th,
 $\Delta ABC : \Delta AEF = AB^2 : AE^2$.

15| ΔPQR -G $PM, QN \perp RS$ ga^g vI q O we^t Z tQ^t Kt itQ |

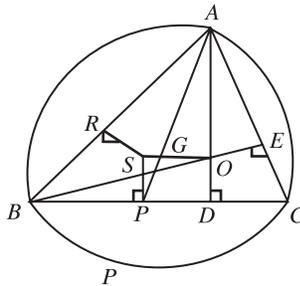
K. O we^t yUi big uk? O we^t y PM tK uk Abcv tZ we f³ Kt it?

L. ΔPQR ntZ $PQ^2 + PR^2 = 2(PM^2 + QM^2)$ m^u K^u c^u Z^u Ki |

M. t^t Lvl th, ΔPQR -Gi evu wZb_u i e tM^o mgwv O we^t yntZ kxle y wZb_u i t^t Zj e tM^o mgwv i wZb_u Y |



16|



l cti i wP t^t S, O h_vvtg cwie tK^t " l j AP ga^g v, $BC = a, AC = b$ Ges $AB = c$

K. OA Ges SP Gi g^t a^t m^u K^u b^u Y^u Ki |

L. t^t Lvl th, S, G, O GKB mij ti Lvq Aew^t Z |

M. $\angle C$ m² tK^u Y ntj $a \cdot CD = b \cdot CE$ mgxKi Y^u c^u Z^u Ki |

PZ_L Aa^ovq R^owgwZK A¼b

K^oúm I i^ovi e^oenvi K^oi w^ow^o kZ^oAbhvqx th w^oP^o A¼b Kiv nq, Zvnb R^owgwZK A¼b | Dccv^o c^ovtYi Rb^o th w^oP^o A¼b Kiv nq Zv h_{vh} (accurate) ni qv Lp Rijx bq | m^oúv^o i t^oq^o R^owgwZK w^oP^o A¼b h_{vh} ni qv LpB c^oqvRb |

Aa^ovq tk^ol w^oq^ov^o –

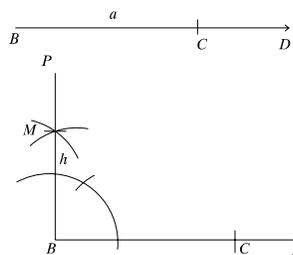
- c^oÉ Z^o I Dcv^oÉi w^of^ow^oÉ^oZ w^o f^oR A¼b Ges A¼tbi h_v Zv hvPvB Ki^oz cvi^ote |
- c^oÉ Z^o I Dcv^oÉi w^of^ow^oÉ^oZ eÉ A¼b Ges A¼tbi h_v Zv hvPvB Ki^oz cvi^ote |

4.1 w^o f^oR m^oúv^oš-KwZcq m^oúv^o :

m^oúv^o 1

w^o f^oRi f^owg, f^owg msj M^oeGKwU t^oKvY I D^oPZv t^o I qv Av^otQ, w^o f^oRwU A¼b Ki^oz nte |

g^ot^obKwi, w^o f^oRi f^owg a, D^oPZv h Ges f^owg msj M^oeGKwU x t^o I qv Av^otQ | w^o f^oRwU A¼b Ki^oz nte |

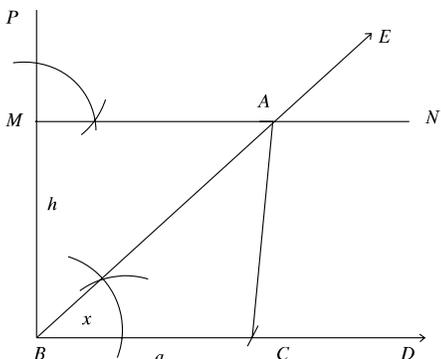
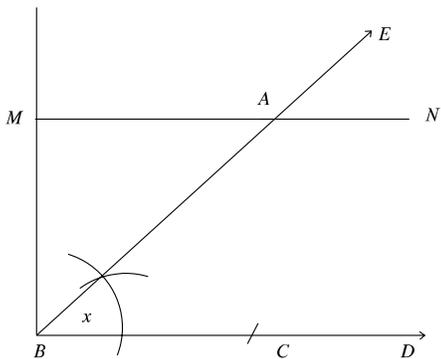
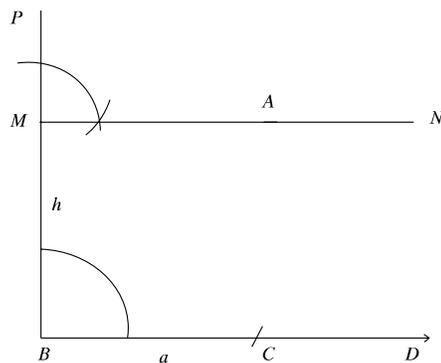


A¼tbi weeiY :

avc 1 : th^otKvb i^owk^o BD t^o t^oK BC = a Ask t^o t^oU w^obB |

avc 2 : B we^o t^oZ BC Gi Dci j^o BP A¼b Kwi Ges BP t^o t^oK BM = h Ask t^o t^oU w^obB |

avc 3 : M we^o t^oZ BC Gi mgv^oš^ovj MN tiLvsk A¼b Kwi |



avc 4 : Avevi B we`fZ cõ È $\angle x$ Ge mgvb Kti $\angle CBE$ A¼b Kwi | BE ti Lvsk MN tK A we`fZ tQ` Kti |

avc 5 : A, C thvM Kwi | Zvntj ABC -B Dwi` ó wî fR |

cõvY : thñZl $MN \parallel BC$ (A¼bvymvñi)

$\therefore ABC$ Gi D" PZv $BM = h$

Avevi , $BC = a$ Ges $\angle ABC = \angle x$

$\therefore \triangle ABC$ -B Dwi` ó wî fR |

we`fkiY : thñZl fwg l fwg msj MæfKvY t` qv AvtQ, mZivs GKwU mij ði Lv t_ðK fwi mgvb Ask tKtU wbtq Zvi GK cõš-cõ È tKvYi mgvb tKvY AvkñZ nte | AZ:ci fwi mñ½ wv` ð tKvY AvbZ Ggb ti Lv` we` yvbYq KiñZ nte | fwg t_ðK Gi D" PZv wî fñRi D" PZvi mgvb nq |

mæúv` 2

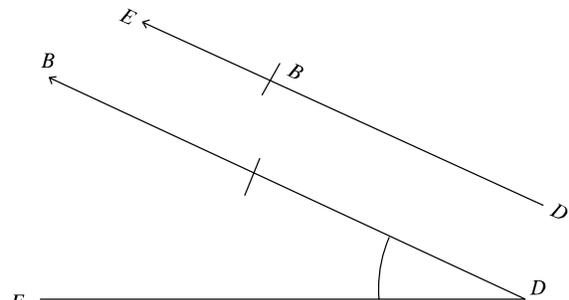
wî fñRi fwg, wkiññKvY l Aci evúñtqi mgwó t` l qv AvtQ | wî fñRiU A¼b KiñZ nte |



gñb Kwi, GKwU wî fñRi fwg a , Aci evúñtqi mgwó s Ges wkiññKvY x t` l qv AvtQ | wî fñRiU A¼b KiñZ nte |

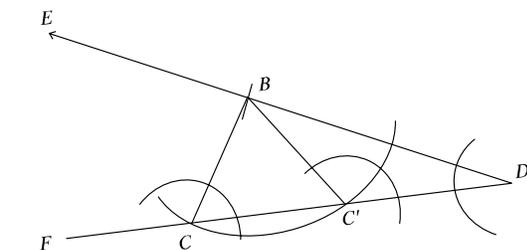
A¼ñbi weeiY :

avc 1 : thñKvñbv i wñkñ DE t_ðK $DB = s$ Ask tKtU wvB |

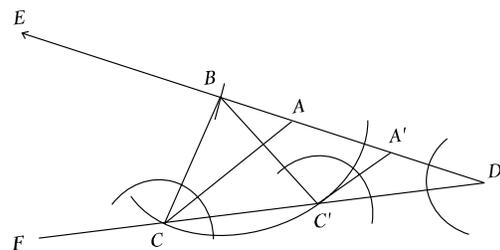


avc 2 : DB ti Lvi D we`fZ $\angle BDF = \frac{1}{2} \angle x$

A¼b Kwi |



avc 3 : B tK tK` ð Kti fwg a Gi mgvb e`vmva wbtq GKwU eñPvc A¼b Kwi hv DF tK $C \mid C'$ we`fZ tQ` Kti | $B, C \mid B, C'$ thvM Kwi |



avc 4 : C we`fZ $\angle BDF$ Gi mgvb $\angle DCA$ Ges C'we`fZ $\angle BDF$ Gi mgvb $\angle DC'A'$ A¼b Kwi | CA | C'A' ti Lv0q BD tK h_vµtg A | A' we`fZ tQ` Kti | Zvntj ABC | $A'BC'$ wî fR0q Dwi' ó wî fR |

c0vY : thtnZi $\angle ACD = \angle ADC = \angle A'C'D = \frac{1}{2}\angle x$ (A¼bvbmvti)

$$\therefore \angle BAC = \angle ADC + \angle ACD = \frac{1}{2}\angle x + \frac{1}{2}\angle x + \angle x = \angle x$$

$$\angle BA'C' = \angle A'DC' + \angle A'CD' = \frac{1}{2}\angle x + \frac{1}{2}\angle x = \angle x$$

Ges $AC = AD, A'C' = A'D$

ABC wî fR $\angle BAC = \angle x, BC = a$ Ges $CA + AB = DA + AB = DB = s$

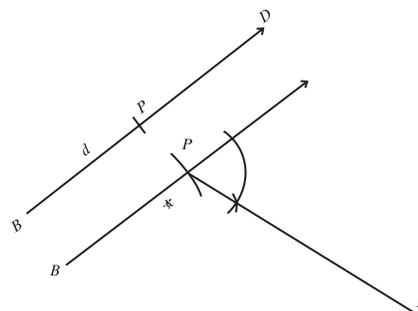
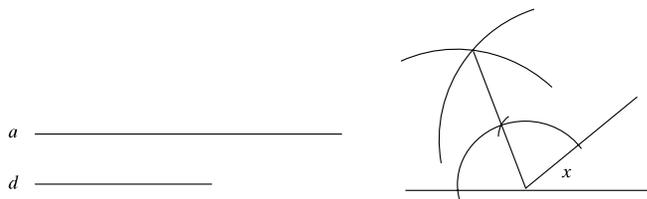
$\therefore \triangle ABC$ -B Dwi' ó wî fR |

Avevi $A'BC'$ wî fR $\angle BA'C' = \angle x, BC' = a$ Ges $C'A' = A'B = DA' + A'B = DB = s$

$\triangle A'BC'$ -B Aci Dwi' ó wî fR |

m0v` 3

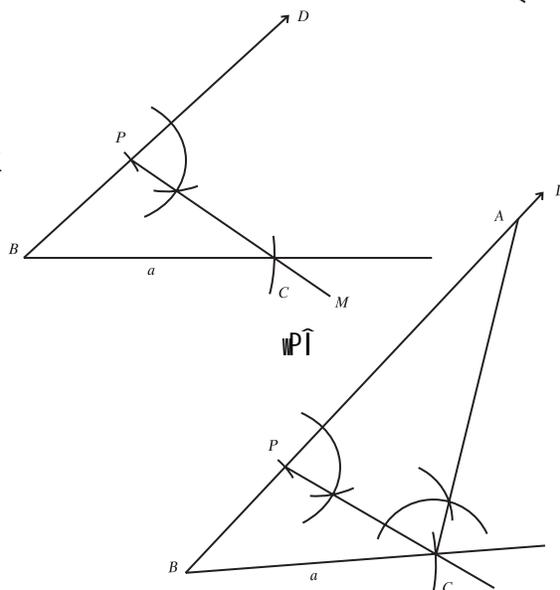
wî fRi fvg, wki tKvY Ges Aci `B ev0i Ašt' t' l qv AvtQ | wî fRw A¼b Ki tZ nte |



gtb Kwi, fvg a | Aci `B ev0i Ašt' d Ges wki tKvY x t' l qv AvtQ | wî fRw A¼b Ki tZ nte | A¼tbi weeiY :

avc 1 : thtKvµbv i wKf BD t_ tK $BP = d$ Ask tK tU w0B |

avc 2 : P we`fZ $\angle x$ Gi m0v tK tKvYi AtafKi mgvb $\angle DPM$ A¼b Kwi |



arc 3 : B tk tK`^a Kti a Gi mgvb e`vma`ib`b`q AwZ eEPvc PM mij ti Lvtk C me`fZ tQ`
Kti |

arc 4 : B I C thvM Kwi |

arc 5 : Avevi, C me`fZ $\angle DPC = \angle PCA$ tKvY
A¼b Kwi thb CA ti Lvsk BD tK A me`fZ tQ`
Kti | Zvntj ABC -B Dwi`ó wí fR |
cõvY :

$$\angle APC = \angle ACP \quad \therefore AP = AC$$

$$\therefore AB - AC = AB - AP = d$$

Avevi $\angle APC = \angle ACP = \angle x$ Ge m`ú`K tKvYi
A`aR |

$$\therefore \angle APC + \angle ACP = \angle x \text{ Gi m`ú`K}$$

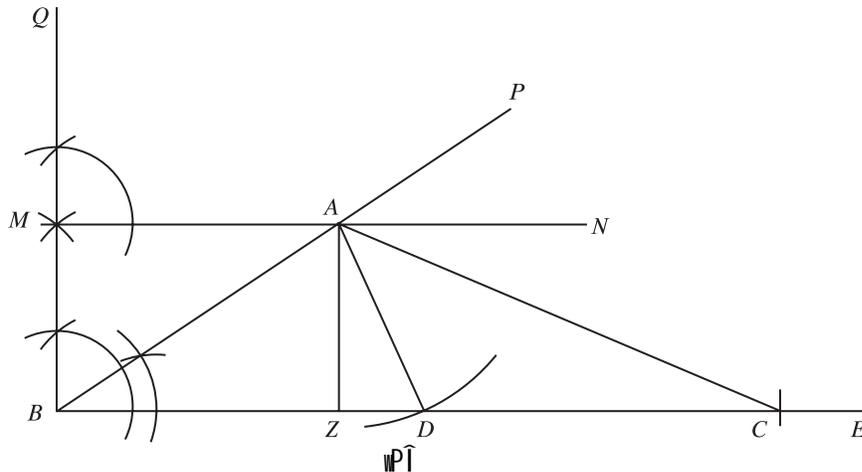
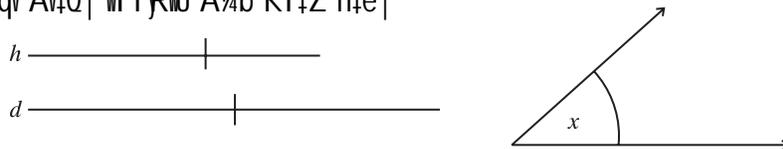
$$= \text{eint}^- \angle CAD = \angle CAB \text{ Gi m`ú`K |}$$

$$\therefore \angle A = \angle CAB = \angle x$$

$$\therefore ABC -B \text{ w`b`Y` wí fR |}$$

m`ú`v` 4

wí fRi D'PZv, fwi Dci ga`gv Ges fwi msj Mæ
GKwU tKvY t` l qv AvtQ | wí fRwU A¼b Ki tZ nte |



g`b Kwi, wí fRi D'PZv h, fwi Dci ga`gv d Ges fwi msj MæGKwU $\angle x$ t` l qv AvtQ | wí fRwU A¼b
Ki tZ nte |

A¼tbi weeiY :

- avc 1 : thtKvfbv iwkt BE Gi B we`fZ $\angle x$ Gi mgvb $\angle EBP$ A¼b Kwi |
- avc 2 : B we`fZ BE tiLvi Dci BQ j α A¼b Kwi |
- avc 3 : BQ t_tK wí fRi D"pZv h Gi mgvb BM Ask tKtU wB |
- avc 4 : M we`fZ BE Gi mgvš+vj Kti MN tiLv A¼b Kwi hv BP tK A we`fZ tQ` Kti |
- avc 5 : A we`fK tK`^a Kti ga'gv d Gi mgvb e"mva@btq GKwU eEPvc A¼b Kwi | H eEPvc BE tK D we`fZ tQ` Kti |
- avc 6 : BE t_tK $BD = DC$ Ask tKtU wB |
- avc 7 : A, C thvM Kwi | Zvntj , & ABC -B Dwí ó wí fR |

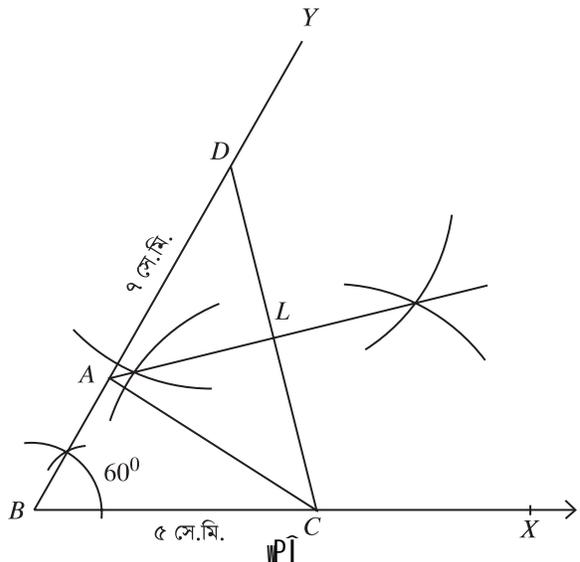
cgvY : A, D thvM Kwi Ges A t_tK BC Gi Dci AZ j α A¼b Kwi |
 GLvfb, MN I BE mgvš+vj Ges MB I AZ DftqB BE Gi Dci j α
 $\therefore MB = AZ = h = D"pZv$
 $BD = DC \therefore D$ we`B BC Gi ga'we`y |
 $\therefore AD = d =$ fvgi Dci Aw¼Z ga'gv, A_fR, BC fvg |
 Avevi, $\angle ABC = \angle x =$ fvg msj MæGKwU tKvY |
 $\therefore ABC$ -B Dwí ó wí fR |
 gše" : $\angle x$ Gi Dci wbfP Kti AtbK t_qtî `BwU wí fR cvl qv thtZ cvti |

D`vniY 1 | wí fRi fvgi $\hat{N}^{\circ} 5$ tm.wg., fvg msj MætkvY 60° Ges Aci `B evúi mgwó 7 tm.wg. |
 wí fRwU A¼b Ki tZ nte |

mgvavb : t`l qv AvtQ fvg $BC = 5$ tm.wg. Aci `B evúi mgwó $AB + AC = 7$ tm.wg, Ges $\angle ABC = 60^{\circ}$ | $\triangle ABC$ A¼b Ki tZ nte |

A¼tbi avcmgn :

- avc 1 : thtKvfbv iwkt BX t_tK $BC = 5$ tm.wg. tKtU wB
- avc 2 : $\angle XBY = 60^{\circ}$ Awk
- avc 3 : BY iwkt t_tK $BD = 7$ tm.wg. tKtU j B |
- avc 4 : C, D thvM Kwi

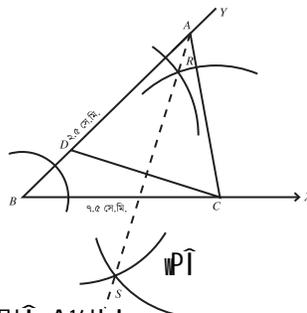


- avc 5 : CD tiLvi j α OLEK Awk hv BD tK A we`fZ tQ` Kti |
- avc 6 : A, C thvM Kwi, Zvntj ABC -B wbtY@ wí fR |
 tbvU : thtnZi AL, CD Gi j α OLEK
 $\therefore AD = AC$

Zvntj $BD = BA + AD = BA + AC = 7 \text{ tm.wg.}$ |

D`vniY 2 : wî fîRi fwi %N°7.5 tm.wg. fwi msj MætkvY 45° Ges Aci `ß evûi AŠt 2.5 tm.wg t` I qv AvtQ | wî fîRi A¼b Ki tZ nte |

mgravb : t` I qv AvtQ fwi $BC = 7.5 \text{ tm.wg.}$, Aci `ß evûi AŠt $AB - AC$ ev $AC - AB = 2.5 \text{ tm.wg.}$ Ges fwi msj MætkvY 45° | wî fîRi Avk tZ nte |



(i) $AB - AC = 2.5 \text{ tm.wg}$ Gi t`I t` A¼b i avC mgra :

1 | thtKv t`v i wk f BX t` tK $BC = 7.5 \text{ tm.wg}$ tK tU vbB |

2 | $\angle YBC = 45^\circ$ A¼b Kwi |

3 | BY i wk f t` tK $BD = 2.5 \text{ tm.wg}$ tK tU vbB |

4 | C, D thvM Kwi |

5 | CD Gi Dci RS j \perp LÊK Avk thb BY tK A we` tZ tQ` Kti |

6 | A, C thvM Kwi

Zvntj ABC -B vbYq wî fîR |

(ii) $AC - AB = 2.5 \text{ tm.wg.}$ a t`i wî fîRi vb tR AsKb Ki |

KvR :

1 | GKv wî fîRi cwi mxgv Ges fwi msj MætkvY 0q t` I qv AvtQ | wî fîRi A¼b Ki |

2 | wî fîRi fwi $BC = 4.6 \text{ tm.wg}$, $\angle B = 45^\circ$ Ges $AB + CA = 8.2 \text{ tm.wg}$ t` I qv AvtQ | wî fîRi Avk tZ nte |

3 | mg tKvYx wî fîRi GK evûi $\hat{\text{N}}^\circ 3.5 \text{ tm.wg}$, Aci evû Ges AvZ fîRi $\hat{\text{N}}^\circ 5.5 \text{ tm.wg}$ t` I qv AvtQ | wî fîRi Avk tZ nte |

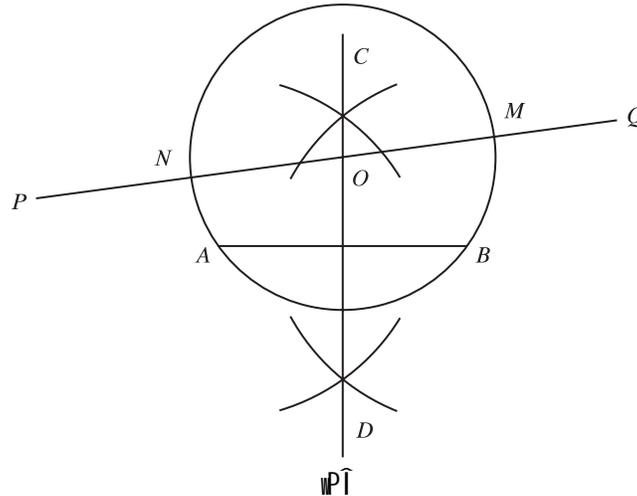
4 | $\triangle ABC$ Gi $BC = 4.5 \text{ tm.wg}$, $\angle B = 45^\circ$ Ges $AB - AC = 2.5 \text{ tm.wg}$ t` I qv AvtQ | $\triangle ABC$ wU A¼b Ki tZ nte |

5 | $\triangle ABC$ Gi cwi mxgv 12 tm.wg , $\angle B = 60^\circ$ Ges $\angle C = 45^\circ$ t` I qv AvtQ | $\triangle ABC$ wU Avk tZ nte |

4.2 eĚ msμvš-A¼b

mꝑúv` 5

Ggb GKwU eĚ A¼b KiřZ nte hv `BwU wbow` Ě we`y w`řq hvq Ges hvi tK`ª GKwU wbow` Ě mij ři Lvq Aew`Z _řřK|



A I B `BwU wbow` Ě we`y Ges PQ GKwU wbow` Ě mij ři Lv| Ggb GKwU eĚ A¼b KiřZ nte hv A I B we`y w`řq hvq Ges hvi tK`ª PQ mij ři Lvi Dci Aew`vb Kři |

avc 1 : A, B řhvM Kwı

avc 2 : AB ři Lvřřki mgwĚLĚK CD A¼b Kwı

avc 3 : CD ři Lvřk PQ ři Lvřřk O we`řřZ řQ` Kři

avc 4 : O tK`ª Kři OA ev OB e`vmva`řbtq Aw¼Z ABMN eĚ Aw¼Z nřj v| hv wbtřř eĚ|

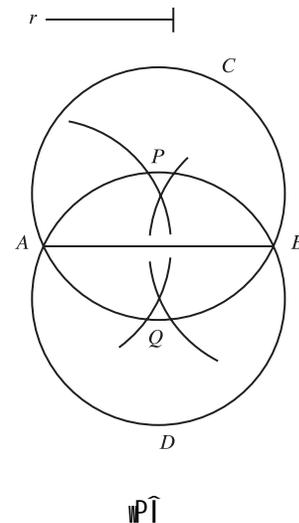
cřřvY : CD ři Lv AB ři Lvi j ĚĚLĚK| mřřivs CD ři Lv`řřřřřřřř we`y A I B řřřř mg`řeZřř A¼b vřřřřřřřř , O we`y řř CD I PQ Gi Dci Aew`Z| Avevi , OA I OB mgvb eřj O tK tK`ª Kři OA ev OB e`vmva`řbtq eĚ Aw¼řř eĚwU A I B we`y w`řq hvře Ges eřřři tK`ª O we`y řř PQ ři Lvi Dci Aew`vb Kře|

∴ O tK tK`ª Kři OA ev OB e`vmva`řbtq Aw¼Z eĚB wbtřř eĚ|

mꝑúv` -6

GKwU wbow` Ě ři Lvřřki mgvb e`vmva`řewkó GKwU eĚ A¼b KiřZ nte hv `BwU wbow` Ě we`y w`řq hvq|

A I B `BwU wbow` Ě we`y Ges r GKwU wbow` Ě ři Lvřřki %N% Ggb GKwU eĚ A¼b KiřZ nte hv A I B we`y w`řq hvq Ges hvi e`vmva`r Gi mgvb nq|



wřř

A¼tbi avcmgn :

- 1| $A \perp B$ thvM Kwi
- 2| $A \perp B$ tK tK[^]aKti r Gi mgvb e'vmva[®]btq AB Gi Dfq cvtk \perp BwU Kti eEPvc AvnK | GK cvtki eEPvc \perp BwU ci \bar{u} i tK P we[^] tZ Ges Aci cvtki eEPvc \perp BwU ci \bar{c} i tK Q we[^] tZ tQ[^] Kti |
- 3| P tK tK[^]aKti PA e'vmva[®]btq ABC eE A¼b Kwi |
- 4| Avevi Q tK tK[^]aKti QA e'vmva[®]btq ABD eE A¼Z ntj v | Zvntj $ABC \perp ABD$ eE \perp BwU cÖZ[^]KwUB vb tY[®] eE |

cÖvY : $PA = PB = r$

$\therefore P$ tK tK[^]aKti PA ev PB e'vmva[®]btq A¼Z ABC eE $A \perp B$ we[^] yw[^] tq hvq Ges e'vmva[®] $PA = r$ nq |

Avevi $QA = QB = r$

Q tK tK[^]aKti QA ev QB e'vmva[®]btq A¼Z ABD eE $A \perp B$ we[^] yw[^] tq hvq Ges e'vmva[®] $QA = r$ |

$\therefore ABC \perp ABD$ eE \perp BwU cÖZwUB Dwí ó eE |

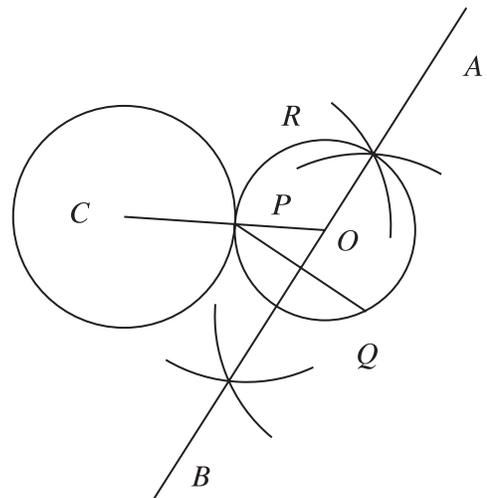
m[^]úv[^] 7

Gifc GKwU eE A¼b Ki tZ nte hv GKwU vbw[^] eE tK vbw[^] we[^] tZ \bar{u} k[®]Kti Ges eE e[^]i evnt[^] tKv[^]vb vbw[^] we[^] yw[^] tq hvq |

g[^]tb Kwi , vbw[^] eE eE tK[^]a C, P H eE e[^]i I ci Aew[^]Z GKwU vbw[^] we[^] yGes Q H eE e[^]i evnt[^] GKwU vbw[^] we[^] y | Gifc GKwU eE A¼b Ki tZ nte hv H eE tK P we[^] tZ \bar{u} k[®]Kti Ges Q we[^] yw[^] tq hvq |

A¼tbi avcmgn :

- avc 1 : P, Q thvM Kwi |
 - avc 2 : PQ Gi j[®]LÜK AB AvnK
 - avc 3 : C, P thvM Kwi
 - avc 4 : e[^]vaZ CP ti Lvsk AB tK O we[^] tZ tQ[^] Kti
 - avc 5 : ' O ' tK tK[^]aKti OP Gi mgvb e'vmva[®]btq A¼Z PQR -B Dwí ó eE |
- cÖvY : O, Q thvM Kwi | AB ti Lvsk ev OB ti Lvsk PQ Gi j[®]LÜK |
- $\therefore OP = OQ$



mZivs O tK tK[^]aKti OP e'vmva[®]btq eE AuKtj Zv Q we[^] yw[^] tq hvte |

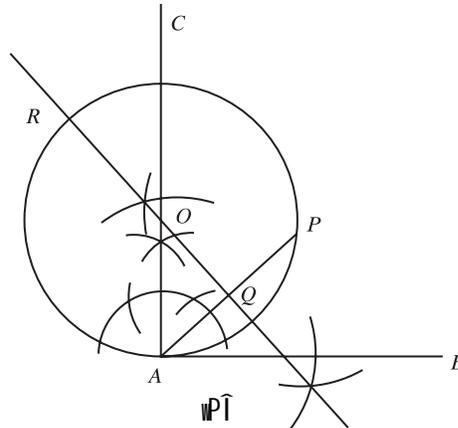
Avevi P we[^] yU \perp BwU eE e[^]i tK[^] t[^]q mshvRK ti Lvi Dci Aew[^]Z Ges P we[^] yDfq eE e[^]i Dci Aew[^]Z A[®] P we[^] tZ eE tQ[^] w[^]ij Z ntqtQ | mZivs eE tQ[^] P we[^] tZ \bar{u} k[®]Kti |

mZivs O tK tK[^]aKti OP e'vmva[®]btq A¼Z eE B Dwí ó eE |

m=úw` 8

Giƒc GKwU eĚ A¼b KiŕZ nte hv GKwU wbow`Ŧ mij ti LvŕK GKwU wbow`Ŧ we`ŕZ ŕúk©Kti Ges ti Lvi tKvŕbv we`yw`ŕq hvq|

gŕb Kwi, AB mij ti Lvŕ A GKwU wbow`Ŧ we`yGes AB ti Lvi ewntŕ P Aci GKwU wbow`Ŧ we`y| Giƒc GKwU eĚ A¼b KiŕZ nte hv AB tK A we`ŕZ ŕúk©Kti Ges P we`yw`ŕq hvq|



A¼ŕbi avcmgn :

avc 1 : AB Gi Dci A we`ŕZ AC j =A¼b Kwi |

avc 2 : P, A thvM Kti Zvi j =LÜK QO A¼b Kwi |

avc 3 : QO Ges AC ti Lvŕq O we`ŕZ tQ` Kti |

avc 4 : O tK tK`ªKti OA e`vmvaŕbtq Aw¼Z eĚwU QO ti LvŕK R te`ŕZ tQ` Kti |

Zvntj APRB Dwi`Ŧ eĚ |

cŕy : O, P thvM Kwi | AP ti Lvi j =LÜK OQ Gi Dci O we`yU AewŕZ |

∴ OA = OP

∴ O tK tK`ªKti OA e`vmvaŕbtq Aw¼Z eĚ P we`yw`ŕq hvq|

Avevi OA e`vmvaŕti Lvi A cŕš-we`ŕZ AB Gi Dci AO j =ŕ |

∴ AB ti Lvsk eĚwUŕtK A we`ŕZ ŕúk©Kti |

∴ O tK tK`ªKti OA e`vmvaŕbtq Aw¼Z eĚwUB wbtYŕ eĚ |

weŕkŕY : thŕnZzeĚwU wbow`Ŧ ti LvŕK wbow`Ŧ we`ŕZ ŕúk©Kti, mŕZivs H ti LwU wbow`Ŧ we`ŕZ =úkŕKi mŕ½ mgŕKvŕY _vKŕe| mŕZivs wbow`Ŧ ti Lvi wbow`Ŧ we`ŕZ j =Aw¼KŕZ nte Ges GB j =B eŕĚi GKwU e`vm nte| Avevi H ti Lvŕ wbow`Ŧ we`y| ewntŕ wbow`Ŧ we`yDfŕqB eŕĚi cwŕwai l cŕi _vKŕe weavq GB we`ŕŕtqi mŕŕhvRK ti Lvi j =LÜK tK`ªw`ŕq hvte|

Zvntj GB j =B wLEK l ceŕw¼Z e`vŕmi tQ` we`y eŕĚi tK`ªnŕe|

D`vniY 1| 2 tm.wg, e`vmvaŕwewkó eŕĚi tK`ªtŕK 5 tm.wg, ŕŕi tKvb wbow`Ŧ we`y tŕK ŕúkŕŕŕtqi ŕŕZj wbtYŕ Ki |

mgvavb : 2 tm.vg. e"vmva"neukó eġĒi tK>`ª 0 Ges wlv`θ P t_ġK 0 we`j `ġZi 5 tm.vg | P we`y t_ġK D³ eġĒ ĩukR AsKb Kġi Zvi %N"bYġ KiġZ nġe |

A¼ġbi avcmgn :

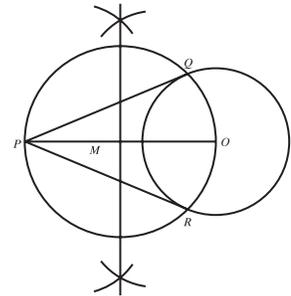
avc 1 : OP tiLvġK wθLwŪZ Kwġ | awġi, wθLwŪZ we`y M .

avc 2 : M -tK tK>`ª Kġi OM e"vmva"bġġ GKwU eġĒ AwK hv O

tKw`K eġĒi Q Ges R we`ġZ tQ` Kġi |

avc 3 : P,Q Ges P,R thvM Kwġ | Zvntġ PQ Ges PR-B wġġYġ ĩukR |

GLb, PQ I PR tK cwegvc Kġi cvB, PQ = PR = 4 · 6 tm.vg.



KvR :

1 | 5 tm.vg., 12 tm.vg. | 13 tm.vg evūneukó GKwU wġ fġRi AġĒ AsKb Kġi Gi e"vmva"bYġ Ki |

2 | 6.5 tm.vg., 7 tm.vg. Ges 7.5 tm.vg. evūneukó GKwU wġ fġRi ewnteġ A¼b Kġi Gi e"vmva"bYġ Ki |

Abġġj bx 4

1.



wPġ

$x = 60$ nġġ $\angle x$ Gi m"úġK tKvġYi AġġKi gvB KZ?

- K. 30°
- L. 60°
- M. 120°
- N. 180°

2. i thġKvġbv ĩġN"wbZwU evū θviv wġ fġR A¼b Kiv hvq bv |

ii ġagvġ e"vmva"Rvbn_vKġġ eġĒ A¼b Kiv hvq |

iii eġĒi tKvb we`ġZ GKwU gvġ ĩukR AwKv hvq |

Dġġi evK`ġġvi tKvbwU mwVK ?

- K. i I ii
- L. ii I iii
- M. i I iii
- N. i, ii I iii

3 | tKvġbv wġ fġRi `βwU tKvY I Zvġ ġi wecixZ evūġġġi Aġġ t` I qv AvġQ, wġ fRwU AwK |

4 | tKvġbv wġ fġRi fvg, fvg msj M"ġKvYġġġi Aġġ I Aġi evūġġġi mgwó t` I qv AvġQ, wġ fRwU AwK |

5 | fvg, wki tKvY I Aġi tKvYġġġi mgwó t` I qv AvġQ, wġ fRwU AwK |

6 | fvg, wki tKvY I Aġi tKvYġġġi Aġġ t` I qv AvġQ, wġ fRwU AwK |

7 | mgġKvYx wġ fġRi AwZfR I Aġi `β evūi mgwó t` I qv AvġQ, wġ fRwU AwK |

- 8| fvg msj MæGKwU tKvY, D'PZv I Aci `β evúí mgwó t` I qv AvtQ, wí fRwU AwK |
- 9| mgtKvYx wí fRi AwZfR I Aci `βwU evúí Aší t` I qv AvtQ, wí fRwU AwK |
- 10| Ggb GKwU eĚ A¼b Ki hv GKwU wbow`θ mij tti Lvtk Gi tKvfbv wbow`θ we`fZ Ges Aci GKwU eĚtk `úkKti |
- 11| Ggb GKwU eĚ A¼b Ki hv GKwU wbow`θ eĚtk Gi tKvfbv wbow`θ we`fZ Ges Aci GKwU eĚtk tKvfbv we`fZ `úkKti |
- 12| Ggb GKwU eĚ A¼b Ki hv GKwU wbow`θ mij tti Lvtk tKvfbv we`fZ Ges GKwU wbow`θ eĚtk Gi tKvfbv wbow`θ we`fZ `úkKti |
- 13| wfbawfbae`vmva`ewkó Gifc wZbwU eĚ AwK thb Zviv ci `úitk ewnt`úkKti |
- 14| tKvfbv eĚEi AB R`v-Gi P thtkvb we`y| P we`yw`tq Aci GKwU R`v CD A¼b Kitz nte| thb $CP^2 = AP \cdot OB$ nq|
15. mgwóevú wí fRi fvg 5 tm.wg. Ges mgvb mgvb evúí `N°6 tm.wg. |
- K. wí fRwU A¼b Ki |
- L. wí fRwU cwi eĚ A¼b Kti e`vmva`bYq Ki |
- M. wí fRwU cwi eĚ A¼b Kti e`vmva`bYq Ki |
- N. Ggb GKwU eĚ A¼b Ki hv cte`AwZ cwi eĚEi e`vmvta`i mgvb GKwU eĚtk P we`fZ `úkKti Ges eĚEi ewnt` tKvb we`yQ w`tq hvq |

cĀg Aa'vq mgxKiY

exRMWYtZ AĀvZ ev Pj i vnk LpB „i“ZcY©Bnv cteP Avtj vPbv Kiv ntqtQ | ev e Rxeṭb Anbw̄ Ṫ tKvṭbv e⁻; msL^v ev e⁻ mgntK eṣṭbvi Rb^ṽ Avgiv x, y, z BZ^{v̄} cĀK e⁻envi Kwi | GB iKg cĀK ev cĀKmgntK Pj K ev AĀvZ i vnk etj | GKwĀK Pj K ev AĀvZ i vnk mgštq i vnkgyj vi mjo nq | thgb, $2x + y, x^2 + z, x + y + 2z$, BZ^{v̄} | Avevi tKvṭbv AĀvZ i vnk ev i vnkgyj v hLb w̄ Ṫ msL^v ev gyṭbi mgvb wj Lv nq ZLb ZvṭK mgxKiY etj | exRMWYtZ mgxKiY LpB „i“ZcY©GKwJ wēlq | Bnvi mvrṭh^ṽ AṭbK ev e⁻ mgm^v mntRB mgvavb Kiv hvq |

Aa'vq tktl v̄kṭv_xPv –

- w̄NvZ mgxKiY $(ax^2 + bx + c = 0)$ mgvavb KiṭZ cviṭe |
- eMṭj wēkó mgxKiY w̄PvY^{v̄}Z KiṭZ cviṭe |
- eMṭj wēkó mgxKiY mgvavb KiṭZ cviṭe |
- mPKxq mgxKiY e^vL^v KiṭZ cviṭe |
- mPKxq mgxKiY mgvavb KiṭZ cviṭe |
- `Ḇ Pj ṭKi GKvZ I w̄NvZ mgxKiṭYi tRvU mgvavb KiṭZ cviṭe |
- ev⁻ewf^{v̄}ĒK mgm^vṭK `Ḇ Pj ṭKi GKvZ I w̄NvZ mgxKiṭY cĀKvK Kṭi mgvavb KiṭZ cviṭe |
- `Ḇ Pj K wēkó mPKxq mgxKiY tRvU mgvavb KiṭZ cviṭe |
- tj LwPṭĪ i mvrṭh^ṽ w̄NvZ mgxKiY $(ax^2 + bx + c = 0)$ mgvavb KiṭZ cviṭe |

5.1 GK Pj K mgwšZ w̄NvZ mgxKiY I Zvi mgvavb

gva^{v̄}gK exRMWYtZ GK Pj ṭKi GKvZ I w̄NvZ mgxKiY Ges `Ḇ Pj ṭKi GKvZ mgxKiY wēlṭq wēk^ṽ Avtj vPbv Kiv ntqtQ | exR^{v̄}tjv gj^ṽ msL^v ntj, GK Pj ṭKi w̄NvZ mgxKiṭYi evgcṭṭK Drcv^ṽṭK wēṭkIY Kṭi mntRB Zvi mgvavb Kiv hvq | w̄Kš^{v̄} me i vnkgyj vṭK mntR Drcv^ṽṭK wēṭkIY Kiv hvq bv | tm Rb^ṽ thṭKvṭbv cĀKvi w̄NvZ mgxKiṭYi mgvavṭbi Rb^ṽ wbgṭj wLZ c×wZw e⁻envi Kiv nq |

GK Pj K mgwšZ w̄NvZ mgxKiṭYi Av^ṽ kPjC $ax^2 + bx + c = 0$. GLvṭb a, b, c ev e⁻ msL^v Ges a Gi gyv KLbB kb^ṽ ntZ cviṭe bv |

Avgiv w̄NvZ mgxKiYwJi mgvavb Kwi,

$$ax^2 + bx + c = 0$$

ev, $a^2x^2 + abx + ac = 0$ [DfqcṭṭK a Ṫviv^{v̄}Y Kṭi]

$$\text{ev, } (ax)^2 + 2(ax)\frac{b}{2} + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + ac = 0 \quad \text{ev, } \left(ax + \frac{b}{2}\right)^2 = \frac{b^2}{4} - ac$$

$$\text{ev, } \left(ax + \frac{b}{2}\right)^2 = \frac{b^2 - 4ac}{4} \qquad \text{ev, } ax + \frac{b}{2} = \pm \frac{\sqrt{b^2 - 4ac}}{2}$$

$$\text{ev, } ax = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2} \qquad \text{ev, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (i)$$

AZGe, x Gi `βwU gvb cvl qv tMj Ges gvb `βwU nt"Q

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad (ii) \qquad \text{Ges} \qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad (iii)$$

Dctii (i) bs mgxKi tY $b^2 - 4ac$ tK wNvZ mgxKi YwU wDvqK etj KviY Bnv mgxKi YwU gj Øtqi aib l cKwZ wDvq Kti |

wDvq tKi Ae`v t f t` wNvZ mgxK tYi gj Øtqi aib l cKwZ

(i) $b^2 - 4ac > 0$ Ges cYEM t j mgxKi YwU gj Øq ev`e, Amgvb l gj` nte |

(ii) $b^2 - 4ac > 0$ wKŠ' cYEM b v n t j mgxKi YwU gj Øq ev`e, Amgvb l Agj` nte |

(iii) $b^2 - 4ac = 0$ n t j mgxKi YwU gj Øq ev`e l ci`ui mgvb nte | G t q t t x = $-\frac{b}{2a}, -\frac{b}{2a}$.

(iv) $b^2 - 4ac < 0$ A`F FYvZK n t j gj Øq Ae v`e nte | G t q t t gj Øq memgq `βwU AbpÜx Riwj ev Kvi wK msL`v nq | G w l t q D'PZI t k w t Z RvbtZ cvi te |

D`vni Y 1 | $x^2 - 5x + 6 = 0$ Gi mgvavb Ki |

mgvavb : $ax^2 + bx + c = 0$ mgxKi tYi mv t` Z j bv Kti G t q t t cvl qv hvq $a = 1, b = -5$ Ges $c = 6$.

AZGe mgxKi YwU mgvavb

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm \sqrt{1}}{2}$$

$$= \frac{5 \pm 1}{2} = \frac{5+1}{2}, \frac{5-1}{2}$$

A`F $x_1 = 3, x_2 = 2$.

D`vni Y 2 | $x^2 - 6x + 9 = 0$ Gi mgvavb Ki |

mgvavb : $ax^2 + bx + c = 0$ mgxKi tYi mv t` Z j bv Kti G t q t t cvl qv hvq $a = 1, b = -6$ Ges $c = 9$.

AZGe mgxKi YwU mgvavb

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 - 36}}{2} = \frac{6 \pm 0}{2}$$

A`F $x_1 = 3, x_2 = 3$.

D`vni Y 3 | mgvavb Ki t $x^2 - 2x - 2 = 0$

mgvavb : Av`k Pfc wNvZ mgxKi tYi mv t` Z j bv Kti cvl qv hvq, $a = 1, b = -2, c = -2$.

AZGe mgxKiYvUj gj Øq

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2}$$

$$\text{ev, } x = \frac{2 \pm 2\sqrt{3}}{2} = \frac{2(1 \pm \sqrt{3})}{2}$$

$$A_{\text{fr}} \quad x_1 = 1 + \sqrt{3}, \quad x_2 = 1 - \sqrt{3}.$$

GLvfb j ¶Yxq th, mvaviY vobqfg gj `msL'vi mrvvth" $x^2 - 2x - 1$ tK Drcv` tK vettkIY Kiv bv tMtj l cØ È mgxKiYvUj mgvavb Kiv mæe ntq†Q |

D`vniY 4 | mgvavb Ki t $3 - 4x - x^2 = 0$

mgvavb : Av` kPfc vØNvZ mgxKi†Yi mvt_ Zj bv K†i cvl qv hvq, $a = -1, b = -4, c = 3$.

AZGe mgxKiYvUj gj Øq

$$x = \frac{-(-1) \pm \sqrt{(-4)^2 - 4 \cdot (-1) \cdot 3}}{2 \cdot (-1)} = \frac{1 \pm \sqrt{16+12}}{-2} = \frac{4 \pm \sqrt{28}}{-2} = \frac{4 \pm 2\sqrt{7}}{-2}$$

$$\text{ev, } x = -\left(2 \pm \sqrt{7}\right)$$

$$A_{\text{fr}} \quad x_1 = -2 - \sqrt{7}, \quad x_2 = -2 + \sqrt{7}.$$

KvR : Dcti i (ii) l (iii) bs m†i i mrvvth" $ax^2 + bx + c = 0$ ntZ x_1 Ges x_2 Gi gvb vby¶ Ki hLb
 (i) $b = 0$, (ii) $c = 0$ (iii) $b = c = 0$ (iv) $a = 1$ Ges (v) $a = 1, b = c = 2p$

Abkxj bx 5.1

m†i i mrvvth" vbtPi mgxKiY, t j vi mgvavb Ki t

- | | | |
|-------------------------|-------------------------|-------------------------|
| 1 $2x^2 + 9x + 9 = 0$ | 2 $3 - 4x - 2x^2 = 0$ | 3 $4x - 1 - x^2 = 0$ |
| 4 $2x^2 - 5x - 1 = 0$ | 5 $3x^2 + 7x + 1 = 0$ | 6 $2 - 3x^2 + 9x = 0$ |
| 7 $x^2 - 8x + 16 = 0$ | 8 $2x^2 + 7x - 1 = 0$ | 9 $7x - 2 - 3x^2 = 0$ |

5.2 | gj vPy mæv Z mgxKiY

Avgiv Rvmb, Pj tKi th gvb ev gvb, t j vi Rb" mgxKi†Yi Dfq c¶ | mgvb nq, H gvb ev gvb, t j vB mgxKi†Yi exR ev gj (Root) Ges H gvb ev gvb, t j vi Øviv mgxKiYvUj v m× nq |

mgxKi†Y Pj tKi eM¶j mæv Z i vK t j Zv tK eM¶K†i eM¶j vPygv³ bZb mgxKiY cvl qv hvq | D³ mgxKiY mgvavb K†i th exR, t j v cvl qv hvq A†bK mgq me, t j v exR cØ È mgxKiYvU†K v m× K†i bv | G aitbi exR Aevš† (Extraneous) exR | mZivs gj vPy mæv Z mgxKiY mgvavb cØµqvq cØB exR, t j v cØ È mgxKi†Yi exR vK bv Zv Aek" B cix¶v K†i t`Lv` iKvi | cix¶vi ci th me exR D³ mgxKiY†K v m× K†i ZvB nte cØ È mgxKi†Yi exR | vbtP K†qKvU D`vniY t` l qv ntj v |

$\text{KvR: } p = \sqrt{\frac{x}{x+16}} \text{ at} \sqrt{\frac{x}{x+16}} + \sqrt{\frac{x+16}{x}} = \frac{25}{12} \text{ mgvKi YvUj mgvavb Kti i'v cix'v Ki}$
--

$$\text{D`vni Y 1 | mgvavb Ki : } \sqrt{8x+9} - \sqrt{2x+15} = \sqrt{2x-6}$$

$$\text{mgvavb : } \sqrt{8x+9} - \sqrt{2x+15} = \sqrt{2x-6}$$

$$\text{ev, } \sqrt{2x+15} + \sqrt{2x-6} = \sqrt{8x+9}$$

$$\text{ev, } 2x+15+2x-6+2\sqrt{2x+15}\sqrt{2x-6} = 8x+9 \text{ [eM'Kti]}$$

$$\text{ev, } \sqrt{2x+15}\sqrt{2x-6} = 2x$$

$$\text{ev, } (2x+15)(2x-6) = 4x^2 \text{ [cpi vq eM'Kti]}$$

$$\text{ev, } 4x^2 + 18x - 90 = 4x^2$$

$$\text{ev, } 18x = 90$$

$$\therefore x = 5$$

$$\text{i'v cix'v : } x = 5 \text{ ntj, evgc'v} = \sqrt{49} - \sqrt{25} = 7 - 5 = 2 \text{ Ges Wwbc'v} = \sqrt{4} = 2$$

$$\therefore \text{wb'tY' mgvavb } x = 5.$$

$$\text{D`vni Y 2 | mgvavb Ki : } \sqrt{2x+8} - 2\sqrt{x+5} + 2 = 0$$

$$\text{mgvavb : } \sqrt{2x+8} = 2\sqrt{x+5} - 2$$

$$\text{ev, } 2x+8 = 4(x+5)+4-8\sqrt{x+5} \text{ [eM'Kti]}$$

$$\text{ev, } 8\sqrt{x+5} = 4x+20+4-2x-8 \text{ [c'v'v'Št Kti]}$$

$$\text{ev, } 8\sqrt{x+5} = 2x+16 = 2(x+8)$$

$$\text{ev, } 4\sqrt{x+5} = x+8$$

$$\text{ev, } 16(x+5) = x^2 + 16x + 64 \text{ [eM'Kti]}$$

$$\text{ev, } 16 = x^2$$

$$\therefore x = \pm\sqrt{16} = \pm 4$$

$$\text{i'v cix'v : } x = 4 \text{ ntj, evgc'v} = \sqrt{16} - 2\sqrt{9} + 2 = 4 - 2 \times 3 + 2 = 0 = \text{Wwbc'v}$$

$$x = -4 \text{ ntj, evgc'v} = \sqrt{-8+8} - 2\sqrt{-4+5} + 2 = 0 - 2 \times 1 + 2 = 0 = \text{Wwbc'v}$$

$$\therefore \text{wb'tY' mgvavb } x = 4, -4.$$

$$\text{D`vni Y 3 | mgvavb Ki : } \sqrt{2x+9} - \sqrt{x-4} = \sqrt{x+1}$$

$$\text{mgvavb : } \sqrt{2x+9} - \sqrt{x-4} = \sqrt{x+1}$$

$$\text{ev, } 2x+9+x-4-2\sqrt{(2x+9)(x+4)} = x+1 \text{ [eM'Kti]}$$

$$\text{ev, } 2\sqrt{2x^2 + x - 36} = 2x + 4$$

$$\text{ev, } \sqrt{2x^2 + x - 36} = x + 2$$

$$\text{ev, } 2x^2 + x - 36 = x^2 + 4x + 4 \text{ [eM}^{\text{Q}}\text{K}^{\text{t}}\text{i]}$$

$$\text{ev, } x^2 - 3x - 40 = 0$$

$$\text{ev, } (x - 8)(x + 5) = 0$$

$$\therefore x = 8 \text{ A_ev } -5$$

i'ux cix'iv : $x = 8$ ntj , evgc'v = $5 - 2 = 3$ Ges Wwbc'v = 3

AZGe, $x = 8$ c' E mgxKi'tYi GKwU exR |

$x = -5$ Mh'YthvM' bq, tKbbv mgxKi'tY $x = -5$ emvtj FYvZ'K msL'vi eM'j Avtm hv ms'AwqZ bq |

\therefore wbt'Y' mgvavb $x = 8$

$$\text{D`vni Y 4 | mgvavb Ki : } \sqrt{(x-1)(x-2)} + \sqrt{(x-3)(x-4)} = -\sqrt{2}$$

$$\text{mgvavb : } \sqrt{(x-1)(x-2)} + \sqrt{(x-3)(x-4)} = \sqrt{2}$$

$$\text{ev, } \sqrt{x^2 - 3x + 2} - \sqrt{2} = -\sqrt{x^2 - 7x + 12}$$

$$\text{ev, } x^2 - 3x + 2 - 2\sqrt{2}\sqrt{x^2 - 3x + 2} + 2 = x^2 - 7x + 12 \text{ [eM}^{\text{Q}}\text{K}^{\text{t}}\text{i]}$$

$$\text{ev, } \sqrt{2x^2 - 6x + 4} = 2x - 4$$

$$\text{ev, } 2x^2 - 6x + 4 = (2x - 4)^2 = 4x^2 - 16x + 16 \text{ [eM}^{\text{Q}}\text{K}^{\text{t}}\text{i]}$$

$$\text{ev, } x^2 - 5x + 6 = 0$$

$$\text{ev, } (x - 2)(x - 3) = 0$$

$$\therefore x = 2 \text{ A_ev } x = 3.$$

i'ux cix'iv : $x = 2$ ntj evgc'v = $\sqrt{2} = \text{Wwbc'v}$

$x = 3$ ntj , evgc'v = $\sqrt{2} = \text{Wwbc'v}$

\therefore wbt'Y' mgvavb $x = 2, 3$

$$\text{D`vni Y 5 | mgvavb Ki : } \sqrt{x^2 - 6x + 15} - \sqrt{x^2 - 6x + 13} = \sqrt{10} - \sqrt{8}$$

$$\text{mgvavb : } \sqrt{x^2 - 6x + 15} - \sqrt{x^2 - 6x + 13} = \sqrt{10} - \sqrt{8}$$

GLb $x^2 - 6x + 13 = y$ ai'tj c' E mgxKi'Y nte

$$\sqrt{y+2} - \sqrt{y} = \sqrt{10} - \sqrt{8}$$

$$\text{ev, } \sqrt{y+2} + \sqrt{8} = \sqrt{y} + \sqrt{10}$$

$$\text{ev, } y + 2 + 8 + 2\sqrt{8y+16} = y + 10 + 2\sqrt{10y} \text{ [eM}^{\text{Q}}\text{K}^{\text{t}}\text{i]}$$

$$\text{ev, } \sqrt{8y+16} = \sqrt{10y}$$

$$\text{ev, } 8y + 16 = 10y \text{ [eM}^{\text{Kti}}]$$

$$\text{ev, } 2y = 16 \text{ ev, } y = 8$$

$$\text{ev, } x^2 - 6x + 13 = 8 \text{ [y Gi gvb eim}^{\text{tq}}]$$

$$\text{ev, } x^2 - 6x + 5 = 0 \text{ ev, } (x-1)(x-5) = 0$$

$$\therefore x = 1 \text{ A_ev } 5.$$

$$\text{i}^{\text{w}} \times \text{ci}^{\text{x}} \text{v} : x = 1 \text{ ntj, evgc}^{\text{v}} = \sqrt{10} - \sqrt{8} = \text{Wwbc}^{\text{v}}$$

$$x = 5 \text{ ntj, evgc}^{\text{v}} = \sqrt{10} - \sqrt{8} = \text{Wwbc}^{\text{v}}$$

$$\therefore \text{wb}^{\text{t}} \text{Y}^{\text{q}} \text{ mgvavb } x = 1, 5$$

$$\text{D}^{\text{vni}} \text{Y } 6 \text{ | mgvavb Ki : } (1+x)^{\frac{1}{3}} + (1-x)^{\frac{1}{3}} = 2^{\frac{1}{3}}$$

$$\text{mgvavb : } (1+x)^{\frac{1}{3}} + (1-x)^{\frac{1}{3}} = 2^{\frac{1}{3}}$$

$$\Rightarrow 1+x+1-x+3(1+x)^{\frac{1}{3}}(1-x)^{\frac{1}{3}} \left\{ (1+x)^{\frac{1}{3}} + (1-x)^{\frac{1}{3}} \right\} = 2 \text{ [Nb Kti]}$$

$$\text{ev, } 2+3(1+x)^{\frac{1}{3}}(1-x)^{\frac{1}{3}} 2^{\frac{1}{3}} = 2$$

$$\text{ev, } 3 \cdot 2^{\frac{1}{3}} (1+x)^{\frac{1}{3}} (1-x)^{\frac{1}{3}} = 0$$

$$\text{ev, } (1+x)^{\frac{1}{3}} (1-x)^{\frac{1}{3}} = 0$$

$$\text{ev, } (1+x)(1-x) = 0 \text{ [Avevi Nb Kti]}$$

$$x = 1 \text{ Ges } x = -1 \text{ Df}^{\text{qB}} \text{ mgxKi Yw}^{\text{t}} \text{K } \text{w}^{\text{m}} \times \text{Kti} \text{ |}$$

$$\therefore \text{wb}^{\text{t}} \text{Y}^{\text{q}} \text{ mgvavb } x = \pm 1$$

Ab}xj bx 5.2

mgvavb Ki :

$$1 \text{ | } \sqrt{x-4} + 2 = \sqrt{x+12}$$

$$2 \text{ | } \sqrt{11x-6} = \sqrt{4x+5} - \sqrt{x-1}$$

$$3 \text{ | } \sqrt{2x+7} + \sqrt{3x-18} = \sqrt{7x+1}$$

$$4 \text{ | } \sqrt{x+4} + \sqrt{x+11} = \sqrt{8x+9}$$

$$5 \text{ | } \sqrt{11x-6} = \sqrt{4x+5} + \sqrt{x-1}$$

$$6 \text{ | } \sqrt{x^2+4x-4} + \sqrt{x^2+4x-10} = 6$$

$$7 \text{ | } \sqrt{x^2-6x+9} - \sqrt{x^2-6x+6} = 1$$

$$8 \text{ | } \sqrt{2x^2+5x-2} - \sqrt{2x^2+5x-9} = 1$$

$$9 \text{ | } 6\sqrt{\left(\frac{2x}{x-1}\right)} + 5\sqrt{\left(\frac{x-1}{2x}\right)} = 13$$

$$10 \text{ | } \sqrt{\left(\frac{x-1}{3x+2}\right)} + 2\sqrt{\left(\frac{3x+2}{x-1}\right)} = 3$$

5.3 mPK mgxKi Y (Indicial Equation)

th mgxKi }y A}vZ Pj K mPKi }c _v}K, Zv}K mPK mgxKi Y etj |

$2^x = 8, 16^x = 4^{x+2} \cdot 2^{x+1} - 2^x - 8 = 0$ BZ }w` mgxKi Y }tj v mPK mgxKi Y thLv}b x A}vZ Pj K | mPK mgxKi Y mgvavb Ki }Z mP}Ki }b}w}j wLZ ag}U c}qB e`envi Kiv nqt

$a \neq 1$ n̄tj $a^x = a^m$ n̄te hw` l tKej hw` $x = m$ nq| G Rb` c0tg mgrKi t̄Yi Dfq c¶t̄K GKB mSLvi NvZ ev kw̄īf̄c cK̄vk Kiv nq :

$$\text{KvR: 1} \mid 4096 \text{ tK } \frac{1}{2}, 2, 4, 8, 16, 2\sqrt{2}, \sqrt[3]{4} \text{ Gi mP̄t̄K cK̄vk Ki} \mid$$

$$2 \mid 729 \text{ tK } 3, 9, 27, 16, \sqrt[5]{9} \text{ Gi mP̄t̄K vj L} \mid$$

$$3 \mid \frac{64}{729} \text{ tK } \frac{3}{2}, \sqrt[3]{\frac{3}{2}} \text{ Gi mP̄t̄K cK̄vk Ki} \mid$$

D`vniY 1 | mgvavb Ki t $2^{x+7} = 4^{x+2}$

mgvavb t $2^{x+7} = 4^{x+2}$

ev, $2^{x+7} = (2^2)^{x+2}$

ev, $2^{x+7} = 2^{2x+4}$

$\therefore x + 7 = 2x + 4$

ev, $x = 3$

\therefore w̄b̄t̄Ȳq̄ mgvavb, $x = 3$.

D`vniY 2 | mgvavb Ki t $3 \cdot 27^x = 9^{x+4}$

mgvavb t $3 \cdot 27^x = 9^{x+4}$

ev, $3 \cdot (3^3)^x = (3^2)^{x+4}$

ev, $3 \cdot 3^{3x} = 3^{2(x+4)}$

ev, $3^{3x+1} = 3^{2x+8}$

$\therefore 3x + 1 = 2x + 8$

ev, $x = 7$

\therefore w̄b̄t̄Ȳq̄ mgvavb $x = 7$

D`vniY 3 | mgvavb Ki t $3^{mx-1} = 3a^{mx-2}$, ($a > 0$, $a \neq 3$, $m \neq 0$)

mgvavb t $3^{mx-1} = 3a^{mx-2}$

ev, $\frac{3^{mx-1}}{3} = a^{mx-2}$ [Dfq c¶t̄K 3 Øviv f̄vM K̄t̄i]

ev, $3^{mx-2} = a^{mx-2}$

ev, $\left(\frac{a}{3}\right)^{mx-2} = 1 = \left(\frac{a}{3}\right)^0$

ev, $mx - 2 = 0$

ev, $mx = 2$

ev, $x = \frac{2}{m}$

$$\therefore \text{wb} \ddagger \text{Y} \text{Q} \text{ mgvavb } x = \frac{2}{m}$$

$$\text{D`vni Y 4 | mgvavb Ki : } 2^{3x-5} \cdot a^{x-2} = 2^{x-3} \cdot 2a^{1-x}, \quad (a > 0 \text{ Ges } a \neq \frac{1}{2})$$

$$\text{mgvavb : } 2^{3x-5} \cdot a^{x-2} = 2^{x-3} \cdot 2a^{1-x}$$

$$\text{ev, } \frac{a^{x-2}}{a^{1-x}} = \frac{2^{x-3} \cdot 2^1}{2^{3x-5}} \quad \text{ev, } a^{x-2-1+x} = 2^{x-3+1-3x+5}$$

$$\text{ev, } a^{2x-3} = 2^{-2x+3} \quad \text{ev, } a^{2x-3} = 2^{-(2x-3)}$$

$$\text{ev, } a^{2x-3} = \frac{1}{2^{2x-3}} \quad \text{ev, } a^{2x-3} \cdot 2^{2x-3} = 1$$

$$\text{ev, } (2a)^{2x-3} = 1 = (2a)^0$$

$$\therefore 2x-3=0 \quad \text{ev, } 2x=3 \quad \text{ev, } x = \frac{3}{2}$$

$$\therefore \text{wb} \ddagger \text{Y} \text{Q} \text{ mgvavb } x = \frac{3}{2}$$

$$\text{D`vni Y 5 | mgvavb Ki t } a^{-x}(a^x + b^{-x}) = \frac{a^2 b^2 + 1}{a^2 b^2}, \quad (a > 0, b > 0 \text{ Ges } ab \neq 1)$$

$$\text{mgvavb t } a^{-x}(a^x + b^{-x}) = 1 + \frac{1}{a^2 b^2}$$

$$\text{ev, } a^{-x} \cdot a^x + a^{-x} \cdot b^{-x} = 1 + \frac{1}{a^2 b^2}$$

$$\text{ev, } 1 + (ab)^{-x} = 1 + (ab)^{-2}$$

$$\text{ev, } (ab)^{-x} = (ab)^{-2}$$

$$\therefore -x = -2$$

$$\text{A}_\text{ff}, x = 2$$

$$\therefore \text{wb} \ddagger \text{Y} \text{Q} \text{ mgvavb } x = 2$$

$$\text{D`vni Y 6 | mgvavb Ki t } 3^{x+5} = 3^{x+3} + \frac{8}{3}$$

$$\text{mgvavb t } 3^{x+5} = 3^{x+3} + \frac{8}{3}$$

$$\text{ev, } 3^x \cdot 3^5 = 3^x \cdot 3^3 + \frac{8}{3}$$

$$\text{ev, } 3^x \cdot 3^6 - 3^x \cdot 3^4 = 8 \quad [\text{c} \text{¶} \text{v} \text{š} \text{† Ges Df} \text{q} \text{ c} \text{†} \text{¶} \text{] } 3 \text{ Øiv } \text{, Y K} \text{†i}]$$

$$\text{ev, } 3^x \cdot 3^4 (3^2 - 1) = 8$$

$$\text{ev, } 3^{x+4} \cdot 8 = 8$$

$$\text{ev, } 3^{x+4} = 1 = 3^0$$

$$\therefore x + 4 = 0 \text{ ev, } x = -4$$

$$\therefore \text{wb}^{\dagger}\text{Y}^{\circledast} \text{ mgvavb } x = -4$$

$$\text{D}^{\text{vni}} \text{Y } 7 | \text{ mgvavb Ki t } 3^{2x-2} - 5 \cdot 3^{x-2} - 66 = 0$$

$$\text{mgvavb t } 3^{2x-2} - 5 \cdot 3^{x-2} - 66 = 0$$

$$\text{ev, } \frac{3^{2x}}{9} - \frac{5}{9} \cdot 3^x - 66 = 0$$

$$\text{ev, } 3^{2x} - 5 \cdot 3^x - 594 = 0 \text{ [Df}^{\circledast} \text{ ct}^{\circledast} | 9 \text{ } \emptyset \text{viv } _{\text{Y}} \text{ K}^{\dagger}\text{i}]$$

$$\text{ev, } a^2 - 5a - 594 = 0 \text{ (} 3^x = a \text{ a}^{\dagger}\text{i)}$$

$$\text{ev, } a^2 - 27a + 22a - 594 = 0$$

$$\text{ev, } (a - 27)(a + 22) = 0$$

$$\text{GLb } a \neq -22, \text{ } \dagger \text{Kbbv } a = 3^x > 0 \text{ m}^{\dagger}\text{Zivs } a + 22 \neq 0$$

$$\text{AZGe, } a - 27 = 0$$

$$\text{ev, } 3^x = 27 = 3^3$$

$$\therefore x = 3$$

$$\text{wb}^{\dagger}\text{Y}^{\circledast} \text{ mgvavb } x = 3$$

$$\text{D}^{\text{vni}} \text{Y } 8 | \text{ mgvavb Ki t } a^{2x} - (a^3 + a)a^{x-1} + a^2 = 0 (a > 0, a \neq 1)$$

$$\text{mgvavb t } a^{2x} - (a^3 + a)a^{x-1} + a^2 = 0$$

$$\text{ev, } a^{2x} - a(a^2 + 1)a^x \cdot a^{-1} + a^2 = 0$$

$$\text{ev, } a^{2x} - (a^2 + 1)a^x + a^2 = 0$$

$$\text{ev, } p^2 - (a^2 + 1)p + a^2 = 0 \text{ (} a^x = p \text{ a}^{\dagger}\text{i)}$$

$$\text{ev, } p^2 - a^2 p - p + a^2 = 0$$

$$\text{ev, } (p - 1)(p - a^2) = 0$$

$$\therefore p = 1 \quad \text{A_ev } p = a^2$$

$$\text{ev, } a^x = 1 = a^0 \quad \text{ev } a^x = a^2$$

$$\therefore x = 0 \quad \therefore x = 2$$

$$\therefore \text{wb}^{\dagger}\text{Y}^{\circledast} \text{ mgvavb } x = 0, 2$$

Abkij bx 5.3

mgvavb Ki :

$$1 | \quad 3^{x+2} = 81$$

$$2 | \quad 5^{3x-7} = 3^{3x-7}$$

$$7 | \quad \frac{5^{3x-5} \cdot b^{2x-6}}{5^{x+1}} = a^{2x-6} (a > 0, b > 0, 5b \neq a)$$

$$8 | \quad 4^{x+2} = 2^{2x+1} + 14$$

3| $2^{x-4} = 4a^{x-6}, (a > 0, a \neq 2)$

9| $5^x + 5^{2-x} = 26$

4| $(\sqrt{3})^{x+5} = (\sqrt[3]{3})^{2x+5}$

10| $3(9^x - 4 \cdot 3^{x-1}) + 1 = 0$

5| $(\sqrt[5]{4})^{4x+7} = (\sqrt[11]{64})^{2x+7}$

11| $4^{1+x} + 4^{1-x} = 10$

6| $\frac{3^{3x-4} \cdot a^{2x-5}}{3^{x+1}} = a^{2x-5} (a > 0)$

12| $2^{2x} - 3 \cdot 2^{x+2} = -32$

5.4 | `B Pj Kwekó wNvZ mgxKiY tRvU

`B Pj Kwekó `Bw GKNvZ mgxKiY A_ev GKw GKNvZ I GKw wNvZ mgxKiY mgštq MvZ tRvUi mgvavb wYq c xwZ gva`wgK exRMvYZ c`#K Avtj vPbv Kiv ntqtQ | GLv`b Gifc `Bw wNvZ mgxKiY mgštq MvZ KuzCq tRvUi mgvavb wYq Avtj vPbv Kiv ntj v |

Dtj L` th, Pj K `Bw x I y ntj $(x, y) = (a, b)$ Gifc AvKv`i tRvUi GKw mgvavb hw` mgxKiY `BwZ x `tj a Ges y `tj b emvtj Zv` i Dfq c¶ mgvb nq |

D`vni Y 1 | mgvavb t $x + \frac{1}{y} = \frac{3}{2}, y + \frac{1}{x} = 3$

mgvavb t $x + \frac{1}{y} = \frac{3}{2}, (i) y + \frac{1}{x} = 3 (ii)$

(i) t_#K $xy + 1 = \frac{3}{2}y (iii) (ii) t_#K, xy + 1 = 3x (iv)$

(iii) I (iv) t_#K $\frac{3}{2}y = 3x$ ev, $y = 2x (v)$

(v) t_#K y Gi gvb (iv) G emtq cvB,

$2x^2 + 1 = 3x$ ev, $2x^2 - 3x + 1 = 0$

ev, $(x-1)(2x-1) = 0 \therefore x = 1$ A_ev $\frac{1}{2}$

(v) t_#K, hLb $x = 1$, ZLb $y = 2$ Ges $x = \frac{1}{2}$, ZLb $y = 1$

\therefore wYq mgvavb $(x, y) = (1, 2), \left(\frac{1}{2}, 1\right)$

D`vni Y 2 | mgvavb Ki t $x^2 = 3x + 6y, xy = 5x + 4y$

mgvavb : $x^2 = 3x + 6y (i) xy = 5x + 4y (ii)$

(i) t_#K (ii) wētqM Kti, $x(x-y) = -2(x-y)$

ev, $x(x-y) + 2(x-y) = 0$

$$\text{ev, } (x-y)(x+2)=0 \therefore x=y \quad (\text{iii})$$

$$\text{ev, } x=-2 \quad (\text{iv})$$

$$(\text{iii}) \mid (\text{i}) \text{ t_tK Avgiv cvB, } y^2=9y \text{ ev, } y(y-9)=0 \therefore y=0 \text{ A_ev } 9$$

$$(\text{iii}) \text{ t_tK, hLb } y=0 \text{ ZLb } x=0 \text{ Ges hLb } y=9, \text{ ZLb } x=9$$

$$\text{Avgiv (iv) \mid (i) t_tK Avgiv cvB, } x=-2 \text{ Ges } 4=-6+6y \text{ ev, } 6y=10 \text{ ev, } y=\frac{5}{3}$$

$$\therefore \text{wbtYq mgvavb } (x, y) = (0, 0), (9, 9), \left(-2, \frac{5}{3}\right)$$

$$\text{D`vni Y 3 \mid mgvavb Ki t } x^2 + y^2 = 61, xy = -30$$

$$\text{mgvavb : } x^2 + y^2 = 61 \quad (\text{i}) \quad xy = -30 \quad (\text{ii})$$

$$(\text{ii}) \text{ tK 2 } \emptyset \text{v iv } \text{Y Kti (i) t_tK wettqM Ki tj Avgiv cvB, } (x-y)^2 = 121 \quad (\text{iii})$$

$$\text{ev, } x-y = \pm 11 \quad (\text{iv})$$

$$(\text{iii}) \mid (\text{iv}) \text{ t_tK,}$$

$$\left. \begin{array}{l} x+y=1 \\ x-y=11 \end{array} \right\} (\text{v}), \quad \left. \begin{array}{l} x+y=1 \\ x-y=-11 \end{array} \right\} (\text{vi}), \quad \left. \begin{array}{l} x+y=-1 \\ x-y=11 \end{array} \right\} (\text{vii}), \quad \left. \begin{array}{l} x+y=-1 \\ x-y=-11 \end{array} \right\} (\text{viii})$$

$$\text{mgvavb Kti cvB,}$$

$$(\text{v}) \text{ t_tK, } x=6, y=-5; \quad (\text{vi}) \text{ t_tK } x=-5, y=6$$

$$(\text{vii}) \text{ t_tK, } x=5, y=-6 \quad (\text{viii}) \text{ t_tK, } x=-6, y=5$$

$$\therefore \text{wbtYq mgvavb } (x, y) = (6, -5), (-5, 6), (5, -6), (-6, 5)$$

$$\text{D`vni Y 4 \mid mgvavb Ki t } x^2 - 2xy + 8y^2 = 8, 3xy - 2y^2 = 4$$

$$\text{mgvavb t } x^2 - 2xy + 8y^2 = 8, \quad (\text{i}) \quad 3xy - 2y^2 = 4 \quad (\text{ii})$$

$$(\text{i}) \text{ Ges (ii) t_tK Avgiv cvB,}$$

$$\frac{x^2 - 2xy + 8y^2}{3xy - 2y^2} = \frac{2}{1} \text{ ev, } x^2 - 2xy + 8y^2 = 6xy - 4y^2$$

$$\text{ev, } x^2 - 8xy + 12y^2 = 0$$

$$\text{ev, } x^2 - 6xy + 2xy + 12y^2 = 0$$

$$\text{ev, } (x-6y)(x-2y)=0 \therefore x=6y \quad (\text{iii}) \text{ A_ev } x=2y \quad (\text{iv})$$

$$(\text{iii}) \text{ t_tK, } x \text{ Gi gvb (ii) G emtq Avgiv cvB,}$$

$$3.6y.y - 2y^2 = 4 \text{ ev, } 16y^2 = 4 \text{ ev, } y^2 = \frac{1}{4} \text{ ev, } y = \pm \frac{1}{2}$$

$$(\text{iii}) \text{ t_tK, } x = 6 \times \left(\pm \frac{1}{2}\right) = \pm 3.$$

Averi (iv) t_#K x Gi gvb (ii) G emtq Avgiv cvB,

$$3.2y \cdot y - 2y^2 = 4 \quad \text{ev, } 4y^2 = 4 \quad \text{ev, } y^2 = 1 \quad \text{ev, } y = \pm 1$$

$$(iv) t_#K x = 2 \times (\pm 1) = \pm 2$$

$$\therefore \text{wb#Y\# mgvavb } (x, y) = \left(3, \frac{1}{2}\right), \left(-3, -\frac{1}{2}\right), (2, 1), (-2, -1)$$

$$D`vni Y 5 | mgvavb Ki t \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2}, x^2 + y^2 = 90$$

$$\text{mgvavb t } \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2} \quad (i) \quad x^2 + y^2 = 90 \quad (ii)$$

(i) t_#K Avgiv cvB,

$$\frac{(x+y)^2 + (x-y)^2}{(x+y)(x-y)} = \frac{5}{2}$$

$$\text{ev, } \frac{2(x^2 + y^2)}{x^2 - y^2} = \frac{5}{2}$$

$$\therefore \frac{2 \times 90}{x^2 - y^2} = \frac{5}{2} \quad [(ii) t_#K x^2 + y^2 = 90 \text{ emtq}]$$

$$\text{ev, } x^2 - y^2 = 72 \quad (iii)$$

$$(ii) + (iii) \text{ wb#j, } 2x^2 = 162 \text{ ev, } x^2 = 81 \quad \text{ev, } x = \pm 9$$

$$\text{Ges } (ii) - (iii) \text{ wb#j, } 2y^2 = 18 \text{ ev, } y^2 = 9 \text{ ev, } y = \pm 3$$

$$\therefore \text{wb#Y\# mgvavb } (x, y) = (9, 3), (9, -3), (-9, 3), (-9, -3)$$

KvR :

D`vni Y 2 Ges 3 Gi mgvavb weKí c×wZ#Z wbY\# Ki |

Ab#kij bx 5.4

mgvavb Ki t

$$1 | (2x+3)(y-1)=14, (x-3)(y-2)=-1$$

$$7 | xy - x^2 = 1, y^2 - xy = 2$$

$$2 | (x-2)(y-1)=3, (x+2)(2y-5)=15$$

$$8 | x^2 - xy = 14, y^2 + xy = 60$$

$$3 | x^2 = 7x + 6y, y^2 = 7y + 6x$$

$$9 | x^2 + y^2 = 25, xy = 12$$

$$4 | x^2 = 73x + 2y, y^2 = 3y + 2x$$

$$10 | \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{10}{3}, x^2 - y^2 = 3$$

$$5 | x + \frac{4}{y} = 1, y + \frac{4}{x} = 25$$

$$11 | x^2 + xy + y^2 = 3, x^2 - xy + y^2 = 7$$

$$6 | y + 3 = \frac{4}{x}, x - 4 = \frac{5}{3y}$$

$$12 | 2x^2 + 3xy + y^2 = 20, 5x^2 + 4y^2 = 41$$

5.5 \mathbb{N} mngxKi \ddot{y} i e \ddot{e} nv

mngxKi \ddot{y} i aviv e \ddot{e} nv K \ddot{t} i \wedge b \wedge b R \ddot{x} etbi enymgm \ddot{v} i mgvavb Kiv hvq | A \ddot{t} bK mgq mgm \ddot{v} q \wedge B \ddot{u} A \ddot{A} vZ iv \wedge ki gvb w \ddot{b} Y \ddot{q} Ki \ddot{t} Z nq | tm \ddot{t} q \ddot{t} i A \ddot{A} vZ iv \wedge ki \wedge B \ddot{u} i gvb x Ges y ev Ab \ddot{t} th \ddot{t} Kvb \wedge B \ddot{u} \wedge Z \ddot{s} ; c \ddot{z} XK ai \ddot{t} Z nq | Z \ddot{v} ici mgm \ddot{v} i kZ \ddot{e} v kZ \ddot{t} v \ddot{t} - \ddot{t} K ci \ddot{u} i Av \ddot{b} f \ddot{p} , m \ddot{z} wZcY \ddot{c} mngxKiY Mv \ddot{b} K \ddot{t} i mngxKiY tR \ddot{v} t \ddot{u} i mgvavb Ki \ddot{t} j B x Ges y A \ddot{A} vZ iv \wedge ki \ddot{t} j \ddot{v} i gvb w \ddot{b} Y \ddot{q} Kiv hvq |

D \ddot{v} niY 1 | \wedge B \ddot{u} eM \ddot{p} q \ddot{t} i \ddot{t} q \ddot{t} i d \ddot{t} j i mgw \ddot{o} 650 eM \ddot{q} U \ddot{v} i | H \wedge B \ddot{u} eM \ddot{p} q \ddot{t} i \wedge B \ddot{u} ev \ddot{u} \ddot{v} iv M \ddot{w} Z AvqZ \ddot{t} q \ddot{t} i \ddot{t} q \ddot{t} i dj 323 eM \ddot{q} U \ddot{v} i n \ddot{t} j, eM \ddot{p} q \ddot{t} i \wedge B \ddot{u} i c \ddot{z} Z \ddot{t} K ev \ddot{u} i cwi gvb KZ? mgvavb : g \ddot{t} b K \ddot{w} i, GK \ddot{u} eM \ddot{p} q \ddot{t} i ev \ddot{u} i cwi gvY x w \ddot{g} U \ddot{v} i Ges Aci \ddot{u} i ev \ddot{u} i cwi gvY y w \ddot{g} U \ddot{v} i |

ck \ddot{g} tZ, $x^2 + y^2 = 650$ (i)

Ges $xy = 323$ (ii)

$\therefore (x + y)^2 = x^2 + y^2 + 2xy = 650 + 646 = 1296$

A \ddot{u} (x + y) = $\pm\sqrt{1296} = \pm 36$

Ges $(x - y)^2 = x^2 + y^2 - 2xy = 650 - 646 = 4$

A \ddot{u} (x - y) = ± 2

th \ddot{t} nZi \wedge N \ddot{c} abvZ \ddot{K} , tm \ddot{t} nZi (x + y) Gi gvb av \ddot{z} Z \ddot{K} n \ddot{t} Z n \ddot{t} e |

$\therefore (x + y) = 36$ (iii)

(x - y) = ± 2 (iv)

th \ddot{v} M K \ddot{t} i, $2x = 36 \pm 2$

$\therefore x = \frac{36 \pm 2}{2} = 18 \pm 1 = 19$ ev, 17

mngxKiY (iii) \ddot{t} - \ddot{t} K cvB, $y = 36 - x = 17$ ev, 19.

\therefore GK \ddot{u} eM \ddot{p} q \ddot{t} i ev \ddot{u} i cwi gvY 19 w \ddot{g} U \ddot{v} i Ges Aci eM \ddot{p} q \ddot{t} i ev \ddot{u} i cwi gvY 17 w \ddot{g} U \ddot{v} i |

D \ddot{v} niY 2 | GK \ddot{u} AvqZ \ddot{t} q \ddot{t} i \wedge N \ddot{c} Z \ddot{v} i c \ddot{t} - \ddot{t} \wedge Y A \ddot{t} c \ddot{q} v 10 w \ddot{g} U \ddot{v} i Kg | AvqZ \ddot{t} q \ddot{t} i \ddot{t} q \ddot{t} i dj 600 eM \ddot{q} U \ddot{v} i n \ddot{t} j, Gi \wedge N \ddot{c} w \ddot{b} Y \ddot{q} Ki |

mgvavb : g \ddot{t} b K \ddot{w} i, AvqZ \ddot{t} q \ddot{t} i \wedge N \ddot{c} = x w \ddot{g} U \ddot{v} i Ges AvqZ \ddot{t} q \ddot{t} i c \ddot{t} ' = y w \ddot{g} U \ddot{v} i

ck \ddot{g} tZ, $2y = x + 10$ (i)

$xy = 600$ (ii)

mngxKiY (i) \ddot{t} - \ddot{t} K cvB, $y = \frac{10 + x}{2}$

mngxKiY (ii) G y Gi gvb ew \ddot{m} tq cvB, $\frac{x(10 + x)}{2} = 600$

ev, $\frac{10x + x^2}{2} = 600$ ev, $x^2 + 10x = 1200$

ev, $x^2 + 10x - 1200 = 0$ ev, $(x + 40)(x - 30) = 0$

m \ddot{z} i vs, $(x + 40) = 0$ A \ddot{u} ev $(x - 30) = 0$

A \ddot{u} , $x = -40$ ev, $x = 30$

WKS' % N° FYvZK nZ cvti bv,

∴ x = 30

∴ AvqZtfti i N° = 30 wglvi |

D`vni Y 3 | B A¼weikó GKwJ msL`vK A¼tqi Ydj Øviv fVM Ki t j fVMdj nq 3. msL`wUj mvt_ 18 thvM Ki t j A¼tq `vb weibgq Kti | msL`wUj wbyq Ki | mgvavb t gtb Kwi , `kK `vbxq A¼ = x Ges GKK `vbxq A¼ y ∴ msL`wUj = 10x + y

cŭg kZtfti , $\frac{10x+y}{xy} = 3$ ev, $10x + y = 3xy$(i)

wZxq kZtfti , $10x + y + 18 = 10y + x$ ev, $9x - 9y + 18 = 0$ ev, $x - y + 2 = 0$ ev, $y = x + 2$(ii)

mgxKi Y (i) G $y = x + 2$ eimtq cvB, $10x + x + 2 = 3 \cdot x(x + 2)$

ev, $11x + 2 = 3x^2 + 6x$

ev, $3x^2 - 5x - 2 = 0$ ev, $3x^2 - 6x + x - 2 = 0$

ev, $3x(x - 2) + 1(x - 2) = 0$

ev, $(x - 2)(3x + 1) = 0$

mZivs $(x - 2) = 0$ A_ ev $(3x + 1) = 0$ ev, $3x = -1$

A_ , $x = 2$ ev, $x = -\frac{1}{3}$

WKS' msL`vi A¼ FYvZK ev fMusk nZ cvti bv |

mZivs $x = 2$ Ges $y = x + 2 = 2 + 2 = 4$

∴ msL`wUj 24

ckgvj v 5.5

- 1 | BwJ eMftfti tftidjt i mgwó 481 eMglvi | H BwJ eMftfti B evú Øviv MwZ AvqZtfti tftidjt 240 eMglvi ntj , eMftfti BwJ cŭZ`K evú cwi gvY KZ ?
- 2 | BwJ avvZK msL`vi eMftfti mgwó 250 | msL`v BwJ Ydj 117 , msL`v BwJ wbyq Ki |
- 3 | GKwJ AvqZtfti KtYp N° 10 wglvi | Bnvi evú tqi thvMdj I wetqvm djt i mgvb N° weikó evú tqi Øviv AwZ AvqZtfti tftidjt 28 eMglvi ntj , cŭg AvqZtfti N° I cŭ' wbyq Ki |
- 4 | BwJ msL`vi eMftfti mgwó 181 Ges msL`v BwJ Ydj 90 , msL`v BwJ eMftfti AŠt wbyq Ki |
- 5 | GKwJ AvqZtfti tftidjt 24 eMglvi | Aci GKwJ AvqZtfti N° I cŭ' cŭg AvqZtfti N° I cŭ' Atcftv h_vµtg 4 wglvi Ges 1 wglvi tewk Ges tftidjt 50 eMglvi | cŭg AvqZtfti N° I cŭ' wbyq Ki |

- 6| GKwU AvqZt¶¶i c¶i wY ^N°Atc¶v 23 wguvi teuk | AvqZt¶¶i t¶¶dj 600 eM¶guvi ntj, Zvi ^N°I c¶'wbY¶ Ki |
- 7| GKwU AvqZt¶¶i cwi mxgv KY¶qi ^¶N¶ mgwó Atc¶v 8 wguvi teuk | t¶¶wU t¶¶dj 48 eM¶guvi ntj, Zvi ^N°I c¶'wbY¶ Ki |
- 8| `B A¼weukó GKwU msL'vK Gi A¼¶qi Ydj ¶viv fV M Ki t j fV M dj 2 nq | msL'wU mv¶ 27 thV M Ki t j A¼¶q ¶vb weibgq K ti | msL'wU wbY¶ Ki |
- 9| GKwU AvqZKvi eMv¶bi cwi mxgv 56 wguvi Ges KY°20 wguvi | H eMv¶bi mgvb t¶¶dj weukó eM¶¶i GK evú i ^N°KZ ?
- 10| GKwU AvqZt¶¶i t¶¶dj 300 eM¶guvi Ges Gi Aa¶wi mxgv GKwU KY°Atc¶v 10 wguvi teuk | t¶¶wU ^N°I c¶'wbY¶ Ki |

5.6 | `B Pj Kweukó mPK mgxKi Y tRvU

ceZP Aa¶v¶q GK Pj Kweukó mPK mgxKi¶Yi mgvavb wbY¶ m¶ú¶K°Av¶j vPbv Kiv ntq¶Q | `B Pj Kweukó mPKxq mgxKi Y tRv¶Ui mgvavb wbY¶ Kiv m¶ú¶K°GLv¶b Av¶j vPbv Kiv ntj v |

D`vni Y 1 | mgvavb Ki : $a^{x+2} \cdot a^{2y+1} = a^{10}$, $a^{2x} \cdot a^{y+1} = a^9 (a \neq 1)$

mgvavb : $a^{x+2} \cdot a^{2y+1} = a^{10}$ (i) $a^{2x} \cdot a^{y+1} = a^9$ (ii)

(i) t_¶K $a^{x+2y+3} = a^{10}$ ev, $x + 2y + 3 = 10$ ev, $x + 2y - 7 = 0$ (iii)

(ii) t_¶K, $a^{2x+y+1} = a^9$ ev, $2x + y + 1 = 9$ ev, $2x + y - 8 = 0$ (iv)

(iii) I (iv) t_¶K eR¶ Yb c×wZ Ab¶v¶i ,

$$\frac{x}{-16+7} = \frac{y}{-14+8} = \frac{1}{1-4}$$

ev, $\frac{x}{-9} = \frac{y}{-6} = \frac{1}{-3}$

ev, $\frac{x}{3} = \frac{y}{2} = 1$

ev, $x = 3, y = 2$

∴ wb¶Y¶ mgvavb $(x, y) = (3, 2)$

D`vni Y 2 | mgvavb Ki : $3^{3y-1} = 9^{x+y}$, $4^{x+3y} = 16^{2x+3}$

mgvavb : $3^{3y-1} = 9^{x+y}$ (i)

ev, $3^{3y-1} = (3^2)^{x+y} = 3^{2x+2y}$

∴ $3y - 1 = 2x + 2y$

$$\text{ev, } 2x - y + 1 = 0 \quad (\text{iii})$$

$$4^{x+3y} = 16^{2x+3} \quad (\text{ii})$$

$$\text{ev, } 4^{x+3y} = (4^2)^{2x+3} \text{ ev, } 4^{x+3y} = 4^{4x+6}$$

$$\text{ev, } x + 3y = 4x + 6 \text{ ev, } 3x - 3y + 6 = 0$$

$$\text{ev, } x - y + 2 = 0 \quad (\text{iv})$$

(iii) | (iv) †_†K eR^a, Yb c×wZ Abym†i ,

$$\frac{x}{-2+1} = \frac{y}{1-4} = \frac{1}{-2+1}$$

$$\text{ev, } \frac{x}{-1} = \frac{y}{-3} = -1$$

$$\text{ev, } x = 1, y = 3$$

$$\therefore \text{wb†Y} \text{ mgvavb } (x, y) = (1, 3)$$

D`vni Y 3 | mgvavb Ki : $x^y = y^x, x = 2y$

$$\text{mgvavb : } x^y = y^x \quad (\text{i}) \quad x = 2y \quad (\text{ii}) \text{ GLv†b } x \neq 0, y \neq 0$$

$$(\text{ii}) \text{ †_†K } x \text{ Gi gvb } (\text{i}) \text{ G ewm†q cvB, } (2y)^y = y^{2y} \text{ ev, } 2^y \cdot y^y = y^{2y}$$

$$\text{ev, } \frac{y^{2y}}{y^y} = 2^y \text{ ev, } y^y = 2^y \therefore y = 2 \quad (\text{ii}) \text{ †_†K, } x = 4$$

$$\therefore \text{wb†Y} \text{ mgvavb } (x, y) = (4, 2)$$

D`vni Y 4 | mgvavb Ki : $x^y = y^2, y^{2y} = x^4, \text{ thLv†b } x \neq 1$

$$\text{mgvavb : } x^y = y^2 \quad (\text{i}), \quad y^{2y} = x^4 \quad (\text{ii})$$

(i) †_†K cvB,

$$(x^y)^y = (y^2)^y \text{ ev, } x^{y^2} = y^{2y} \quad (\text{iii})$$

(iii) | (ii) †_†K cvB, $x^{y^2} = x^4$

$$\therefore y^2 = 4 \text{ ev, } y = \pm 2$$

$$\text{GLb } y = 2 \text{ ntj } (\text{i}) \text{ †_†K cvB, } x^2 = 2^2 = 4 \text{ ev, } x = \pm 2$$

$$\text{Averi, } y = -2 \text{ ntj, } (\text{i}) \text{ †_†K cvB, } (x)^{-2} = (-2)^2 = 4$$

$$\text{ev, } \frac{1}{x^2} = 4 \text{ ev, } x^2 = \frac{1}{4} \text{ ev, } x = \pm \frac{1}{2}$$

$$\therefore \text{wb†Y} \text{ mgvavb } (x, y) = (2, 2), (-2, 2), \left(\frac{1}{2}, -2\right), \left(-\frac{1}{2}, -2\right)$$

D`vni Y 5 | mgvavb Ki : $8 \cdot 2^{xy} = 4^y$, $9^x \cdot 3^{xy} = \frac{1}{27}$

mgvavb : $8 \cdot 2^{xy} = 4^y$ (i), $9^x \cdot 3^{xy} = \frac{1}{27}$ (ii)

(i) t_#K cvB, $2^3 \cdot 2^{xy} = (2^2)^y$ ev, $2^{3+xy} = 2^{2y}$ $\therefore 3+xy = 2y$ (iii)

(ii) t_#K cvB, $(3^2)^x \cdot 3^{xy} = \frac{1}{3^3}$ ev, $3^{2x+xy} = 3^{-3}$ $\therefore 2x+xy = -3$ (iv)

(iii) t_#K (iv) wqtqM Kti cvB, $3-2x = 2y+3$ ev, $-x = y$

(v) t_#K y Gi gvb (iii) G ewntq cvB, $3-x^2 = -2x$

ev, $x^2 - 2x - 3 = 0$ ev, $(x+1)(x-3) = 0$

$\therefore x = -1$ A_ev $x = 3$

$x = -1$ ntj (v) t_#K cvB, $y = 1$; $x = 3$ ntj (v) t_#K cvB, $y = -3$

\therefore wbtYq mgvavb $(x, y) = (-1, 1), (3, -3)$

D`vni Y 6 | mgvavb Ki : $18y^x - y^{2x} = 81$, $3^x = y^2$

mgvavb : $18y^x - y^{2x} = 81$, (i) $3^x = y^2$ (ii)

(i) t_#K cvB, $y^{2x} - 18y^x + 81 = 0$ ev, $(y^x - 9)^2 = 0$

ev, $y^x - 9 = 0$ ev, $y^x = 3^2$ (iii)

(ii) t_#K cvB, $(3^x)^y = (y^2)^y$ ev, $3^{x^2} = y^{2x}$ (iv)

(iii) t_#K cvB, $(yx)^2 = (3^2)^2$ ev, $y^{2x} = 3^4$ (v)

(iv) l (v) t_#K cvB, $3^{x^2} = 3^4$ $\therefore x^2 = 4$ ev, $x = \pm 2$

$x = 2$ ntj (ii) t_#K cvB, $y^2 = 9$ ev, $y = \pm 3$

$x = -2$ ntj (iii) t_#K cvB, $y^{-2} = 9$ ev, $y^2 = \frac{1}{9}$ ev, $y = \pm \frac{1}{3}$

\therefore wbtYq mgvavb $(x, y) = (2, 3), (2, -3), \left(-2, \frac{1}{3}\right), \left(-2, -\frac{1}{3}\right)$

Abkij bñ5-6

mgvavb Ki :

1 | $2^x + 3^y = 31$

$2^x - 3^y = -23$

2 | $3^x = 9^y$

$5^{x+y+1} = 25^{xy}$

3 | $3^x \cdot 9^y = 81$

$2x - y = 8$

4| $2^x \cdot 3^y = 18$
 $2^{2x} \cdot 3^y = 36$

5| $a^x \cdot a^{y+1} = a^7$
 $a^{2y} \cdot a^{3x+5} = a^{20}$

6| $y^x = x^2$
 $x^{2x} = y^4$ } $y \neq 1$

7| $y^x = 4$
 $y^2 = 2^x$

8| $4^x = 2^y$
 $(27)^{xy} = 9^{y+1}$

9| $8y^x - y^{2x} = 16$
 $2^x = y^2$

5.7 tj LwPŕŕi mrvvth" wŦNvZ mgxKiY $ax^2 + bx + c = 0$ Gi mgvavb

wŦNvZ mgxKiY $ax^2 + bx + c = 0$ Gi mgvavb Avgiv BŕZvcŕe© exRMwYZxq c×wZŕZ wkŕLwQ| GLb tj LwPŕŕi mrvvth" Bnvi mgvavb c×wZ Avŕj vPbv Kiv nŕe|

gŕb Kwi $y = ax^2 + bx + c$. Zvntj x Gi th mKj gvŕbi Rb" $y = 0$ nŕe A_ŕ tj LwPŕŕi x -AŕŕŕK tŦ" Kiŕe, x Gi H mKj gvŕb-B $ax^2 + bx + c = 0$ mgxKiYwŦi mgvavb|

D`vniY 1| tj LwPŕŕi mrvvth" $x^2 - 5x + 4 = 0$ Gi mgvavb Ki |

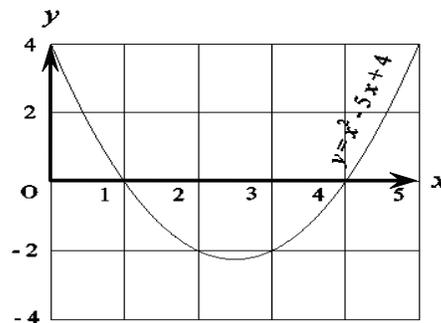
mgvavb : gŕb Kwi , $y = x^2 - 5x + 4$.

x Gi KŕqKwŦ gvŕbi Rb" y Gi gvŕb wŦYŕ Kŕi GB mgxKiŕYi tj ŕLi KŕqKwŦ wŦ" j ŕvbsK wŦYŕ Kwi :

x	0	1	2	2.5	3	4	5
y	4	0	-2	-2.25	-2	0	4

Dcti i mviwŦZ cŦB wŦ" j v OK KwŦŕ ŕvcb Kŕi mgxKiYwŦi tj LwPŕŕi AŦb Kwi | ŕLv hvq th tj LwPŕŕi x -AŕŕŕK (1, 0) | (4, 0) wŦ" ŕZ tŦ" KŕiŕQ|

mŕZivs, mgxKiYwŦi mgvavb $x = 1$ ev $x = 4$.



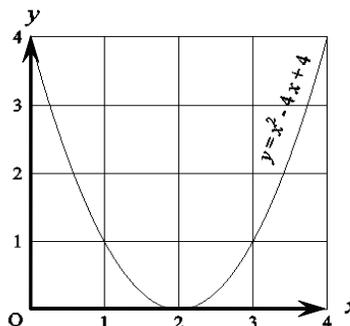
D`vniY 2| tj LwPŕŕi mrvvth" $x^2 - 4x + 4 = 0$ Gi mgvavb Ki |

mgvavb : gŕb Kwi , $y = x^2 - 4x + 4$.

x Gi KŕqKwŦ gvŕbi Rb" y Gi gvŕb wŦYŕ Kŕi tj LwPŕŕi Rb" KŕqKwŦ wŦ" j ŕvbsK wŦYŕ Kwi t

x	0	1	1.5	2	2.5	3	4
y	4	1	0.25	0	0.25	1	4

Dctii mviwY ntZ c0B we`y,tjv QK KvMfR `vcb Kti mgxKiYwUi tj LwPti A4b Kwi | tj LwPti t`Lv hvq th Bnv x-AqitK (2, 0) we`fZ `uk©KtiitQ | thtnZi w0NvZ mgxKiitYi `BwU gj `vfk, tmtnZi mgxKiYwUi mgvavb nte $x = 2, x = 2$.



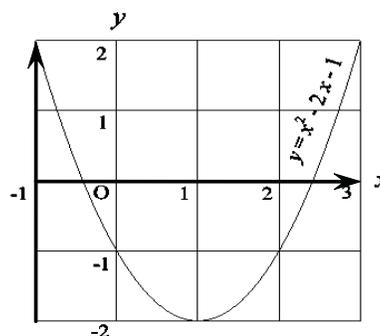
D`vniY 3 | tj LwPti i mrvth` mgvavb Ki t $x^2 - 2x - 1 = 0$

mgvavb : gfb Kwi , $y = x^2 - 2x - 1$.

mgxKiYwUi tj LwPti A4tbi Rb` x Gi KtqKwU gvb wbtq Zv` i Abjfc y Gi gvb wby© Kwi :

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	2	0.25	-1	-1.75	-2	-1.75	-1	0.25	2

mviwYfZ `wcz we`y,tjv QK KvMfR `vcb Kti mgxKiYwUi tj LwPti A4b Kwi | t`Lv hvq th tj LwPtiw x-AqitK tgvUvgUfvte (-0.4, 0) | (2.4, 0) we`fZ tQ` KtiitQ | mZivs, mgxKiYwUi mgvavb $x = -0.4$ (Avmbø ev $x = 2.4$ (Avmbø|



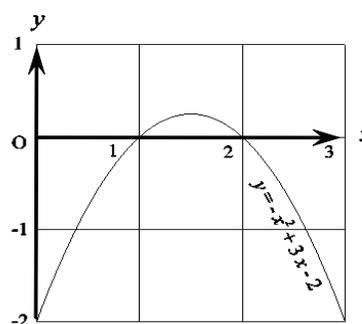
D`vniY 4 | $-x^2 + 3x - 2 = 0$ Gi gj 0q tj LwPti i mrvth` wby© Ki |

mgvavb : gfb Kwi , $y = -x^2 + 3x - 2$.

x Gi KtqKwU gvtbi Rb` y Gi gvb wby© Kti c0B mgxKiitYi tj tLi KtqKwU we`j `vbsK wby© Kwi :

x	0	0.5	1	1.5	2	2.5	3
y	-2	-0.75	0	0.25	0	-0.75	-2

c0B we`y,tjv QK KvMfR `vcb Kti mgxKiYwUi tj LwPti A4b Kwi | t`Lv hvq th tj LwPtiw x - AqitK Dci (1, 0) | (2, 0) we`y w`tq wMtqitQ | mZivs mgxKiYwUi mgvavb $x = 1$ ev $x = 2$.



Abjxj bx 5.7

1) $ax^2 + bx + c = 0$ Ges a, b, c ev⁻e msL^{vi} ntj $x^2 - x - 12 = 0$ mgxKi tY b Gi gvb tKvbU?

K. 0 L. 1

M. -1 N. 3

2) $16^x = 4^{x+1}$ mgxKi YU i mgvavb tKvbU?

K. 2 L. 0

M. 4 N. 3

3) $x^2 - x + 13 = 0$ ntj mgxKi YU i GKU gj tKvbU?

K. $\frac{-1 + \sqrt{-51}}{2}$ L. $\frac{-1 - \sqrt{51}}{2}$

M. $\frac{1 + \sqrt{-51}}{2}$ N. $\frac{1 + \sqrt{51}}{2}$

4) $y^x = 9, y^2 = 3^x$ ntj mWK mgvavb tKvbU?

K. $(2, 3), (-2, \frac{1}{9})$ L. $(2, -2), (3, \frac{1}{9})$

M. $(2, \frac{1}{9}), (-2, 3)$ N. $(-2, -\frac{1}{9}), (2, 3)$

vb tPi Z t i w f w E t Z 5 I 6 bs c k i e D E i ` v l :

` B U abv Z K c Y ms L vi e t M P A S t 11 Ges , Y d j 30 |

5) msL^{vi} ` B U w K w K ?

K. 1 Ges 30 L. 2 Ges 15

M. 5 Ges 6 N. 5 Ges -6

6) msL^{vi} ` B U i e t M P mgw K Z ?

K. 1 L. 5

M. 61 N. $\sqrt{41}$

7) GKU msL^{vi} I H msL^{vi} , Yv Z K w e c i x Z msL^{vi} mgw 6 | m e t e mgxKi YU Mvb Ki t j n q-

i $x + \frac{1}{x} = 6$

ii $x^2 + 1 = 6x$

iii $x^2 - 6x - 1 = 0$

vb tPi tKvbU mWK ?

K. i I ii L. i I iii

M. ii I iii N. i, ii I iii

8) $2^{px-1} = 2q^{px-2}$ Gi mgvavb tKvbU?

K. $\frac{p}{2}$ L. p

M. $-\frac{p}{2}$ N. $\frac{2}{p}$

tj LwPŕĪ i mvrvtĥ" wbtPi mgrKiY ſj vi mgrvab Ki :

$$9 | \quad x^2 - 4x + 3 = 0$$

$$10 | \quad x^2 + 2x - 3 = 0$$

$$11 | \quad x^2 + 7x = 0$$

$$12 | \quad 2x^2 - 7x + 3 = 0$$

$$13 | \quad 2x^2 - 5x + 2 = 0$$

$$14 | \quad x^2 + 8x + 16 = 0$$

$$15 | \quad x^2 + x - 3 = 0$$

$$16 | \quad x^2 = 8$$

17 | GKwU msL"vi eŕMŕ wŰ ſY msL"vU i 5 ſY t_ŕK 3 Kg | wKŠ' H msL"vU i eŕMŕ 3 ſY msL"vU i 5 ſY t_ŕK 3 tenk |

K. DĪ xcŕKi Z_ ſj vi mvrvtĥ" mgrKiY MVb Ki |

L. mŕ cŕqM Kŕi 1g mgrKiYwU mgrvab Ki |

M. 2q mgrKiYwU tj LwPŕĪ i mvrvtĥ" mgrvab Ki |

18 | Rbve AvkdvK Avj xi Rŕgi tŕĪ dj 0.12 tn± i | RŕgU i Aaŕwi mxgv Gi GKwU KYŕAŕcŕŕv 20 wglvi

tenk | wZwb Zwi Rŕg t_ŕK k'vgevej wBKU GK ZZxqisk wŕwŕ Kŕi b | k'vg evej Rŕgi ŕŕ Nŕ, cŕŕ' Aŕcŕŕv 5 wglvi tenk |

K. DĪ xcŕKi Avŕj vŕK `ŕwU mgrKiY MVb Ki |

L. AvkdvK Avj xi Rŕgi ŕŕ Nŕ I cŕŕ' wŕYŕ Ki |

M. k'vgevej RŕgU i KŕYŕ ŕŕ Nŕ I cwi mxgv wŕYŕ Ki |

I ô Aa`vq AmgZv

mgxKiY ev mgZv m^utK^oAvgt`i aviYv ntqtQ| wKŠ' ev`e Rxeþb AmgZvi I GKUv we`Z I „i“ZcY^o fngKv i tqtQ| ``bw`b Rxeþb cKwZtZ Avgiv hZwKQz`wL Zvi tKvbwUi tŕŕtB GK RvZxq `þwU e`i ev RxeRŠi ev `þRb gvbþI i thtKvþbv aiþbi cwi gvc úeú GK cvl qv hvq bv| GgbwK t`LtZI GKB i Kg nq bv| dtj Avgvt`i AmgZvi aviYv cŕqvRb nq|

Aa`vq tktI wKŕv_xŕv –

- GK I `þ Pj tKi GK NvZ wewkó AmgZv e`vL`v Ki tZ cvi te|
- `þ Pj Kwekó mij AmgZv MVb I mgvavb Ki tZ cvi te|
- ev`erfweK MwYwZK mgm`vq AmgZv e`envi Kti mgvavb Ki tZ cvi te|
- GK I `þ Pj Kwekó AmgZvtK tj LwPŕt i mrvvth` mgvavb Ki tZ cvi te|

AmgZv

gtb Kwi GKwU Kwtki QvT msL`v 200 Rb| `ŕfvweKfvte t`Lv hvq th, H Kwtk mew`b mKtj Dcw`Z `vK bv| GKwU wbow`w`þb Dcw`Z QvT msL`v x ntj Avgiv wj LtZ cwi $0 < x \leq 200$ | GKB fvte Avgiv t`wL th, tKvþbvI GKwU wlogwšZ Abþvþb mevB Dcw`Z nq bv| tcvkvK-cwi`Q` I Ab`vb` AþbK tfvM`cY` `Zwi tZ cwi`vi fvte AmgZvi aviYv cŕqvRb nq| `vj vb `Zwi i tŕŕt, cy`K gy`ŕYi tŕŕt Ges Gi Kg Avi I AþbK tŕŕt Dcv`vb`_tj v mwK cwi gvtY wbyŕ Kiv hvq bv weavq cŕg chŕq Abgvþbi wfvE tZ Dcv`vb`_tj v µq ev msMh Ki tZ nq| AZGe t`Lv hvte`Q` th, Avgvt`i ``bw`b Rxeþb AmgZvi aviYvUv LþB „i“ZcY^o

mgxKiY msµvš`Ztwm× ev weiamgn AmgZvi tŕŕtI cŕhvR`| i aye`wZµg ntj v Amgvb i wktK mgvb mgvb FyVZK msL`v Øviv `Y ev fvM Ki tZ AmgZvi w`K cvtè hvq|

4<6 AmgZwU j ŕI Kwi |

∴ 4+2 < 6 + 2 ev, 6< 8

[Dfqcŕŕ 2 thvM Kti]

Z`*c 2 < 4

[Dfqcŕŕ t`tK 2 weŕqvM Kti]

Z`*c 4<12

[DfqcŕŕtK 2 Øviv `Y Kti]

Z`*c 2<3

[DfqcŕŕtK 2 Øviv fvM Kti]

AmgZwUi DfqcŕŕtK -2 Øviv `Y Ki tZ Avj v`vfvte cvl qv hvq -8 Ges -12

GLvþb -8>-12, tZgwb -2>-3 {DfqcŕŕtK -4 Øviv fvM Kti}

mvavi Yfvte ej v hvq, hw` a < b nq, Zte,

$a + c < b + c$	c Gi thtKvfbv gvfbv Rb"
$a - c < b - c$	c Gi thtKvfbv gvfbv Rb"
$ac < bc$	c Gi abvZK gvfbv Rb"
$\frac{a}{c} < \frac{b}{c}$	c Gi abvZK gvfbv Rb"
$\text{wKŠ' } ac > bc$	c Gi FYvZK gvfbv Rb"
$\frac{a}{c} > \frac{b}{c}$	c Gi FYvZK gvfbv Rb"

KvR: 1 | tZvgv` i tkivYi th mKj QvI-QvIxi D"PIv 5 dtUi tPtq tewk Ges 5 dtUi tPtq Kg Zv` i D"PIv
 AmgZvi gva`tg cKvk Ki |
 2 | tKvfbv cixvivi tgvU baf 1000 ntj , GKRB cixvixp cÜB baf AmgZvi gva`tg cKvk Ki |

D`vni Y 1 | mgvavb Ki I mgvavb tmUw msL`vfi Lvq t`Lvl : $4x + 4 > 16$

mgvavb : t` I qv AvtQ, $4x + 4 > 16$

$\therefore 4x + 4 - 4 > 16 - 4$ [Dfqcv t`tk 4 wetaqm Kti]

ev, $4x > 12$

ev, $\frac{4x}{4} > \frac{12}{4}$ [Dfqcv tk 4 Øviv fvM Kti]

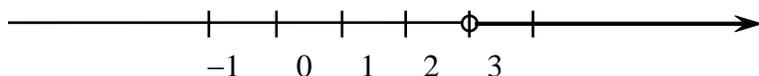
ev, $x > 3$

\therefore wbtYq mgvavb $x > 3$

GLvfb mgvavb tmU, $S = \{x \in R : x > 3\}$

mgvavb tmUw wbtP AwZ msL`vfi Lvq t`Lvfbv ntjv | 3 Atcvv eo mKj ev`e msL`v cÜE AmgZvi

mgvavb Ges mgvavb tmU, $S = \{x \in R : x > 3\}$



D`vni Y 2 | mgvavb Ki Ges mgvavb tmU msL`vfi Lvq t`Lvl : $x - 9 > 3x + 1$

mgvavb : t` I qv AvtQ, $x - 9 > 3x + 1$

$\therefore x - 9 + 9 > 3x + 1 + 9$

ev, $x > 3x + 10$

ev, $x - 3x > 3x + 10 - 3x$

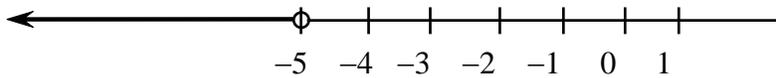
ev, $-2x > 10$

ev, $\frac{-2x}{-2} < \frac{10}{-2}$ [Dfqcv tk FYvZK msL`v -2 Øviv fvM Kivq]

ev, $x < -5$ AmgZvi w`K cvtë tMfQ]

∴ wɔtYq mgvavb $x < -5$

GLvɔb mgvavb tɔU $S = \{x \in R : x < -5\}$, A_ŕ -5 Aɔcŕv tQvU mKj ev e msL'v cŕE AmgZvi mgvavb |



we: a mgxKiɔyi mgvavb thgb GKwU mgxKiY (mgZv) Őviv cKvk cvq, tZgwb AmgZvi mgvavb GKwU AmgZv Őviv cKvk cvq | AmgZvi mgvavb tɔU (mvaviYZ) ev e msL'vi Amxg DcɔtɔU |

$a \geq b$ Gi A_ŕ, $a > b$ A_ev $a = b$

A_ŕ i ay $a < b$ nɔj B $a \geq b$ wq_v nq |

AZGe, $4 > 3$ Ges $4 = 4$ BwU DvB mZ |

D`vniY 3 | mgvavb Ki t $a(x + b) < c$, [$a \neq 0$]

mgvavb : a abvZK nɔj, $\frac{a(x + b)}{a} < \frac{c}{a}$, DfqcŕtK a Őviv fvM Kɔi cvB,

$$x + b < \frac{c}{a} \quad \text{ev, } x < \frac{c}{a} - b$$

a FYvZK nɔj GKB cŕvq cvB, $\frac{a(x + b)}{a} > \frac{c}{a}$

$$\text{ev, } x + b > \frac{c}{a} \quad \text{ev, } x > \frac{c}{a} - b$$

∴ wɔtYq mgvavb : (i) $x < \frac{c}{a} - b$, hw` $a > 0$ nq,

$$(ii) x > \frac{c}{a} - b, \text{ hw` } a < 0 \text{ nq |}$$

we: a hw` kb` Ges c hw` abvZK nq, Zte x Gi thɔKvɔbv gvɔbi Rb` AmgZvU mZ` nte | wKŠ` a hw` kb` Ges c FYvZK nq, Zte AmgZvU i tKɔbv mgvavb vKte bv |

ckg v 6.1

AmgZv, tɔj v mgvavb Ki Ges msL'v i qLvq mgvavb tɔU t` Lv | :

$$1 | y - 3 < 5 \quad 2 | 3(x - 2) < 6 \quad 3 | 3x - 2 > 2x - 1 \quad 4 | z \leq \frac{1}{2}z + 3$$

$$5 | 8 \geq 2 - 2x \quad 6 | x \leq \frac{x}{3} + 4 \quad 7 | 5(3 - 2t) \leq 3(4 - 3t) \quad 8 | \frac{x}{3} + \frac{x}{4} + \frac{x}{5} > \frac{47}{60}$$

AmgZvi e"envi

mgiKiYi mivvth" tZvgiv mgm"v mgravb Ki tZ wktLQ | GKB cxiZtZ AmgZvi m"uKZ mgm"vi mgravb Ki tZ cvi te |

D`vniY 1 | tKvfbv cixvq evsj v 1g l 2q cT i gv tctqQ h_vmtg 5x Ges 6x baf Ges KgKg tctqQ 4x Ges 84 baf | tKvfbv cT tKD 40 Gi wbtP cvqub | evsj v wltq KgKg ntqQ cDg Ges i gv ntqQ wZxq | x Gi gvb m"e" AmgZvi gva"tg cKvk Ki |

mgravb : i gv tctqQ tgvU $5x + 6x$ baf Ges KgKg tctqQ $4x + 84$ tgvU baf |

ckgtZ, $5x + 6x < 4x + 84$

ev, $5x + 6x < 4x + 84$ ev, $7x < 84$

ev, $x < \frac{84}{7}$ ev, $x < 12$

wKs, $4x \geq 40$ [cB meba baf 40] ev, $x \geq 10$

\therefore AmgZvi gva"tg wj Lv hvq $10 \leq x \leq 12$

D`vniY 2 | GKRB QvT 5 UvKv `ti x wJ tcvYj Ges 8 UvKv `ti $(x + 4)$ wJ LvZv wktbtQ | tgvU gj " AbaY97 UvKv ntj , mebaK KqW tcvYj wktbtQ?

mgravb : x wJ tcvYj i `vg 5x UvKv Ges $(x + 4)$ wJ LvZvi `vg $8(x + 4)$ UvKv |

ckgtZ, $5x + 8(x + 4) \leq 97$ ev, $5x + 8(x + 4) \leq 97$

ev, $13x \leq 97 - 32$ ev, $13x \leq 65$

ev, $x \leq \frac{65}{13}$ ev, $x \leq 5$

\therefore QvT wJ mebaK 5 wJ tcvYj wktbtQ |

KvR : 140 UvKv tKvR `ti tWvFW x tKvR Avtcj wktbtj b | wZwb wtpZvK 1000 UvKvi GKlvb tvU w` tj b | wtpZv 50 UvKvi x Lvbv tvUmn evKx UvKv tdi Z w` tj b | mgm"wuK AmgZvi gva"tg cKvk Ki Ges x Gi m"e" gvb wBYK Ki |

ckgtj v 6.2

1N5 chS-mgm"v , tj v AmgZvi gva"tg cKvk Ki Ges x Gi gvb m"e" gvb wBYK Ki |

1 | GK evj K NÈvq x wK. wg. tetM 3 NÈv nuUj Ges NÈvq $(x + 2)$ wK. wg. tetM $\frac{1}{2}$ NÈv t`šovj Ges Zvi AwZpvš-c_29 wK. wg. Gi Kg |

2 | GKwU tewWvS-G tivR 4x tKvR Pvj Ges $(x - 3)$ tKvR Wvj j vM Ges Pvj l Wvj wgtj 40 tKvRi tenk j vM bv |

- 3| 70 UvKv tKwR `ti tmvnie mvfne x tKwR Avg wKbtj b| wepmZvtK 500 UvKvi GKlvbv tlvU w`tj b| wepmZv 20 UvKvi x Lvbv tlvUmn ewK UvKv tdi Z w`tj b|
- 4| GKwU Mvwo 4 NÈvq hvq x wK. wg. Ges 5 NÈvq hvq (x + 120) wK. wg. | MvwoUi Mo MvZteM NÈvq 100 wK. wg. Gi tevk bq|
- 5| GK UKiv KvMfRi tTdj 40 eMtm. wg. | Zv t_tK x tm. wg. `xN Ges 5 tm. wg. cUwekó AvqZvKvi KvMR tKtU tbi qv ntj v|
- 6| cTti eqm gvtqi eqtmi GK-ZZxqsk | wczv gvtqi tPtq 6 eQti i eo | wZbRtbi eqtmi mgwó AbpY90 eQi | wczvi eqm AmgZvi gva`tg cKvk Ki |
- 7| tRwb 14 eQi eqtm Rvbi eqm cixTlv w`tqvQj | 17 eQi eqtm tm Gm.Gm.wm cixTlv w`te | Zvi eZgvb eqm AmgZvq cKvk Ki |
- 8| GKlvb tRU tctbi MvZ cZ tmfKtU mefak 300 wgvUv | tcvU 15 wK. wg. hvl qvi ctqvRbxq mgq AmgZvq cKvk Ki |
- 9| XvKv t_tK tRi vi wegvb ct_ `tZj 5000 wK.wg | tRU wegvb mteP MvZteM NÈvq 900 wK. wg. | wKs' XvKv t_tK tRi v hvevi ct_ cZKj w`tK NÈvq 100 wK. wg. tetM evqcvtni m`x nZ nq | XvKv t_tK tRi vi weivZnxb DÇqfbi cqvRbxq mgq GKwU AmgZvi gva`tg cKvk Ki |
- 10| ceZr`cKkè m` a`ti, tRi v t_tK XvKv tdivi ct_ DÇqfbi cqvRbxq mgq GKwU AmgZvi gva`tg cKvk Ki |
- 11| tKvfbv avZK cymsL`vi 5 ,Y, msL`vUv w`Y Ges 15 Gi mgwó AtcTlv tQvU | msL`vUv m`e` gvb AmgZvq cKvk Ki |

`B Pj Kwekó mij GKvZ AmgZv

Avgiv `B Pj Kwekó $y = mx + c$ (hvi mvariY AvKvi $ax + by + c = 0$) AvKv`i mij mgxKiYi tj LwP A/b KiZ wktLwQ (mBg tkYxi exRMwYZ cy`K `ðe") | Avgiv t`tLwQ th, G iKg cZ`K tj LwP`B GKwU mij tiLv |

vbwvqZ x, y mgZtj $ax + by + c = 0$ mgxKiYi tj LwP`i thtKvfbv we`j vbwv mgxKiYw`K w`x Kti A` mgxKiYwU evgctT x l y Gi cwietZ`h_v`tg H we`j fR l tKwU emvtj Gi gvb kb` nq | Ab`w`tK, tj LwP`i evBti tKvfbv we`j vbwv`B mgxKiYw`K w`x Kti bv A` H we`j fR tKwU Rb` $ax + by + c$ Gi gvb kb` AtcTlv eo ev tQvU nq | mgZj t` tKvfbv we`y P Gi fR l tKwU Øviv $ax + by + c$ iwki x l y tK h_v`tg cZ`vcb Kiti iwkwU th gvb nq, ZvtK P we`tZ iwkwU gvb ejv nq Ges D^3 gvb`K mvariYZ $f(P)$ Øviv wbt`R Kiv nq | P we`ytj Lw`Z ntj $f(P) = 0$, P we`ytj LwP`i emt`ntj $f(P) > 0$ A_ev $f(P) < 0$

ev`te tj LwPti ewnt`mKj we`ytj L 0v`v`Bw Aaztj wef³ nq t GKw Aaztj i c0Z`K we`yP Gi Rb` f(P) > 0; Aci Aaztj i c0Z`K we`yP Gi Rb` f(P) < 0.

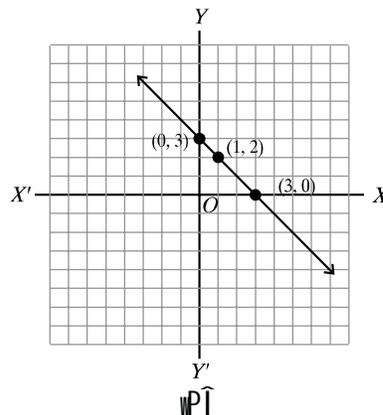
ejv evuj, tj tLi Dci Aew`Z c0Z`K we`yP Gi Rb` f(P) = 0

D`vni Y 1 | $x + y - 3 = 0$ mgxKi YwJ we`Pbv KwI | mgxKi YwJ t`tK cvl qv hvq t

$$y = 3 - x$$

x	0	3	1
y	3	0	2

Ges (x,y) mgZtj QK KwMtr t0vU eM`tj i evui` NqK GKK ati mgxKi YwJi tj LwPti wbaifc nq:



GB tj LwPti ti Lv mgM0Zj wUtK wZbwJ Ask c`K Kti | h_v:

(1) ti Lvi (K) wPwyZ cvtki we`yngn

(2) ti Lvi (L) wPwyZ cvtki we`yngn (3) ti Lw`Z we`yngn |

GLvfb (K) wPwyZ Asktk tj L ti Lvi 0Dcti i Ask0 I (L) wPwyZ Asktk tj L- ti Lvi 0wbtpi Ask0 ejv hvq |

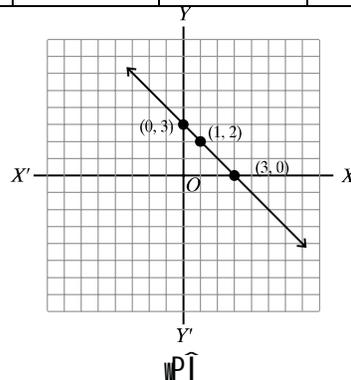
`B Pj Kwewk0 Amgzvi tj LwPti

D`vni Y 2 | $x + y - 3 > 0$ A_ev $x + y - 3 < 0$ Amgzvi tj LwPti Askb Ki |

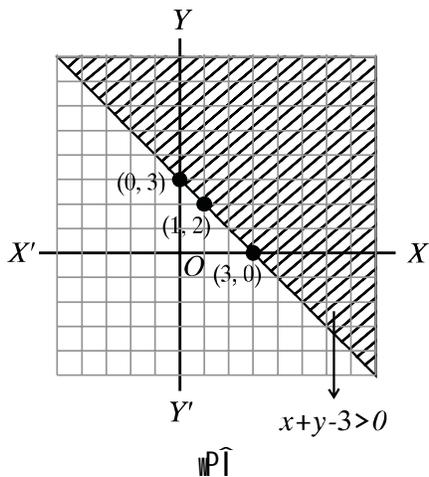
mgvavb : Dctiv³ Amgzv0tqi tj LwPti Askb Kitz c0tgB QK KwMtr $x + y - 3 = 0$ mgxKi YwJi tj LwPti Askb KwI |

$x + y - 3 = 0$ mgxKi Y t`tK cvB

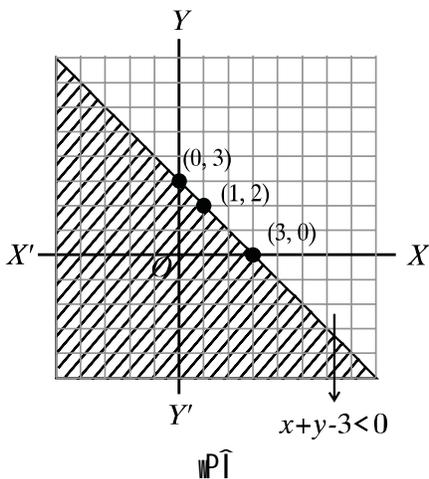
x	0	3	1
y	3	0	2



$x + y - 3 > 0$ AmgZvi tj LwPÎ A¼tbi Rb" D³ AmgZvq gj we`y(0, 0) Gi gvb emvtj Avgiv cvB $-3 > 0$ hv mZ" bq| KvtrB, AmgZvi QvqwPÎ nte $x + y - 3 = 0$ ti Lvi th cvtk gj we`yi tqtQ Zvi weci xZ cvtk |



$x + y - 3 < 0$ AmgZvi tj LwPÎ Asktbi Rb" D³ AmgZvq gj we`y(0, 0) Gi gvb emvtj cvl qv hvq $\bar{N}3 < 0$ hv AmgZvfk mmx Kti ev gvb mZ" | KvtrB, G Ae`vq AmgZvi QvqwPÎ nte ti LwUi th cvtk gj we`yi tqtQ tm cvtk |



D`vni Y 3 | $2x - 3y + 6 \geq 0$ AmgZvi mgvarb tmþUi eYØv `vI | wPwÎ Z Ki |

mgvarb : Avgiv cÛtg $2x - 3y + 6 = 0$ mgxKi þYi tj LwPÎ A¼b Kwii |

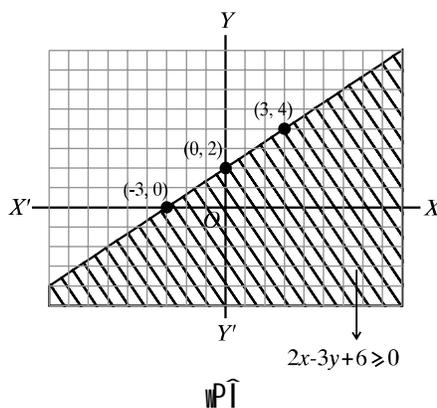
mgxKi YwU t_þK cvl qv hvq :

$$2x - 3y + 6 \text{ ev } y = \frac{2x}{3} + 2$$

G tj LwPÎ w`Z KtqKuU `vbw¼ :

x	0	-3	3
y	2	0	4

~vbwqZ QK KvMfRi qiz Zg eMfP evui ~NfK GKK ari (0, 2), (-3, 0), (3, 4) we`y,tj v ~vcb Kfi mgxKi YvUi tj LwP^ A4b Kwi |



GLb gj we`y (0, 0) tZ $2x - 3y + 6$ iwiki gvb 6, hv abvZK | mZivs tj LwP^ ti LwUi th cvtk gj we`y i tqfQ tmB cvtki mKj we`y j Rb`B $2x - 3y + 6 > 0$

AZGe, $2x - 3y + 6 > 0$ AmgZvi mgvavb tmU $2x - 3y + 6 > 0$ mgxKi fYi tj LwP^ w`Z mKj we`y j Ges tj LwP^ i th cvtk gj we`y Aew`Z tmB cvtki mKj we`y j ~vbwq mgstq MwZ |

GB mgvavb tmfUi tj LwP^ Dcti i wP^ i wPwYZ AskUKzhvi gta` tj LwP^ ti LwUi Ašf® |

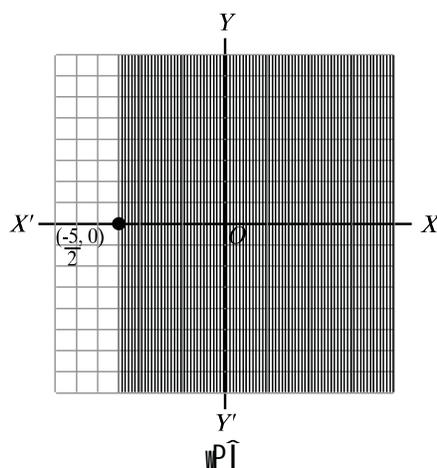
D`vni Y 4 | x, y mgZtj, $-2x < 5$ AmgZvi tj LwP^ A4b Ki |

mgvavb : $-2x < 5$ AmgZvUfK Gfvte tj Lv hvq |

$$2x + 5 > 0 \quad \text{ev, } 2x > -5 \quad \text{ev, } x > -\frac{5}{2}$$

GLb ~vbwqZ x, y mgZtj $x = -\frac{5}{2}$ mgxKi fYi tj LwP^ A4b Kwi | QK KvMfRi qiz Zg eMfP evui

~NfK w0_yfK GKK ari $(-\frac{5}{2}, 0)$ we`y w` tq y Afq i mgvstvj Kfi tj LwP^ ti LwU A4b Kiv ntj v |



GB tj LwPÎ ti Lvi Wvb cvtk gj we`y Aew`Z Ges gj we`jZ $x = 0$ hv, $> -\frac{5}{2}$

mZivs tj LwPÎ ti Lvi Wvb cvtki mKj we`j vbr¼B c0 È AmgZvi mgvavb (tj LwPÎ ti Lvi we`y, tj v we`eP` bq) | mgvavb tmËUi tj LwPÎ Dcti i wPÎ i wPwýZ AskUKz (hvi gta` tj LwPÎ ti LwU Ašf® bq) | D`vniY 5 | $y \leq 2x$ AmgZvi tj LwPÎ A¼b Ki |

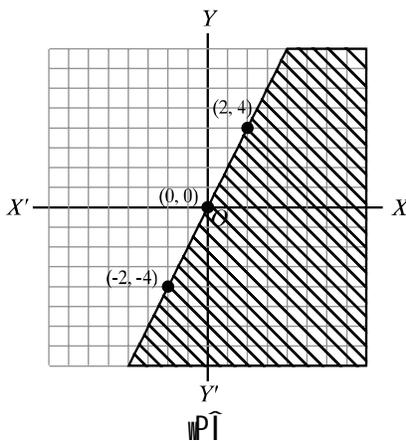
mgvavb : $y \leq 2x$ AmgZwUËK $y - 2x \leq 0$ AvKvËi tj Lv hvq |

GLb $y - 2x = 0$ A_vr $y = 2x$

mgxKiËYi tj LwPÎ A¼b Kwi | mgxKiYwU t_ËK cvB,

x	0	2	-2
y	0	4	-4

vbr¼wqZ QK KvMËRi ¶i Zg eËM® ` N®K GKK aËi (0, 0), (2, 4), (-2, -4) we`y, tj vËK vcb KËi tj LwPÎ ti LwU A¼b Kiv nËj v |



(1, 0) we`y tj LwPÎ ti Lvi 0bËPi Astk0 AvËQ | GB we`jZ $y - 2x = 0 - 2 \times 1 = -2 < 0$

mZivs tj LwPÎ ti LwU | Zvi wËËPi Ask [A_® th Astk (1, 0) we`yU Aew`Z] mgšËq MvZ mgZËj i AskUKB c0 È AmgZvi tj LwPÎ |

D`vniY 6 | $2x - 3y - 1 \geq 0$ Ges $2x + 3y - 7 \leq 0$ AmgZv `BwU hMcr mgvavb wPwýZ Ki |

mgvavb : c0_Ëg $2x - 3y - 1 = 0$ (i)

Ges $2x + 3y - 7 = 0$ (ii)

mgxKiY `BwU tj LwPÎ A¼b Kwi |

(i) t_ËK cvB,

$3y = 2x - 1$ ev, $y = \frac{2x - 1}{3}$

GLvËb,

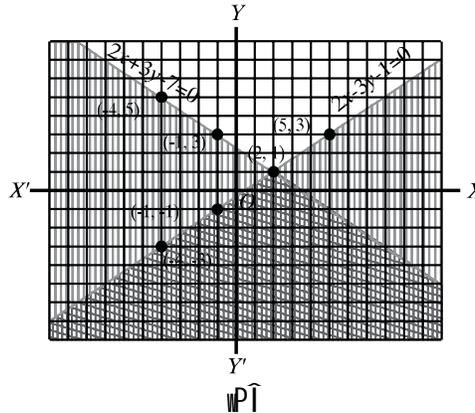
x	5	-4	-1
y	3	-3	-1

(ii) t₁K cvB, $3y = -2x + 7$ ev, $y = \frac{-2x + 7}{3}$

GLvfb,

x	-1	2	-4
y	3	1	5

GLb v₁v₁v₁QK KvM₁Ri ¶i Zg etM₁ evüi ^ N₁K GKK at₁ (5, 3), (-4, -3), (-1, -1) ve₁ y₁ t₁ v₁ v₁cb K₁ti $2x - 3y - 1 = 0$ mgxKi t₁Yi t₁ LwP₁ ti Lv Ges (-1, 3), (2, 1), (-4, 5) ve₁ y₁ t₁ v₁ v₁cb K₁ti $2x + 3y - 7 = 0$ mgxKi t₁Yi t₁ LwP₁ ti Lv A₁¼b Kw₁ |



gj ve₁ y(0, 0) t₁ Z $2x - 3y - 1$ i vki gvb -1, hv FYvZK | m₁Zivs $2x - 3y - 1 = 0$ Gi t₁ LwP₁ ti Lvi th cvtk gj ve₁ y Aev₁Z tmB cvtki mKj ve₁ j Rb₁ $2x - 3y - 1 < 0$ Ges Aci cvtki mKj ve₁ j Rb₁ $2x - 3y - 1 > 0$; AZGe t₁ LwP₁ ti LwLmn Zvi ÖbtP₁ mZt₁ i w₁Pw₁Y₁Z Ask $2x - 3y - 1 > 0$ A mgZvi t₁ LwP₁ | Avevi, (0, 0) t₁ Z $(2x - 3y - 7)$ i vki gvb -7, hv FYvZK | m₁Zivs $2x + 3y - 7 = 0$ Gi t₁ LwP₁ ti Lvi th cvtk gj ve₁ y Aev₁Z tmB cvtki mKj ve₁ j Rb₁ $2x + 3y - 7 < 0$, AZGe t₁ LwP₁ ti LwLmn Zvi ÖbtP₁ mgZt₁ i w₁Pw₁Y₁Z Ask $2x + 3y - 7 \leq 0$ A mgZvi t₁ LwP₁ | AZGe, GB ti Lv `Bw₁Üi msvkó Ask mn GB `B₁f₁v₁te w₁Pw₁Y₁Z Astki t₁Q₁vskB A mgZv `B₁w₁Üi h₁M₁cr mgvav₁t₁bi t₁ LwP₁ | w₁P₁t₁ M₁yp₁f₁v₁te w₁Pw₁Y₁Z AskB (mxg₁v₁t₁ L₁mn) GB t₁ LwP₁ |

Abkxj bx 6-3

- 1) $5x + 5 > 25$ A mgZw₁Üi mgvav₁b tmU t₁Kv₁Üü?
 - K. $S = \{x \in \mathbb{R} : x > 4\}$
 - L. $S = \{x \in \mathbb{R} : x < 4\}$
 - M. $S = \{x \in \mathbb{R} : x \leq 4\}$
 - N. $S = \{x \in \mathbb{R} : x \geq 4\}$
- 2) $x + y = -2$ mgxKi Yw₁Üt₁Z x Gi t₁Kv₁Y gv₁t₁bi Rb₁ $y = 0$ n₁te?
 - K. 2
 - L. 0
 - M. 4
 - N. -2
- 3) $2xy + y = 3$ mgxKi Yw₁Üi m₁w₁K v₁vb₁sK t₁Kv₁Ü₁ t₁v₁?
 - K. (1, -1), (2, -1)
 - L. (1, 1), (2, -1)
 - M. (1, 1), (-2, 1)
 - N. (-1, 1), (2, -1)

wbtge AmgZwU t_tK 4 | 5 baf c0ke DEi `vl :

$$x \leq \frac{x}{4} + 3$$

4 | AmgZwUi mgvavb tmU tKvbW?

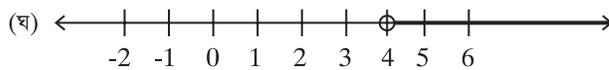
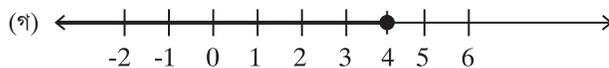
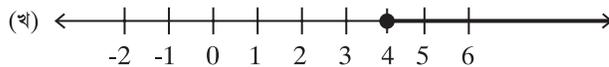
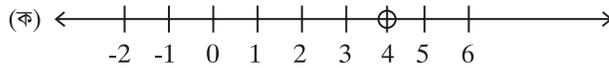
K. $S = \{x \in \mathbb{R} : x > 4\}$

L. $S = \{x \in \mathbb{R} : x < 4\}$

M. $S = \{x \in \mathbb{R} : x \leq 4\}$

N. $S = \{x \in \mathbb{R} : x \geq 4\}$

5 | AmgZwUi mgvavb tmUti msL`v ti Lv tKvbW?



wbtge Abt`Q`wU cto 6 | 7 baf c0ketj vi DEi `vl :

GKRb QvT x 10.00 UvKv `ti x wU tcvYj 6.00 UvKv `ti (x+3)wU LvZv wKtbtQ | me,tj v wgtj tgvU gjt` AbvY` 14.00 UvKv |

6 | mgm`wUi AmgZvq c0kvk tKvbW ?

i $10x + 6(x+3) \leq 114$

ii $10x + 6(x+3) \geq 114$

iii $10x + 6(x+3) < 114$

wbtpi tKvbW mW/K ?

K. i

L. ii

M. iii

N. i | ii

7 | QvT wU meWak KZwU tcvYj wKbj ?

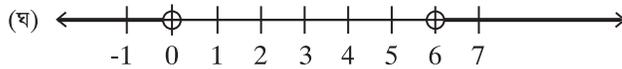
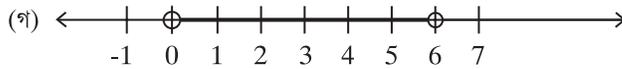
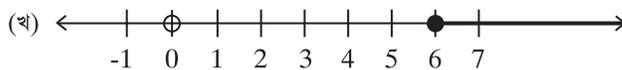
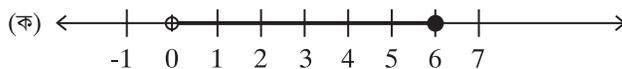
K. 1 wU

L. 3 wU

M. 5 wU

N. 6 wU

8 | mgm`wU msL`v ti Lvq tKvbW c0hvR` nte?



9 | wbtæi c0Z`K AmgZvi mgvavb tmUti tj LuPT` A/b Ki :

(i) $x - y > -10$

(ii) $2x - y < 6$

(iii) $3x - y \geq 0$

(iv) $3x - 2y \leq 12$

(v) $y < -2$

(vi) $x \geq 4$

$$(vii) y > x + 2 \quad (viii) y < x + 2$$

$$(ix) y \geq 2x \quad (x) x + 3y < 0$$

10| wbtPi c0Z`K AmgZvhMjtj i mgvavb tmfUi tj LwPÎ A¼b Ki :

$$(i) x - 3y - 6 < 0 \text{ Ges } 3x + y + 2 < 0$$

$$(ii) x + y - 4 \leq 0 \text{ Ges } 2x - y - 3 \geq 0$$

$$(iii) x - y + 3 > 0 \text{ Ges } 2x - y - 6 \geq 0$$

$$(iv) x + y - 3 > 0 \text{ Ges } 2x - y - 5 > 0$$

$$(v) x + 2y - 4 > 0 \text{ Ges } 2x - y - 3 > 0$$

$$(vi) 5x + 2y > 11 \text{ Ges } 7x - 2y > 3$$

$$(vii) 3x - 3y > 5 \text{ Ges } x + 3y \leq 9$$

$$(viii) 5x - 3y - 9 > 0 \text{ Ges } 3x - 2y \geq 5$$

11| nhi Z kvnRvj vj wegvb e`i t_#K wv¼vcj wegvb ct_i `#Zi 1793 wK.wg. | evsj vt`k wegvbtbi mtePP MwZteM 500 wK.wg./NÈv | wKŠ' nhi Z kvnRvj vj wegvb e`i t_#K wv¼vcj hvevi ct_ c0ZKtj 60 wK.wg./NÈv tetM evqyc0vtni m=§Lxb nq |

K. DÍ xctKi mgm`wUi c0qvRbxq mgq t NÈv atí mgm`wU#K AmgZvq t` Lvl |

L. nhi Z kvnRvj vj wegvbe`i t_#K wv¼vcj wegvbe`i chŠ-weiwZnxb DÇvq#bi c0qvRbxq mgq (K) AmgZv mgxKi Y t_#K wbyq Ki Ges msL`v ti Lvq t` Lvl |

M. wv¼vcj t_#K nhi Z kvnRvj vj wegvbe`i tdivi ct_ weiwZnxb DÇvq#bi c0qvRbxq mgq#K x atí mgm`wU#K AmgZvi gva`tg cKvk Kti tj #Li mrvv#h` mgvavb Ki |

12| `BwU msL`vi 1g msL`wUi 3 ,Y t_#K 2q msL`wUi 5 ,Y wefqvM Kiti 5 A#c¶lv enÈi nq | Avevi 1g msL`v t_#K 2q msL`vi 3 ,Y wefqvM Kiti AbpY9 nq |

K. DÍ xctKi mgm`v ,tj vt#K AmgZvq t` Lvl |

L. 1g msL`wUi 5 ,Y, Bnvi w0 ,Y Ges 15 Gi mgwó A#c¶lv tQvU ntj msL`wUi m=te` gvb AmgZvq cKvk Ki |

M. K bs G c0B AmgZv hMjtj i mgvavb tmfUi tj LwPÎ A¼b Ki |

mBq Aa'vq Amxg aviv Infinite Series

maviv Y exRMWYtZ Abjg I mnxg aviv m'utK'nek` Avtj vPbv Kiv ntqtQ | wKš' Abjg I Amxg avivi gta` GKUv cZ'q m'utK'itqtQ | Abjgti c` ,tj vi mvt_ MvYwZK wPy e'envi Kti Amxg aviv cvl qv hvq | G Aa'vtq Amxg aviv wbtq Avtj vPbv Kiv nte |

Aa'vq tktl wK'v_xPv -

- Abjgti aviv e'vL'v KitZ cvi te |
- Amxg aviv wPyZ KitZ cvi te |
- Amxg ,tYvEi avivi mgwó_vKvi kZ'e'vL'v KitZ cvi te |
- Amxg ,tYvEi avivi mgwó_wbYq KitZ cvi te |
- AveE` kvgK mSL'vtK Abš_ ,tYvEi avivq cKvk Ges maviv Y fMstK ifcvšt KitZ cvi te |

Abjg

wbtPi m'utK'w j q' Kwii :

1	2	3	4	5	<i>n</i>
↓	↓	↓	↓	↓		↓	
1	4	9	16	25	<i>n</i> ²

GLvtb cZ'K 'fweK mSL'v *n* Zvi eM'vL'v *n*² Gi mvt_ m'utK'Z | A_ 'fweK mSL'vi tmU *N* = {1, 2, 3, 4,} t_tK GKw wbtgi gva'tg Zvi eM'vL'vi tmU {1, 4, 9, 16,} cvl qv hvq | GB mRv'v eM'vL'vi tmUw GKw Abjg | mZ'v, KZK ,tj v iwik GKUv wtkl wbtg mgvštq Ggbf'vte mRv'v nq th cZ'K iwik Zvi c'ep c` I c'ti c'ti mvt_ Kxfvte m'utK'Z Zv Rvbv hvq | Gfvte mRv'v iwik ,tj vi tmUtK Abjg (Sequence) ej v nq |

Dcti m'utK'wtK d'vskb etj Ges $f(n) = n^2$ wj Lv nq | GB Abjgti maviv Y c` n^2 . th'Kv'v Abjgti c` mSL'v Amxg | AbjgwU maviv Y c'ti m'v'v' wj Lvi c'wZ ntj v $\{n^2\}$, $n = 1, 2, 3, \dots$ ev, $\{n^2\}_{n=1}^{+\infty}$ ev, $\{n^2\}$.

Abjgti c'g iwikK c'g c`, wZxq iwikK wZxq c`, ZZxq iwikK ZZxq c` BZ'w` ej v nq | Dcti evY 1, 4, 9, 16, ... Abjgti c'g c` = 1, wZxq c` = 4, BZ'w` |

wbtP Abjgti Pvi wU D`vni Y t` I qv ntj v :

$$\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \dots, \frac{1}{2^n}, \dots$$

$$3, 1, \tilde{N}1, \tilde{N}3, \dots, (5 - 2n), \dots$$

$$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \dots, \frac{n}{2n-1}, \dots$$

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{17}, \dots, \frac{1}{n^2+1}, \dots$$

KvR : 1| wbtPi Abjug, tj vi mavi Y c` wYq Ki :

(i) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \dots$ (ii) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$

(iii) $\frac{1}{2}, \frac{1}{2}, \frac{3}{2^3}, \frac{4}{2^4}, \dots$ (iv) $1, \sqrt{2}, \sqrt{3}, 2, \dots$

2| c0 E mavi Y c` ntZ wbtPi Abjug, tj v tj L :

(i) $1+(-1)^n$ (ii) $1-(-1)^n$ (iii) $1+\left(-\frac{1}{2}\right)^n$ (iv) $\frac{n^2}{\sqrt[n]{\pi}}$ (v) $\frac{\ln n}{n}$ (vi) $\cos\left(\frac{n\pi}{2}\right)$

3| tZg iv ctZ` tK GKw Kti Abjutgi mavi Y c` wj tL Abjugw tj L |

aviv

tKvbtv Abjutgi c` ,tj v cici 0+0 wPy 0v iv h3 Kij GKw aviv (Series) cvl qv hvq| thgb, 1+4+9+16+..... GKw aviv | Avevi $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ GKw aviv | Gi cici `Bw ct` i AbcvZ mgvb | mZivs, thtKvbtv avivi cici `Bw ct` i gta` mautKp Dci wbfP Kti aviwUi `enkó` | aviv, tj vi gta` ,i "ZcY GKw aviv ntjv ,tYvEi aviv | Avevi, tKvbtv avivi iwk ev ct` i msL`vi Dci wbfP Kti avivtK `Bfvte fvM Kiv hvq| h_v-

- (i) mmxg aviv (Finite Series), (ii) Amxg aviv (Infinite Series)

mmxg avivtK mvš-aviv Ges Amxg avivtK Abš-aviv | ej v nq | mmxg aviv mautKp gva` wK exRMZYZ cy` tK Avtj vPbv Kiv ntqtQ | GLvtb Amxg aviv mautKp Avtj vPbv Kiv nte |

Amxg aviv (Infinite Series)

ev` e msL`vi GKw Abjug $u_1, u_2, u_3, \dots, u_n, \dots$ ntj $u_1 + u_2 + u_3 + \dots + u_n + \dots$ tK ev` e msL`vi GKw Amxg aviv ej v nq | GB aviwUi n Zg c` u_n |

Amxg avivi Avsikk mgwó (Partial Sum of Infinite Series)

- $u_1 + u_2 + u_3 + \dots + u_n + \dots$ Abš-avivi
- 1g Avsikk mgwó $S_1 = u_1$
- 2q Avsikk mgwó $S_2 = u_1 + u_2$
- 3q Avsikk mgwó $S_3 = u_1 + u_2 + u_3$

.....

 \therefore n Zg AvsikK mgwó $S_n = u_1 + u_2 + u_3 + \dots + u_n$ A_@, tKvfbv Amxg avivi n Zg AvsikK mgwó nƒ”Q avi vUj cŭg n msL”K ($n \in N$) cƒ` i mgwó |

D`vni Y 1 | cŭ Ě Amxg aviv`BvUj AvsikK mgwó vbyĠ Ki |

(K) $1 + 2 + 3 + 4 + \dots$

(L) $1 - 1 + 1 - 1 + \dots$

mgvavb : (K) avivUj cŭg c` a = 1 Ges mvavi Y Ašř d = 1. AZGe avivUj GKvU mgvšř aviv |

$$\begin{aligned} \therefore \text{mgwó } S_n &= \frac{n}{2} \{2 \cdot 1 + (n-1) \cdot 1\} && [\because S_n = \frac{n}{2} \{2a + (n-1)d\}] \\ &= \frac{n}{2} \{2 + n - 1\} \\ &= \frac{n(n+1)}{2} \end{aligned}$$

Dcř i D`vni tY n Gi wevfbovgv eivmřq cvB,

$$S_{10} = \frac{10 \times 11}{2} = 55$$

$$S_{1000} = \frac{1000 \times 1001}{2} = 500500$$

$$S_{10000} = \frac{100000 \times 100001}{2} = 5000050000$$

.....

GřġřĤ, n Gi gvb hZ eo Kiv nq, S_n Gi gvb ZZ eo nq | mživs cŭ Ě Amxg avivUj tKvfbv mgwó bvB |

mgvavb : (L) $1 - 1 + 1 - 1 + \dots$ Amxg avivUj

1g AvsikK mgwó $S_1 = 1$

2q AvsikK mgwó $S_2 = 1 - 1 = 0$

3q AvsikK mgwó $S_3 = 1 - 1 + 1 = 1$

4_ AvsikK mgwó $S_4 = 1 - 1 + 1 + 1 - 1 = 0$

.....

Dcř i D`vni Y t_řK ř Lv hvq th, n weřRvo msL`v nřj n Zg AvsikK mgwó $S_n = 1$ Ges n řRvo msL`v nřj n Zg AvsikK mgwó, $S_n = 0$.

GLvřb ř Lv hvř”Q th, cŭ Ě avivUj řġřĤ, Ggb tKvfbv vboř”Ě msL`v cvl qv hvq bv hvřK avivUj mgwó ej v nq |

Amxg ,†YvËi avivi mgwó (Sum of Infinite Series in Geometric Progression)

$a + ar + ar^2 + ar^3 + \dots$,†YvËi avivwUi cŭg c` a Ges mvavi Y AbpcvZ r .

mZivs, avivwUi n Zg c` = ar^{n-1} , thLv†b $n \in \mathbb{N}$ Ges $r \neq 1$ ntj avivwUi n Zg AvsukK mgwó

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$= a \cdot \frac{r^n - 1}{r - 1}, \text{ hLb } r > 1$$

$$\text{Ges } S_n = a \cdot \frac{1 - r^n}{1 - r}, \text{ hLb } r < 1$$

j ¶| Kwí :

(i) $|r| < 1$ ntj, A_¶, $-1 < r < 1$ ntj, n Gi gvb ep× Kitj ($n \rightarrow \infty$ ntj) $|r^n|$ Gi gvb nwm cvq Ges n Gi gvb ht_ó eo Kitj $|r^n|$ Gi gvb 0 Gi KvQvKwQ nq| A_¶ r^n Gi cŭšxq gvb (Limiting Value) 0 nq| dtj S_n Gi cŭšxq gvb,

$$\begin{aligned} S_n &= \frac{a(1 - r^n)}{1 - r} = \frac{a}{1 - r} - \frac{ar^n}{1 - r} \\ &= \frac{a}{1 - r} \end{aligned}$$

G†¶††, $a + ar + ar^2 + \dots$ Amxg avivwUi mgwó $S_\infty = \frac{a}{1 - r}$

(ii) $|r| > 1$ ntj, A_¶, $r > 1$ A_ev $r < -1$ ntj, n Gi gvb ep× Kitj $|r^n|$ Gi gvb ep× cvq Ges n tK ht_ó eo K†i $|r^n|$ Gi gvb ht_ó eo Kiv hvq| mZivs Ggb †Kv†bv wov` 0 mSL`v S cvl qv hvq bv, hv†K S_n Gi cŭšxq gvb aiv hvq|

A_¶, G†¶†† Amxg avivwUi †Kv†bv mgwó bvB|

(iii) $r = -1$ ntj, S_n Gi cŭšxq gvb cvl qv hvq bv|

†Kbbv, n †Rvo mSL`v ntj $(-1)^n = 1$ Ges n we†Rvo mSL`v ntj $(-1)^n = -1$

G†¶†† avivwUi n†e, $a - a + a - a + a - a + \dots$

mZivs, GB Amxg avivwUi †Kv†bv mgwó bvB|

$|r| < 1$ A_¶, $-1 < r < 1$ ntj, $a + ar + ar^2 + \dots$ Amxg ,†YvËi avivwUi mgwó $S = \frac{a}{1 - r}$.

r Gi Ab` mKj gv†bi Rb` Amxg avivwUi mgwó _vK†e bv|

gše" : Amxg ,tYvEi avivi mguótk (hw` _vfk) S_∞ vj tL cKvk Kiv nq Ges Gfk avivUj AmxgZK mguó ej v nq |

A_ŕ, $a + ar + ar^2 + ar^3 + \dots$,tYvEi avivUj AmxgZK mguó, $S_\infty = \frac{a}{1-r}$, hLb $|r| < 1$

KvR : 1 | vbtPi cftZ`K tŕtŕĤ GKvU Amxg ,tYvEi avivi cġg c` a Ges mavi Y AbjvZ r t` l qv AvtQ |
 avivU tj L Ges hw` Gi AmxgZK mguó _vfk Zv-l vbyġ Ki :

- (i) $a = 4, r = \frac{1}{2}$ (ii) $a = 2, r = -\frac{1}{3}$ (iii) $a = \frac{1}{3}, r = 3$
- (iv) $a = 5, r = \frac{1}{10^2}$ (v) $a = 1, r = -\frac{2}{7}$ (vi) $a = 81, r = -\frac{1}{3}$.

2 | tZvgiv cftZ`K GKvU Kti Amxg ,tYvEi aviv tj L |

D`vni Y 2 | vbtPi Amxg ,tYvEi avivi AmxgZK mguó (hw` _vfk) vbyġ Ki |

- (1) $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots$
- (2) $1 + 0.1 + 0.01 + 0.001 + \dots$
- (3) $1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \frac{1}{4} + \dots$

mgyavb (1) : GLvfb, avivUj cġg c` , $a = \frac{1}{3}$ Ges mavi Y AbjvZ $r = \frac{1}{3^2} \times \frac{3}{1} = \frac{1}{3} < 1$

$$\begin{aligned} \therefore \text{avivUj AmxgZK mguó, } S_\infty &= \frac{a}{1-r} \\ &= \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2} \end{aligned}$$

mgyavb (2) : GLvfb, cġg c` $a = 1$ Ges mavi Y AbjvZ $r = \frac{0.1}{1} = \frac{1}{10} < 1$

$$\therefore \text{avivUj AmxgZK mguó, } S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{10}{9} = 1\frac{1}{9}$$

mgyavb (3) : GLvfb, cġg c` $a = 1$, mavi Y AbjvZ $r = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} < 1$

$$\therefore \text{avivUj AmxgZK mguó, } S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1} = 3.414 \text{ (Avmbġ)}$$

fcŠYtcjYK ` kmg†Ki mvaviY fMvst†k i evŠi

$$D`vniY 3 | (K) : 0 \cdot \dot{5} = 0 \cdot 555 \dots \dots \dots$$

$$= 0 \cdot 5 + 0 \cdot 05 + 0 \cdot 005 + \dots \dots \dots$$

avi vU GKvU Amxg ,†YvEi aviv hvi 1g c` $a = 0 \cdot 5$ Ges mvaviY AbjcvZ $r = \frac{0 \cdot 05}{0 \cdot 5} = 0 \cdot 1$

$$\therefore 0 \cdot \dot{5} = \frac{a}{1-r} = \frac{0 \cdot 5}{1-(0 \cdot 1)} = \frac{0 \cdot 5}{0 \cdot 9} = \frac{5}{9}$$

$$(L) \cdot \dot{12} = \cdot 12121212 \dots \dots \dots$$

$$= \cdot 12 + \cdot 0012 + \cdot 000012 + \dots \dots \dots$$

GB Amxg ,†YvEi avivU i 1g c` $a = \cdot 12$ Ges mvaviY AbjcvZ $r = \frac{\cdot 0012}{\cdot 12} = \cdot 01$

$$\therefore \cdot \dot{12} = \frac{a}{1-r} = \frac{\cdot 12}{1-(\cdot 1)} = \frac{\cdot 12}{\cdot 99} = \frac{4}{33}$$

$$(M) 1 \cdot \dot{231} = 1 \cdot 231231231 \dots \dots \dots$$

$$= 1 + (\cdot 231 + \cdot 000231 + \cdot 000000231 + \dots \dots \dots)$$

GLv†b, eÜbxi Af`Š†i i avivU GKvU Amxg ,†YvEi aviv

hvi 1g c` $a = \cdot 231$ Ges mvaviY AbjcvZ $r = \frac{\cdot 000231}{\cdot 231} = \cdot 001$

$$\therefore 1 \cdot \dot{231} = 1 + \frac{a}{1-r}$$

$$= 1 + \frac{\cdot 231}{1-(\cdot 001)} = 1 + \frac{231}{999} = \frac{410}{333}$$

Abjxj bx 7

1. 1, 3, 5, 7, avivU i 12 Zg c` †KvbwU ?

K. 12

L. 13

M. 23

N. 25

2. †K†bv Abj†tgi n Zg c` = $\frac{1}{n(n+1)}$ Gi 3q c` †KvbwU ?

K. $\frac{1}{3}$

L. $\frac{1}{6}$

M. $\frac{1}{12}$

N. $\frac{1}{20}$

3. †K†bv Abj†tgi n Zg c` = $\frac{1-(-1)^n}{2}$ n†j 20 Zg c` †KvbwU ?

K. 0

L. 1

M. -1

N. 2

4 tKvb Abjuti n Zg c` $U_n = \frac{1}{n}$ Ges $U_n < 10^{-4}$ ntj n Gi gvb nte-

i $n < 10^3$

ii $n < 10^4$

iii $n > 10^4$

wbtpi tKvbW mWVK ?

K. i l ii

L. i l iii

M. i l iii

N. i, ii l iii

wbtpie aviwU j q Ki Ges (5-7) baf cOkie DEi `vl | $4, \frac{4}{3}, \frac{4}{9}, \dots$

5. aviwUi 10 Zg c` tKvbW ?

K. $\frac{4}{3^{10}}$

L. $\frac{4}{3^9}$

M. $\frac{4}{3^{11}}$

N. $\frac{4}{3^{12}}$

6. aviwUi cUg 5 cf` i mgwó KZ?

K. $\frac{160}{27}$

L. $\frac{484}{81}$

M. $\frac{12}{9}$

N. $\frac{20}{9}$

7. aviwUi AmxgZK mgwó KZ?

K. 0

L. 5

M. 6

N. 7

8| cU E Abjuti 10 Zg c`, 15 Zg c` Ges r Zg c` wbyq Ki :

(K) 2, 4, 6, 8, 10, 12,.....

(L) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$

(M) Abjuti n Zg c` $= \frac{1}{n(n+1)}, n \in N$

(N) 0, 1, 0, 1, 0, 1,.....

(O) $5, \frac{5}{3}, \frac{5}{9}, \frac{5}{27}, \frac{5}{81}, \dots$

(P) Abjuti n Zg c` $= \frac{1 - (-1)^{3n}}{2}$

9| GKW Abjuti n Zg c` $u_n = \frac{1}{n}$

(K) $u_n < 10^{-5}$ ntj, n Gi gvb wKi e nte ?

(L) $u_n > 10^{-5}$ ntj, n Gi gvb wKi e nte ?

(M) u_n Gi cUq gvb (n ht` "Q eo ntj) m`utK ej v hvq ?

10| MmYvZK Avtavn c×wZi mrvvth` t` Lvl th, $r \neq 1$ ntj, s,tYvEi aviv

$$a + ar + ar^2 + ar^3 + \dots \text{ Gi } n \text{ Zg AvsukK mgwó, } S_n = a \cdot \frac{1 - r^n}{1 - r}$$

11| cŃ Ę Amxg s,tYvEi avivi (AmxgZK) mgwó hw` _vtK, Zte Zv wbYŃ Ki :

- (K) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$
- (L) $\frac{1}{5} - \frac{2}{5^2} + \frac{4}{5^3} - \frac{8}{5^4} + \dots$
- (M) $8 + 2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots$
- (N) $1 + 2 + 4 + 8 + 16 + \dots$
- (O) $\frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{1}{8} + \left(-\frac{1}{16}\right) + \dots$

12| wbtPi aviv s,tjvi cŃg n msL`K ct` i thvMdj wbYŃ Ki | G s,tjvi AmxgZK mgwó AvtQ wK? bv _vKtj e`vL`v`vl |

- (K) $7 + 77 + 777 + \dots$
- (L) $5 + 55 + 555 + \dots$

13| x -Gi Dci wK kZ©Avtvc Kijtj $\frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{(x+1)^3} + \dots$ Amxg avivUj

(AmxgZK) mgwó _vKte Ges tmB mgwó wbYŃ Ki |

14| cŃ Ę tcŠb:cwbK`kmgK s,tj vtK gj`xq fMstK cKvk Ki :

- (K) $\cdot 27$ (L) $2 \cdot 305$ (M) $\cdot 0123$ (N) $3 \cdot 0403$

15| GKwU Abvtgi n Zg c` $U_n = \frac{1}{n(n+1)}$

K. avivUj wbYŃ Kti mvari Y AbcvZ wbYŃ Ki |

L. avivUj 15 Zg c` Ges 1g 10 ct` i mgwó wbYŃ Ki |

M. avivUj AmxgZK mgwó wbYŃ Ki Ges n Gi gvb ht_ó tŃvU ntj U_n Gi cŃšxq gvb mwtK©K ejv hvq?

16| wbtgic avivUj j ¶ Ki :

$$\frac{1}{2x+1} + \frac{1}{(2x+1)^2} + \frac{1}{(2x+1)^3} + \dots$$

K. $x = 1$ ntj avivUj wbYŃ Ki Ges cŃB avivUj mvari Y AbcvZ KZ?

L. K bs G cŃB avivUj 10Zg c` Ges 1g 10wU ct` i mgwó wbYŃ Ki |

M. cŃ Ę avivUj x Gi Dci Kx kZ©Avtvc Kijtj avivUj AmxgZK mgwó _vKte Ges mgwó wbYŃ Ki |

Aóg Aa'vq wî tKvYngwZ

wî tKvYngwZ kãwJ wefklY Ki tñ cvl qv hvq ðwî tKvYð Ges ðngwZð | wî tKvY ej tZ wZbwJ tKvY Ges wngwZ ej tZ cwigvc eßvq | BstñwRtZ wî tKvYngwZtK 'Trigonometry' tj Lv nq | GwJ wefkl tYI cvl qv hvq 'Trigon' Ges 'Metry' | 'Trigon' MðK kã ðvñ wZbwJ tKvY hv wî fR Ges 'Metry' ðvñ cwigvc tevsvq |

mvavi Yfvte wî tKvYngwZ ej tZ wZbwJ tKvYi cwigvc tevsvq | e'envwi K cðqvRtb wî fRi wZbwJ tKvY I wZbwJ evði cwigvc Ges Gt i mv t_ m'úwKZ weftqi Avtj Pbv t_ tKB wî tKvYngwZi m fcvZ nq | thgb, GKwJ Mv tQi Qvqvi mrvvth" MvQwJi D" PZv, b`xi GKcvti `wotq b`xi we`hi wbyq, tKvYvKvi Rvngi tñ t dj wbyq BZ`w` tñ t wî tKvYngwZi e'envwi AwZcðPxb I Rbw cð | G Qvov, MvYtZi ev weAvtbi cðZwJ kvLvq wî tKvYngwZi e'vcK e'envwi itqtQ | ZvB wî tKvYngwZi Avtj vPbv MvYtZi GKwJ AZxe , i"ZcYweiq wntmte mpcðZwðZ | wî tKvYngwZi Avtj vPbv `ßwJ kvLvq wef³ | GKwJ mgZj xq wî tKvYngwZ (Plane Trigonometry) Ges Ab`wJ tMvj Kxq wî tKvYngwZ (Spherical Trigonometry) | eZðvb Avtj vPbv i'agv t mgZj xq wî tKvYngwZi gta` m'gve x_vKte |

Aa'vq tktl wKñv_ñv—

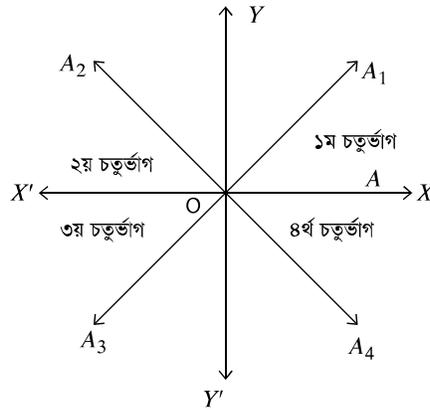
- ti wWqvb cwigv tci avi Yv e'vL`v Ki tZ cvi te |
- ti wWqvb cwigvc I wWwMðcwigv tci cvi `úwi K m'úKwbyq Ki tZ cvi te |
- Pvi wJ PZfñM wî tKvYngwZK AbjcvZmgñni wPy wbt`R Ki tZ cvi te |
- Abjv`2π tKvYi wî tKvYngwZK AbjcvZ wbyq Ki tZ cvi te |
- -θ tKvYi wî tKvYngwZK AbjcvZ wbyq Ki tZ cvi te |
- cYmsL`v n(n ≤ 4) Gi Rb` $\left(\frac{n\pi}{2} \pm \theta\right)$ tKvYi wî tKvYngwZK AbjcvZ wbyq I cðqvM Ki tZ cvi te |
- mnR wî tKvYngwZK mgxKi tYi mgvavb Ki tZ cvi te |

8.1 R`vngwZK tKvY I wî tKvYngwZK tKvY

R`vngwZK tKvY Ges wî tKvYngwZK tKvYi Avtj vPbvi m'yeat`_Avgiv XY mgZtj ci`úi mg tKvY tQ` Kti Gifc GKtRvov mij tñ Lv XOX' Ges YOY' A`b KwJ | ti Lvðq O we` tZ tQ` Kivq (wP t 8.1) th Pvi wJ mg tKvY DrcbæntqtQ Zv t` i cðZ`KwJi Af`st tK GKwJ PZfñM (Quadrant) ej v nq |

OX ti Lv t_ tK Nwoi KwJvi wecixZ w` tK Ni tZ _vKtj cðg mg tKvYi (∠XOY GK mg tKvY) Af`s` i tK cðg PZfñM (First quadrant) Ges GKbfvte Ni tZ _vKtj wðZxq (∠YOX'), ZZxq (XOY') Ges PZL` (Y'OX) mg tKvYi Af`s` mgñtK h_vµtg wðZxq, ZZxq I PZL` PZfñM ej v nq (wP t 8.1) |

R'vgnwZK avi Yv Abymvfi `BwU wfbœi wkf GKwU we`fZ wgvj Z ntj H we`fZ GKwU tKvY DrcbœntqtQ etj aiv nq| wkŠ' wî tKvYwgnwZtZ GKwU w`i i wkf mvtct¶| Aci GKwU NYqgvb i wkf wewfbœAe`vfb wewfbœ tKvY wefepbv Kiv nq|



ৱপী : 8.1

gfb KwI, OA GKwU NYqgvb i wkf Ges GuU i i`tZ OX w`i i wkf Ae`vb t`K Nwoi KuUv thw` tK Nfi Zvi weciwZ (Anticlockwise) w` tK Nt`Q| OA i wkf c`tg OA₁, Ae`vfb Gtm XO A₁, m`tKvY Drcbœkt`i Ges c`g PZf¶M `vK Ges ct`i hLb OX Gi mvt` j wfvte OY Ae`vfb Avtm ZLb XOY tKvYi cwigvc 90° ev GK mgtKvY nq| OA i wkf GKB w` tK AvI wkQz Nfi hLb OA₂, Ae`vfb Avtm ZLb XO A₂, tKvYU `j tKvY| GKBfvte Nfi hLb OA i wkf OX Gi wk weciwZ w` tK OX' Ae`vfb `vK, ZLb DrcbœtKvY XO X' GKwU mij tKvY ev `B mgtKvY| OA i wkf hLb m`uY¶c Nfi wk AvMi Ae`vfb Avtm A` OX mvt` wgvj Z nq ZLb tgvU DrcbœtKvYi cwigvY `B mij tKvY hv Pvi mgtKvY nq|

R'vgnwZtZ tKvYi Avtj vPbv `B mij tKvY chS-mwvZ ivLv nq Ges Gifc R'vgnwZK I wî tKvYwgnwZK tKvYi gta` tKvfb cv`R` bvB| wkŠ' hw` gfb Kiv nq th, OA i wkf m`uY¶c GKevi Nivi ci AvI wkQz tenk Nfi XO A₁, Ae`vfb tMj, ZLb Drcbœ XO A₁, tKvYi cwigvY Pvi mgtKvY A`¶v epEi| Gifc NY¶bi dtj wî tKvYwgnwZtZ AvI epEi tKvY DrcbœntZ cvti| wkŠ' mgZj R'vgnwZtZ Pvi mgtKvYi tPtq tenk avi Yv Kiv hvq bv|

OA i wkf Aw` Ae`vb XO X' tKvYtK R'vgnwZtZ tKvY etj MY` Kiv nq bv, wkŠ' wî tKvYwgnwZtZ XO X' tKvYi cwigvY kb` aiv nq|

8.3 abvZ¶K I FYvZ¶K tKvY :

Dctiv³ Avtj vPbvq Avgiv OA i wkf K (ৱপী 8.1) Nwoi KuUvi weciwZ w` tK NwittqW Ges OA i wkf Øvi wewfbœPZf¶M DrcbœtKvYmgnatK abvZ¶K tKvY wnmvte wefepbv KtiwQ| myZivs tKvfb i wkf K Nwoi KuUvi weciwZ w` tK (Anticlockwise) Nivtj DrcbœtKvYtK abvZ¶K (Positive) tKvY ej v nq Ges tKvfb i wkf K Nwoi KuUvi w` tK (Clockwise) Nivtj DrcbœtKvYtK FYvZ¶K ((Negative) tKvY ej v nq|

ZvB, Dcti i Avtj vPbv t`K ej v hvq GKwU abvZ¶K tKvYi cwigvc 90° A`¶v Kg ntj 1g PZf¶M `vKte| Avevi 360° I 450° gta` `vKtj I tKvYU 1g PZf¶M `vKte| GKBfvte tKvfb abvZ¶K

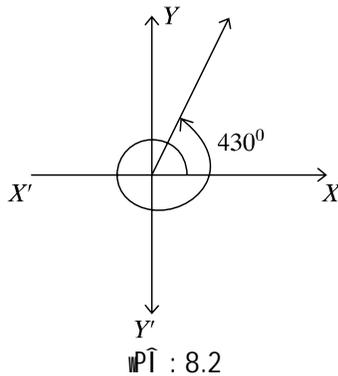
tKvYi gvb 180° I 270° gta" _vKtj tKvYU 3q PZfM, 90° t_tK 180° Gi gta" _vKtj 2q PZfM Ges 270° I 360° Gi gta" _vKtj 4_°PZfM _vKte| Abjfcvte GKwU FvZK tKvYi cwigvc -90° t_tK 0° gta" _vKtj 4_°PZfM, -180° t_tK -90° Gi gta" 3q PZfM, -270° t_tK -180° Gi gta" 2q PZfM I -360° t_tK -270°Gi gta" _vKtj 1g PZfM _vKte| 0° I 360° ev Gi thtKvrbv cYmswL`K ,wYZK XOX' tiLvi Ges 90° I 270° Gt`i thtKvrbv cYmswL`K ,wYZK YOY' tiLvi (wPÎ 8.1) Dci Ae`vb Kiti|

wPÎ : 8.1 bs wPÎ $\angle AOA_1$, 1g PZfM, $\angle AOA_2$, 2q PZfM, $\angle AOA_3$, 3q PZfM Ges $\angle AOA_4$, 4_°PZfM Ae`vb Kiti|

D`vniY 1| (i) 430° I (ii) 545° tKvY0tqi Ae`vb tKvY PZfM wbyQ Ki|

$$430^\circ = 360^\circ + 70^\circ = 4 \times 90^\circ + 70^\circ$$

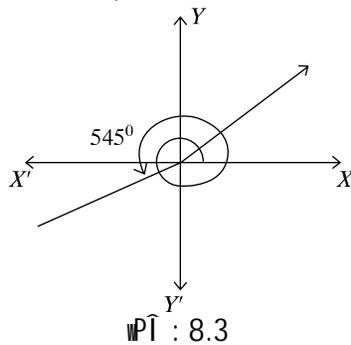
430° tKvYU avZK tKvY Ges 4 mgtkvY Atc¶v eo wKŠ' 5 mgtkvY Atc¶v tQvU| mZivs 430° tKvYU DrcbæKivi Rb` tKvrbv iwK#K 4 mgtkvY ev GKevi mæúY°Nivi ci Avi I 70° Ni_tZ ntqtQ (wPÎ : 8.2)| ZvB 430°tKvYU 1g PZfM Ae`vb Kiti|



(ii) $545^\circ = 540^\circ + 5^\circ = 6 \times 90^\circ + 5^\circ$

545° tKvYU avZK Ges 6 mgtkvY Atc¶v epEi wKŠ' 7 mgtkvY Atc¶v ¶i`Zi| 545° tKvYU Drcbæ KitiZ Nwoi Kuvvi wecixZ w`tK tKvrbv iwK#K 6 mgtkvY ev GKevi mæúY°Nfi Aw` Ae`vfb Avmvi ci Avi I `ß mgtkvYi tPtq 5° teuk Ni_tZ ntqtQ (wPÎ : 8.3)|

mZivs 545°tKvYU ZZxq PZfM Ae`vb Kiti|

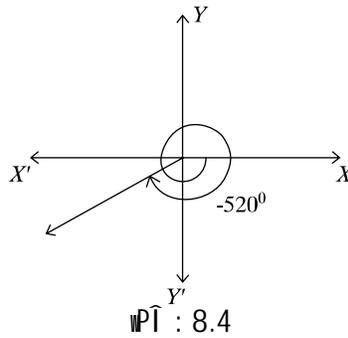


KvR : 330°, 535°, 777° | 1045° tKvYmgñ tKvY PZf^oM Ae⁻vb Kti Zv wP^oTmn t⁻LvI |

D⁻vniY 2 | (i) -520° | (iii) 550° tKvY0q tKvY PZf^oM AvtQ wby^o Ki |

(i) $-520^\circ = -450^\circ - 70^\circ = -5 \times 90^\circ - 70^\circ$

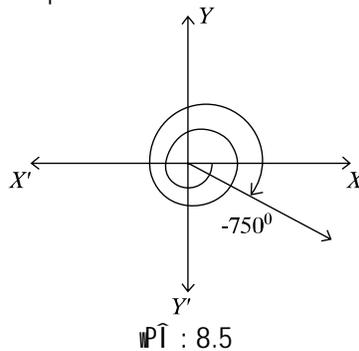
-520° GKwJ FYvZ^oK tKvY | -520° tKvYwJ Drcb^oKi tZ tKv^obv i w^oK^oK Nwoi KuUvi w⁻tK GKevi m^ouY^o N^oi GKB w⁻tK Avt^oiv GK mg^otKvY ev 90° Ges 70° N^oi ZZxq PZf^oM Avm^otZ ntq^otQ (wP^oT : 8.4) | mZi vs, -540° tKvYwJ ZZxq PZf^oM Ae⁻vb Ki tQ |



(ii) $-750^\circ = -720^\circ - 30^\circ = -8 \times 90^\circ - 30^\circ$

-750° tKvYwJ FYvZ^oK tKvY Ges Nwoi KuUvi w⁻tK ^oB^oevi m^ouY^o (8 mg^otKvY) N^oivi ci GKB w⁻tK AvI 30° Ni tZ ntq^otQ (wP^oT 8.5) |

∴ -750° tKvYwJi Ae⁻vb PZ^oL^o PZf^oM |



KvR : -100°, -365°, -720° | 1320° tKvYmgñ tKvb PZf^oM AvtQ, wP^oTmn wby^o Ki |

8.4 | tKvY cw^ogv^ot^oci GKK

tKv^otbv tKv^ot^oYi gv^ob ev cw^ogv^oY wby^oq m^ovavi YZ ^oB^o c^oKvi GKK (Unit) c^ow^oZ e⁻en^ovi Kiv nq :

- (1) IvUgj K c^ow^oZ (Sexagesimal System) |
- (2) e^oE^oxq c^ow^oZ (Circular System) |

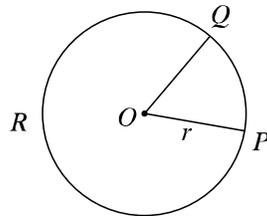
(1) IvUgj K c^ow^oZ : IvUgj K c^ow^oZ tZ mg^otKvY tK tKvY cw^ogv^ot^oci GKK aiv nq | GB c^ow^oZ tZ GK mg^otKvY hv 90° tK mg^ovb 90 f^ov^ot^oM w^of^o3 Kti c^oZ f^ov^ot^oK GK w^ow^oM^o (1° = One degree) aiv nq |

GK w^ow^oM^o K mg^ovb 60 f^ov^oM Kti c^oZ f^ov^ot^oK GK w^ogv^obU (1' = One Minute) Ges GK w^ogv^obU tK mg^ovb 60 f^ov^oM Kti c^oZ f^ov^ot^oK GK t^om^ot^oK^o (1'' = One Second) aiv nq |

$$\begin{aligned} A_{\text{P}}, \quad 60'' \text{ (tm\text{K}\text{U})} &= 1' \text{ (wgnbU)} \\ 60' \text{ (wgnbU)} &= 1^{\circ} \text{ (wvwm\text{U})} \\ 90^{\circ} \text{ (wvwm\text{U})} &= 1 \text{ mg\text{t}\text{K}\text{v}\text{Y}} \end{aligned}$$

e\text{E}xq c\text{x}\text{w}\text{Z} m\text{u}\text{t}\text{K}\text{R}\text{v}\text{b}\text{i} c\text{t}\text{e}\text{q}\text{i} w\text{v}\text{q}\text{v}\text{b} m\text{u}\text{t}\text{K}\text{R}\text{v}\text{b} \text{ i Kvi |}

ti w\text{v}\text{q}\text{v}\text{b} : t\text{K}\text{v}\text{t}\text{b}\text{v} e\text{t}\text{E}\text{i} e\text{v}\text{m}\text{v}\text{a}\text{P} \text{ mgvb Pvc H e\text{t}\text{E}\text{i} t\text{K}\text{t}\text{h} t\text{K}\text{v}\text{Y} \text{ Drcb\text{e}\text{K}\text{t}\text{i} tmB t\text{K}\text{v}\text{Y}\text{t}\text{K} \text{ GK ti w\text{v}\text{q}\text{v}\text{b} t\text{K}\text{v}\text{Y} \text{ e\text{t}\text{j} |}



w\text{P}\text{T} : 8.6

w\text{P}\text{T} \text{ } \text{PQR} \text{ e\text{t}\text{E}\text{i} t\text{K}\text{t}\text{h} \text{ } O \text{ e\text{t}\text{E}\text{i} e\text{v}\text{m}\text{v}\text{a}\text{P} \text{ } OP = r \text{ Ges e\text{v}\text{m}\text{v}\text{a}\text{P} \text{ mgvb Pvc } PQ \text{ | } PQ \text{ Pvc t\text{K}\text{t}\text{h} \text{ } O \text{ t\text{Z} } \angle POQ \text{ Drcb\text{e}\text{K}\text{t}\text{i} t\text{Q} \text{ | } D^3 \text{ t\text{K}\text{v}\text{t}\text{Y}\text{i} \text{ cwi gvYB} \text{ GK ti w\text{v}\text{q}\text{v}\text{b} \text{ | } A_{\text{P}} \text{ } \angle POQ \text{ GK ti w\text{v}\text{q}\text{v}\text{b} \text{ |}

(2) e\text{E}xq c\text{x}\text{w}\text{Z} : e\text{E}xq c\text{x}\text{w}\text{Z}\text{t}\text{Z} \text{ GK ti w\text{v}\text{q}\text{v}\text{b} (Radian) t\text{K}\text{v}\text{Y}\text{t}\text{K} \text{ t\text{K}\text{v}\text{Y} \text{ cwi gv\text{t}\text{c}\text{i} \text{ GKK aiv nq} \text{ |}

t\text{K}\text{v}\text{t}\text{Y}\text{i} w\text{v}\text{m}\text{U} \text{ cwi gvc \text{I} ti w\text{v}\text{q}\text{v}\text{b} \text{ cwi gv\text{t}\text{c}\text{i} \text{ m\text{u}\text{t}\text{K}\text{R}\text{v}\text{b}\text{Y}\text{q}\text{i} \text{ Rb} \text{ w\text{b}\text{t}\text{g}\text{u}\text{e}\text{^3} \text{ c}\text{U}\text{Z}\text{v}\text{m}\text{g}\text{v} \text{ Ges t\text{K}\text{v}\text{t}\text{Y}\text{i} \text{ e\text{E}xq} \text{ cwi gvc} \text{ m\text{u}\text{t}\text{K}\text{R}\text{v}\text{b}\text{v} \text{ c}\text{U}\text{q}\text{v}\text{R}\text{b} \text{ |}

c\text{U}\text{Z}\text{v} 1 : t\text{h}\text{t}\text{K}\text{v}\text{t}\text{b}\text{v} \text{ } \text{B}\text{U} \text{ e\text{t}\text{E}\text{i} \text{ } \text{c}\text{w}\text{i} \text{ w\text{a} \text{ | } e\text{v}\text{t}\text{m}\text{i} \text{ Abc\text{v}\text{Z} \text{ mgvb} \text{ |}

c\text{U}\text{v}\text{Y} : g\text{t}\text{b} \text{ Kwi, c\text{U} \text{E} \text{e\text{E} \text{ } \text{B}\text{U} \text{ mg\text{t}\text{K}\text{w}\text{K} \text{ Ges Df\text{t}\text{q}\text{i} t\text{K}\text{t}\text{h} \text{ } O \text{ | } e\text{p}\text{E}\text{i} \text{ e\text{E}\text{w}\text{U}\text{i} \text{ cwi w\text{a} } P \text{ | } e\text{v}\text{m}\text{v}\text{a}\text{P} \text{ Ges } \text{q}\text{t}\text{Z}\text{i} \text{ e\text{E}\text{w}\text{U}\text{i} \text{ cwi w\text{a} } p \text{ | } e\text{v}\text{m}\text{v}\text{a}\text{P} \text{ } r \text{ (w\text{P}\text{T} : 8.7) \text{ | } \text{GLb} \text{ e\text{p}\text{E}\text{i} \text{ e\text{E}\text{w}\text{U}\text{i} \text{ } n \text{ m}\text{S}\text{L}\text{K} \text{ (} n > 1 \text{) mgvb} \text{ f}\text{v}\text{t}\text{M} \text{ w\text{e}\text{f}\text{^3} \text{ Kwi \text{ | } t\text{K}\text{t}\text{h} \text{ } \text{q}\text{t}\text{Z}\text{i} \text{ m\text{v}\text{t}\text{ } w\text{e}\text{f}\text{^3} \text{ w\text{e}\text{v}\text{y}\text{t}\text{j}\text{v} \text{ thvM} \text{ K}\text{i}\text{t}\text{j} \text{ } \text{q}\text{t}\text{Z}\text{i} \text{ e\text{E}\text{w}\text{U}\text{i} \text{ } n \text{ m}\text{S}\text{L}\text{K} \text{ mgvb} \text{ f}\text{v}\text{t}\text{M} \text{ w\text{e}\text{f}\text{^3} \text{ n\text{t}\text{e} \text{ | } Df\text{q} \text{ e\text{t}\text{E} \text{ w\text{e}\text{f}\text{^3} \text{ w\text{e}\text{v}\text{y}\text{t}\text{j}\text{v} \text{ c}\text{i} \text{ } \text{u}\text{i} \text{ mshy}\text{^3} \text{ Kwi \text{ |}

d\text{t}\text{j} \text{ c}\text{U}\text{Z}\text{K} \text{ e\text{t}\text{E} \text{ } n \text{ m}\text{S}\text{L}\text{K} \text{ e}\text{v}\text{U}\text{w}\text{e}\text{n}\text{k}\text{o} \text{ GK}\text{U} \text{ m}\text{j}\text{g} \text{ e}\text{u}\text{f}\text{R} \text{ A}\text{S}\text{w}\text{j} \text{ } \text{L}\text{Z} \text{ n\text{t}\text{j}\text{v} \text{ (e\text{p}\text{E}\text{i} \text{ e\text{t}\text{E} } ABCD \text{.....} \text{ | } \text{q}\text{t}\text{Z}\text{i} \text{ e\text{t}\text{E} } abcd \text{.....} \text{) \text{ |}

\text{GLb} \text{ } \triangle OAB \text{ Ges } \triangle OAB \text{ m}\text{K}, \text{ KviY, } \angle AOB \text{ Ges } \angle AOB \text{ [m}\text{v}\text{a}\text{v}\text{Y} \text{ t\text{K}\text{v}\text{Y}] \text{ Ges Df}\text{q} \text{ w}\text{I} \text{ f}\text{R} \text{ mgv}\text{U}\text{e}\text{v}\text{U} \text{ e\text{t}\text{j} \text{ e}\text{v}\text{U} \text{ m}\text{S}\text{j} \text{ M}\text{e}\text{t}\text{K}\text{v}\text{Y} \text{ } \text{t}\text{j}\text{v} \text{ mgvb} \text{ |}

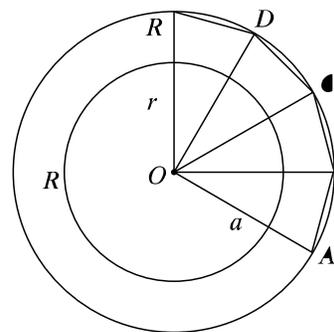
$$\therefore \frac{AB}{ab} = \frac{OA}{oa} = \frac{OB}{ob} = \frac{R}{r}$$

Abj\text{f}\text{c}\text{f}\text{v}\text{t}\text{e},

$$\frac{BC}{bc} = \frac{R}{r}, \frac{CD}{cd} = \frac{R}{r}, \text{ BZ}\text{w}\text{w} \text{ |}$$

$$\therefore \frac{AB}{ab} = \frac{BC}{bc} = \frac{CD}{cd} = \frac{R}{r}$$

$$\therefore \frac{AB + BC + CD + \dots}{ab + bc + cd + \dots} = \frac{R + R + R + \dots}{r + r + r + \dots} = \frac{NR}{nr} = \frac{R}{r} = \frac{2R}{2r} \dots (1) \quad \text{w\text{P}\text{T} : 8.7}$$



n hw` ht_ó eo nq ($n \rightarrow +\infty$) Zvntj AB, BC, CD, \dots ti Lvskmgñ AZ`Š-¶i` nte Ges gtb nte mevB e¶Ei tQvU tQvU Pvc |

mYZivs Gt¶¶tÎ,

$$AB + BC + CD + \dots = \text{enEi e¶Ei cwi wa } P$$

$$\text{Ges } ab + bc + cd + \dots = \text{¶i` Zi e¶Ei cwi wa } p$$

\therefore mgxKiY (1) ntZ cvB&

$$\frac{P}{p} = \frac{2R}{2r}$$

$$A_{\text{¶}}, \frac{P}{2R} = \frac{p}{2r}$$

$$A_{\text{¶}}, \frac{\text{enEi e¶Ei cwi wa}}{\text{enEi e¶Ei e'vm}} = \frac{\text{¶i` Ei e¶Ei cwi wa}}{\text{¶i` Ei e¶Ei e'vm}}$$

\therefore thtKv¶bv `Bw e¶Ei cwi wa l e'vtmi AbjcvZ mgvb | (cöwYZ)

cÖZÁv (1) Gi Avtj v¶K gše` l Abjmvš-:

gše` : 1 | thtKv¶bv e¶Ei cwi wa l e'vtmi AbjcvZ memgq mgvb l GKB a`e msL`v | G a`e msL`wU¶K wMK eY`π (cvB) Øviv cKvk Kiv nq | π GKwU Agj` msL`v Ges `kug¶K cKvk Kij GwU GKwU AŠ-nxb A¶cSbtcbK msL`v ($\pi = 3 \cdot 1415926535897932 \dots$) |

gše` 2 : mvariYZ Pvi `kugK `vb chS-π Gi Avmbægvb $\pi = 3 \cdot 1416$ e`envi Kiv nq | KwúDUvti mnvth` π Gi gvb mnm`j ¶waK `kugK `vb chS-wbY¶Z ntq¶Q | thtnZi π Gi Avmbægvb e`envi Kiv nq tmtnZi DËi l nte Avmbæ ZiB DËi i cv¶k ØcØqØ tj Lv Aek` KZ` | cieZ¶ mg`-Kv¶R Ab` tKv¶bv jfc ej v bv vK¶j Pvi `kugK `vb chS-π Gi Avmbægvb $3 \cdot 1416$ e`envi Kiv nte |

Abjmvš-: e¶Ei e'vmva`r' ntj , cwi wa nte $2\pi r$ |

cÖZÁv 1 Gi Avtj v¶K Avgiv Rwb,

$$\frac{\text{cwi wa}}{\text{e'vm}} = \text{a`emsL`v } \pi$$

$$\text{ev, cwi wa} = \pi \times \text{e'vm}$$

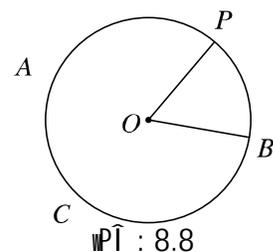
$$= \pi \times 2r \quad [\text{e'vm} = 2r]$$

$$= 2\pi r$$

\therefore r e'vmva`wukó thtKv¶bv e¶Ei cwi wa $2\pi r$ |

cÖZÁv 2 : e¶Ei tKv¶bv Pvc Øviv DrcbetK`r' tKvY H e¶Pvtci mgvbvcwZK |

gtb Kwi , ABC e¶Ei tK`a O Ges e'vmva`OB | P e¶Ei Dci Ab` GKwU w`y | dtj BP e¶Ei GKwU Pvc Ges $\angle POB$ e¶Ei GKwU tK`r' tKvY |



Zvntj , tK>`r' $\angle POB$, Pvc BP . Gi mgvbcwZK nte|

A_ŕ, tK>`r' $\angle POB \propto$ Pvc BP |

cŕZÁv 3 : ti wWqvb tKvY GKwJ a'e tKvY|

wełkl wePb : gtb Kwi , O tK>`lewkó ABC eĚ $\angle POB$ GKwJ ti wWqvb tKvY| cŕvY KiĚZ nte th, $\angle POB$ GKwJ a'e tKvY|

A¼b : OB ti Lvstki (e'vmvtaŕ) Dci OA j r'Avnk|

cŕvY :

OA j r'eĚEi cwi wałK A we`ĚZ tQ` Kti |

Zvntj Pvc $AB =$ cwi vai GK-PZ_ŕsk

$$= \frac{1}{4} \times 2\pi r = \frac{\pi r}{2}$$

Ges Pvc $PB = e'vmva^{\circ}r$ [$\angle POB =$ Gi ti wWqvb]

cŕZÁv 2 t_łK cvB,

$$\frac{\angle POB}{\angle AOB} = \frac{\text{Pvc } PB}{\text{Pvc } AB}$$

$$\begin{aligned} \therefore \angle POB &= \frac{\text{Pvc } PB}{\text{Pvc } AB} \times \angle AOB = \frac{r}{\frac{\pi r}{2}} \times \text{GK mgłKvY} [OA \text{ e'vmva}^{\circ}\text{Ges } OB \text{ Gi Dci j r}'] \\ &= \frac{2}{\pi} \text{ mgłKvY} | \end{aligned}$$

thtnZimgłKvY | π a'eK tmtnZi $\angle POB$ GKwJ a'eK tKvY| (cŕwYZ)

8.5 tKvłYi eĚxq cwi gvc

msÁv : eĚxq c×wZĚZ (*Circular System*) A_ŕ, ti wWqvb GKłK tKvłbv tKvłYi cwi gvcłK Zvi eĚxq cwi gvc (*Circular measure*) ej v nq|

gtb Kwi , $\angle MON$ thłKvłbv GKwJ tKvY hvi eĚxq cwi gvc wBYŕ

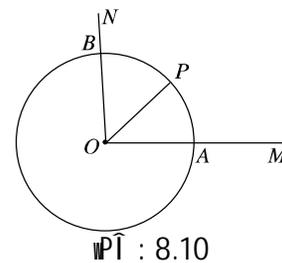
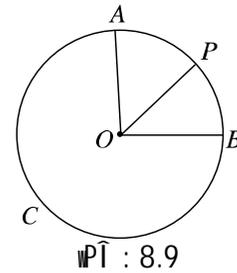
KiĚZ nte| O we`łK tK>`a Kti $OA = r$ e'vmvawłtq GKwJ eĚ

A¼b Kwi |

eĚw $OM \perp ON$ tK h_vłtg A | B we`łZ tQ` Kti | Zvntj

AB Pvc Ōiv DrcbetK>`r' tKvY $\angle AOB$ | e'vmva^{\circ}r Gi mgvb Kti AP Pvc wB (Pvc | e'vmva^{\circ}GKB GKłK nłZ nte)|

Zvntj , $\angle AOP = 1$ ti wWqvb|

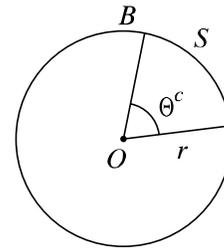


awii Pvc $AB = S$.

côZÁv 2 Abhivqx,

$$\frac{\angle MON}{\angle AOP} = \frac{\text{Pvc } AB}{\text{Pvc } AP} = \frac{\text{Pvc } AB}{\text{e'vma' } OA} = \frac{s}{r}$$

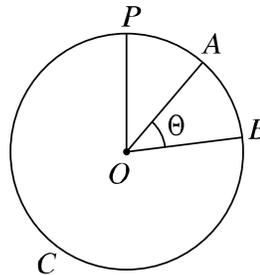
$$\begin{aligned} \therefore \angle MON &= \frac{s}{r} \times \angle AOP \\ &= \frac{s}{r} \times 1 = \text{tiwWqvb} = \frac{s}{r} \text{ tiwWqvb} \end{aligned}$$



WPI : 8.11

$\therefore \angle MON$ Gi eÉxq cwi gvc $\frac{s}{r}$, thLvtb tKvYwU Zvi kxl e'vma' tK'ª Kti Ges r e'vma' tbtq Aw4Z eÉÉ s cwi gvY Pvc LwUZ Kti |

côZÁv 4 | r e'vmta' tKvYv eÉÉ s tN' tKvYv Pvc tK'ª θ cwi gvY tKvY DrcbaeKi tj $s = r\theta$ nte |



WPI : 8.12

wetkl wbePb : gtb Kwii, O tK'ª tK'ª ABC eÉÉi e'vma' $OB = r$ GKK, Pvc $AB = s$ GKK Ges AB Pvc Øviv DrcbaeK'ª tKvY $AOB = \theta^c$ | cõvY Ki tZ nte th, $s = r\theta$ |

A¼b : O w'ª tK'ª Kti OA ev OB Gi mgvb e'vma' tbtq ABC eÉÉ A¼b Kwii | B w'ª tK'ª Kti OB Gi mgvb e'vma' tK'ª BP Pvc Awk thb Zv ABC eÉÉi cwi w'ªK P w'ª tZ tQ' Kti | $O : P$ thvM Kwii |

cõvY : A¼b Abvnti $\angle POB = 1^c$

Avgi v Rvnb, tKvYv eÉÉ Pvc Øviv DrcbaeK'ª tKvY H eÉÉ Pvc i mgvbcwZK |

$$\therefore \frac{\text{Pvc } AB}{\text{Pvc } PB} = \frac{\angle AOB}{\angle POB}$$

$$\text{ev } \frac{s \text{ GKK}}{r \text{ GKK}} = \frac{\theta^c}{1^c}$$

$$\text{ev } \frac{s}{r} = \theta$$

$$\therefore s = r\theta. \text{ [cõvY Z]}$$

8-6 tKv†Yi wWwMÖcwi gvc I ti wWqvb (eĚxq) cwi gvtci m=úK[©]

cĪZÁv 3 (wPĪ 8.9) Gi ti wWqvb tKv†Yi eYĖvq Avgiv cvB,

$$1 \text{ ti wWqvb} = \frac{2}{\pi} \text{ mg†KvY}$$

$$A_{\text{P}}, 1^c = \frac{2}{\pi} \text{ mg†KvY} \quad [1 \text{ ti wWqvb} = 1^c]$$

$$\therefore 1 \text{ mg†KvY} = \left(\frac{\pi}{2}\right)^c$$

$$\text{ev, } 90^0 = \left(\frac{\pi}{2}\right)^c$$

$$\therefore 1^0 = \left(\frac{\pi}{180}\right)^c \text{ Ges } 1^c = \left(\frac{180}{\pi}\right)^0$$

j ¶Yxq :

$$(i) 90^0 = 1 \text{ mg†KvY} = \frac{\pi}{2} \text{ ti wWqvb} = \left(\frac{\pi}{2}\right)^c$$

$$A_{\text{P}}, 180^0 = 2 \text{ mg†KvY} = \pi \text{ ti wWqvb } \pi^c.$$

(ii) IvUgj K I eĚxq c×wZ†Z GKwU tKv†Yi cwi gvc h_vµtg D^0 I R^c ntj

$$D^0 = \left(D \times \frac{\pi}{180}\right)^c = R^c$$

$$A_{\text{P}}, D \times \frac{\pi}{180} = R$$

$$\text{ev, } \frac{D}{180} = \frac{R}{\pi}.$$

Dctiv³ Avtj vPbv t_†K eúj e"eüZ tKvYmg†ni wWwMÖI ti wWqvtbi m=úK[†] I qv ntj v :

$$(i) 1^0 = \left(\frac{\pi}{180}\right)^c$$

$$(ii) 30^0 = \left(30 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{6}\right)^c$$

$$(iii) 45^0 = \left(45 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{4}\right)^c$$

$$(iv) 60^0 = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

$$(v) \quad 90^\circ = \left(90 \times \frac{\pi}{180^\circ}\right)^c = \left(\frac{\pi}{2}\right)^c$$

$$(vi) \quad 180^\circ = \left(180 \times \frac{\pi}{180^\circ}\right)^c = \pi^c$$

$$(vii) \quad 360^\circ = \left(360 \times \frac{\pi}{180^\circ}\right)^c = (2\pi)^c$$

e'envi K t¶t¶ ti wWqv b cZxK (c) mvaY Z wj Lv nq bv | mst¶t¶c (ti wWqv b cZxK Dn' ti tL)

$$1^\circ = \frac{\pi}{180}, \quad 30^\circ = \frac{\pi}{6}, \quad 45^\circ = \frac{\pi}{4}, \quad 60^\circ = \frac{\pi}{3}, \quad 90^\circ = \frac{\pi}{2}, \quad 180^\circ = \pi, \quad 360^\circ = 2\pi \quad \text{BZ'w' |}$$

$$\text{'be' } 1 : 1^\circ = \left(\frac{\pi}{180}\right)^c = 0.01745^c \quad \text{AvmæcuP `kugK `vb chS}$$

$$1^c = \left(\frac{180}{\pi}\right)^0 = 57.29578^c \quad (\text{AvmæcuP `kugK `vb chS}) = 57^\circ 17' 44.81''.$$

Gt¶t¶ π Gi Avmbægvb 3.1416 e'envi Kiv ntqtQ |

'be' 2 : bxtPi mg-D`vniY Ges mg-mgm'vi π Gi Avmbægvb Pvi `kugK `vb (π = 3.1416) chS-
e'envi Kiv nte | π Gi Avmbægvb e'eüZ ntj DEti Aek'B ūcŭŭ K_wU wj LtZ nte |

D`vniY 3 |

$$(i) \quad 30^\circ 12' 36'' \text{ tK ti wWqv t b cKvk Ki |}$$

$$(ii) \quad \frac{3\pi}{13} \text{ tK wWwŭŭ wgvbU | tmKtŭ cKvk Ki |}$$

mgvavb :

$$(i) \quad 30^\circ 12' 36'' = 30^\circ \left(12 \frac{36}{60}\right)' = 30^\circ \left(12 \frac{3}{5}\right)' = 30^\circ \left(\frac{63}{5}\right)'$$

$$= \left(30 \frac{36}{5 \times 60}\right)^0 = \left(30 \frac{21}{100}\right)^0 = \left(\frac{3021}{100}\right)^0$$

$$= \frac{3021}{100} \times \frac{\pi}{180} \text{ ti wWqv b } \left[\because 1^\circ = \frac{\pi^c}{180} \right]$$

$$= \frac{3021\pi}{18000} = .5273 \text{ ti wWqv b (cŭŭ)}$$

$$\therefore 30^\circ 12' 36'' = .5273^c \text{ (cŭŭ)}$$

$$(ii) \quad \frac{3\pi}{13} = \frac{3\pi}{13} \times \frac{180}{\pi} \text{ wWwŭŭ } \left[\because 1^c = \frac{180}{\pi} \right]$$

$$= \frac{540}{13} \text{ mmm}$$

$$= 41^{\circ}32'18.46''.$$

$$\therefore \frac{3\pi}{13} \text{ ti mqv} = 41^{\circ}32' \cdot 46''.$$

D`vni Y 4 | GKwU wî fRi wZbU tKvYi AbcvZ 3:4:5 ; tKvY wZbU eĚxq gvb KZ ?

mgvavb : awi , tKvY wZbU h_vµtg $3x^c, 4x^c \mid 5x^c$ |

cċqġZ, $3x^c + 4x^c + 5x^c = \pi^c$ [wî fRi wZb tKvYi mgwó 2 mgġKvY = π^c]

$$\text{ev, } 12x^c = \pi^c$$

$$\text{ev, } x = \frac{\pi}{12}$$

\therefore tKvY wZbU h_vµtg

$$3x^c = \left(\frac{3\pi}{12}\right)^c = \left(\frac{\pi}{4}\right)^c = \frac{\pi}{4}$$

$$4x^c = \left(\frac{4\pi}{12}\right)^c = \left(\frac{\pi}{3}\right)^c = \frac{\pi}{3}$$

$$5x^c = \left(\frac{5\pi}{12}\right)^c = \frac{5\pi}{12}$$

$$\text{DĚi : } \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{12}.$$

D`vni Y 5 | GKwU PvKv 1.75 wġġ wġUvi c_ thġZ 40 evi Nġi | PvKwU e`vmva^qKZ ?

mgvavb : awi PvKvi e`vmva^r wġUvi |

$$\therefore \text{PvKvi cwi} = 2\pi r \text{ wġUvi } (\pi = 3.1416)$$

Avgiv Rwb PvKwU GKevi Nġġ Zvi cwi wai mgvb `ġZġ AwZµg Kġi |

$$\begin{aligned} \therefore 40 \text{ evi Nġġ PvKwU tgvU AwZµvš-`ġZġ} &= 40 \times 2\pi r \text{ wġ.} \\ &= 80\pi r \text{ wġUvi} \end{aligned}$$

$$\therefore \text{cċqġZ, } 80\pi r = 1750 \text{ [1 wġ. = 1000 wġUvi]}$$

$$\text{ev, } r = \frac{1750}{80\pi} = \frac{1750}{80 \times 3.1416} \text{ wġUvi}$$

$$= 6.963 \text{ wġUvi (cċq)} |$$

$$\text{DĚi : PvKvi e`vmva}^{\text{q}} \text{ 6.963 wġUvi (cċq)} |$$

D`vniY 6 | cW_exi e`vma°6440 Wktj wglvi | XvKv I Rvgvj cji tKt`a 2° tKvY DrcbæKtj XvKv I Rvgvj cji i`iZi wbyq Ki |

mgvavb : e`vma°= r = 6440 Wk.wg.

$$\begin{aligned} cW_exi \text{ tKt`a DrcbæKvY } \frac{\theta}{\pi} &= 2^\circ = 2 \times \frac{\pi^\circ}{180} \\ &= \frac{\pi}{90} \text{ ti wWqvb |} \end{aligned}$$

$$\begin{aligned} \therefore s = \text{Pvtci } \hat{N}^\circ = XvKv \text{ I Rvgvj cji i`iZi} &= r\theta = 6440 \times \frac{\pi}{90} \text{ Wk.wg.} \\ &= \frac{644\pi}{90} \text{ Wk.wg.} \\ &= 224.8 \text{ Wk.wg. (c} \hat{u} \text{q)} \end{aligned}$$

DËi : 224.8 Wk.wg. (cûq) |

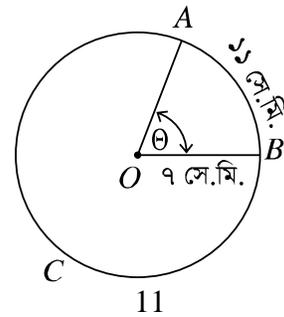
D`vniY 7 | tKvfbv eËËi e`vma°7 tm.wg. | eËËi 11 tm.wg. `xN°Pvtci tKt`a mZtKvYi cwi gvc wbyq Ki |

mgvavb : awi, ABC eËËi e`vma°OB = 7 tm.wg. Ges Pvc AB = 11 tm.wg. | AB Pvtci tKt`a mZtKvYi cwi gvY θ wbyq Ki tZ nte |

Avgiv Rwb, s = rθ

$$\begin{aligned} \text{ev, } \theta &= \frac{s}{r} = \frac{11 \text{ tm.wg.}}{7 \text{ tm.wg.}} \\ &= 1.57 \text{ ti wWqvb (c} \hat{u} \text{q)} \end{aligned}$$

DËi : 1.57 ti wWqvb (cûq) |



D`vniY 8 | Gnmvb mvBtKtj Pto eËvKvi ct_ 10 tmtKtÛ GKwU eËPvc AwZµg Kti | hv` PvcwU tKt`a 28° tKvY DrcbæKti Ges eËËi e`vm 180 wglvi nq, Zte Gnmvbi MwZteM wbyq Ki |

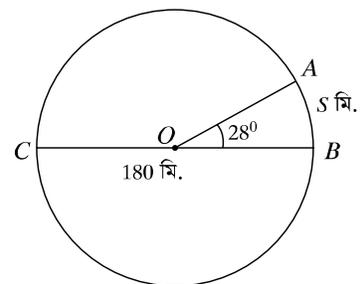
mgvavb : awi, Gnmvb ABC eËËi B we`yt_tK hvÎv Kti 7 tmtKË cti cwi wai Dci A we`tZ Avtm | Zvntj AB Pvc Øviv DrcbæKt`a tKvY ∠AOB = 28°

$$OB = e`vma^\circ = \frac{180}{2} \text{ wglvi} = 90 \text{ wglvi}$$

awi, Pvc AB = S wglvi

Avgiv Rwb,

$$S = r\theta = 90 \times 28^\circ \text{ wglvi}$$



$$\begin{aligned}
 &= 90 \times 28 \times \frac{\pi}{180} \text{ wglvi} \\
 &= 14\pi \text{ wglvi} \\
 &= 14 \times 3.1416 \text{ wglvi (c\ddot{a}q)} \\
 &= 43.98 \text{ wglvi (c\ddot{a}q)}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Gnmv\ddot{t}bi MvZ\ddot{t}eM} &= \frac{43.98}{10} \text{ wglvi / tm\ddot{t}K\ddot{U}} = 4.398 \text{ wglvi / tm\ddot{t}K\ddot{U}} \\
 &= 4.4 \text{ wglvi / tm\ddot{t}K\ddot{U} (c\ddot{a}q)}
 \end{aligned}$$

D\ddot{E}i : 4.4 wglvi / tm\ddot{t}K\ddot{U} (c\ddot{a}q)

D`vniY 9 | 540 wK\ddot{t}j wglvi ` \ddot{t}i GKvU we\` \ddot{t}Z \ddot{t}Kv\ddot{t}bv cvnvo 7' \ddot{t}KvY Drcb\ddot{e}K\ddot{t}i | cvnvo\ddot{U}i D" PZv wby\ddot{e} Ki |

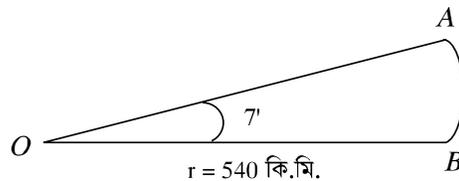
mgvavb : g\ddot{t}b Kvi, AB cvnvo\ddot{U}i cv` we\` y A \ddot{t} \ddot{t}K 540 wK.wg. ` \ddot{t}i O we\` \ddot{t}Z cvnvo\ddot{U} 7' \ddot{t}KvY Drcb\ddot{e} K\ddot{t}i | Zvnt\ddot{t}j $AO = r = e`v\ddot{m}va^{\circ} = 540 \text{ wK.wg.}$

$$\ddot{t}K\` \ddot{t}KvY \text{ } AOB = 7' = \left(\frac{7}{60}\right)^{\circ} = \frac{7\pi}{60 \times 180} \text{ ti wWqvb |}$$

Pvc AB cvnvo\ddot{U}i D" PZv = s wK.wg.

Avgi v Rvnb,

$$\begin{aligned}
 S &= r\theta = 540 \times \frac{7\pi}{60 \times 180} \text{ wK.wg.} \\
 &= \frac{7 \times 3.1416}{20} \text{ wK.wg (c\ddot{a}q)} \\
 &= 1.1 \text{ wK.wg. (c\ddot{a}q) |}
 \end{aligned}$$



D\ddot{E}i : cvnvo\ddot{U}i D" PZv 1.1 wK.wg. (c\ddot{a}q) ev 1100 wglvi (c\ddot{a}q) |

Ab\ddot{t}kj bx 8.1

K`vj K\ddot{t}j Ui e`envi K\ddot{t}i w\ddot{b}\ddot{t}g\ddot{e} mgm`v\` \ddot{t}j vi mgvavb wby\ddot{e} Ki | mg` - \ddot{t}\ddot{t}\ddot{t} \pi Gi Avmb\ddot{e}gyb Pvi ` kvgK `vb ch\ddot{S} - e`envi Ki ($\pi = 3.1416$) |

1 | (K) | ti wWqv\ddot{t}b c\ddot{K}vk Ki :

- (i) 75°30' (ii) 55°54'53" (iii) 33°22'11"

1 | (L) | wWw\ddot{W}Z c\ddot{K}vk Ki :

- (i) $\frac{8x}{13}$ ti wWqv\ddot{b}
- (ii) 1.3177 ti wWqv\ddot{b}
- (iii) 0.9759 ti wWqv\ddot{b}

- 2| GKwJ tKvYtK IvUgj K I eExq c×wZtZ h_vµtg D^0 I R^c Øviv cKvk Kiv ntj , cØvY Ki th $\frac{D}{180} = \frac{R}{\pi}$.
- 3| GKwJ PvKvi e°vma^{©2} wglvi 3 tm.wg. ntj , PvKvi cwiwai Avmbøgyb Pvi `kugK `vb chS_wbYq Ki |
- 4| GKwJ Mvwoi PvKvi e°vm 0.84 wglvi Ges PvKwJ cØZ tmKtÜ 6 evi Nti | MvwoiJi MwZteM wbYq Ki |
- 5| tKvfbv wî fRi tKvY wZbwJi AbjvZ 2:5:3 ¶iz Zg I enEg tKvYi eExq gvb KZ ?
- 6| GKwJ wî fRi tKvY ,tjv mgvšt tKvYfyß Ges enEg tKvYwJ ¶iz Zg tKvYi wØ_Y | tKvY ,tjvi tiwWqv b cwi gvc KZ ?
- 7| c_w_exi e°vma^{©6440} wK.wg. | XvKv I PÆMög c_w_exi tKt`^a 5° tKvY DrcbæKti | XvKv I PÆMötgi `tZjKZ ?
- 8| c_w_exi e°vma^{©6440} wK.wg. | tUKbvd I tZZwj qv c_w_exi tKt`^a 10°6'3" tKvY DrcbæKti | tUKbvd I tZZwj qvi ga°eZP` tZjKZ ?
- 9| kvnt` GKwJ mvBtKtj Pto eEvKvi ct_ 11 tmKtÜ GKwJ eEPvc AwZµg Kti | hw` PvcwJ tKt`^a 30° tKvY DrcbæKti Ges eEi e°vm 201 wglvi nq, Zte kvnt` i MwZteM KZ ?
- 10| c_w_exi e°vma^{©6440} wK.wg. | c_w_exi Dcti th `BwJ `vb tKt`^a 32" tKvY DrcbæKti Zvt` i `tZjKZ ?
- 11| mKvj 9.30 Uvq Nvwi NvUvi KuvI I wgvbtUi KuvI AšMZ tKvYtK tiwWqv b cKvk Ki |
[mstKZ : GK Ni tKt`^a $\frac{360^0}{60} = 6^0$ wvMötKvY DrcbæKti | 9.30 Uvq NÈvi KuvI I wgvbtUi KuvI gta° e°eavb $\left(15 + 2\frac{1}{2}\right)$ ev $17\frac{1}{2}$ Ni]
- 12| GK e°v³ eEvKvi ct_ NvUvq 6 wK.wg. tetM t`što 36 tmKtÜ th eEPvc AwZµg Kti Zv tKt`^a 60° tKvY DrcbæKti | eEi e°vm wbYq Ki |
- 13| 750 wKtj wglvi `ti GKwJ we`tZ tKvfbv cvvvo 8' tKvY DrcbæKti | cvvvoJi D°PZv wbYq Ki |

8.7 wî tKvYwZK AbjvZmgñ (*Trigonometric Ratios*)

wî tKvYwZi GB Astk cØtg m¶tKvYi t¶tÎ wî tKvYwZK AbjvZmgñ (sine, cosine, tangent, secant, cosecant, cotangent) m°útk°Avtj vPbv Kiv nte | m²tKvYi AbjvZmgñi gva°tg thtKvfbv tKvYi wî tKvYwZK AbjvZmgñ wbYq i tKškj Avtj vPbv Kiv nte | AbjvZ mgñi cvi°úwi K m°úK Ges wevfbaPZfM Gt` i wPy wK nte tm m°útk°e°vL°v Kiv nte | wî tKvYwZK AbjvZ msµvš-KwZcq Atf` m°útk°avi Yv t` lqv nte | GQvovI Av`k°tKvYmgñi $\left(0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ wî tKvYwZK AbjvZ Ges AYjvZmgñi mtev° ev mefbggyb A_® gvtbi cwiwa m°útk°Avtj vPbvI GB Astk Ašf° _vKte |

“be” : wĭ tKvYvgnwZK AbjcvZmgntK mstġtġc wj Lv nq | thgb :

sine $\theta = \sin \theta$, cosine $\theta = \cos \theta$, tangent $\theta = \tan \theta$,
 secant $\theta = \sec \theta$, cosecant $\theta = \operatorname{cosec} \theta$, cotangent $\theta = \cot \theta$

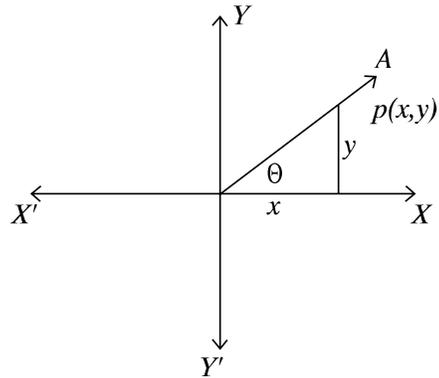
(L) GB Astk Avgiv thtKvġbv tKvġYi Rb“ wĭ tKvYvgnwZK AbjcvZmgnt wbyġ Kie | tm Rb“ Avgvġ`i tKvYvUj cġgZ ev Av`kAe`vb (Standard position) Rvbv `iKvi | KvġZġxq mgZġj ġġ wv`y tġK Wbvġ tK A_ġr abvZġK w K x-AġġtK Avw` iwkġati tKvYvU Aġb Kiġj B Gi Av`kAe`vb cvl qv hvq | GLvġb θ tK Avgiv wĭ tKvYvgnwZK tKvY wmvġte wetePbv Kie A_ġr θ tKvġYi cwi gvY wbv`ġ mxgvi ġġa`_vKte bv |

ġġb Kwġ, KvġZġxq Zġj X'OX tiLv x-Aġġ Y'OY tiLv y-Aġġ Ges O wv`yġġ wv`y | NYġqgvb iwkġ OA abvZġK x-Aġġ A_vġr OX iwkġtġK`i` Kġi Nwġi KuUvi weciZ w`ġK (anticlockwise) Nġi OA Ae`vb θ tKvY DrcbġKġiġġ (wġġ 8.14) |

OX tK θ tKvġYi Awġ evġ (initial side) Ges OA tK cġġKevġ (terminal side) ej v nq | OA cġġK evġi Dci O wv`ywfbae $P(x, y)$ GKwU wv`ywb | Zvntġ OX tġK wv`yUj j^{\wedge} tZġ y, OY tġK Gi j^{\wedge} tZġ x Ges $\angle OQP$ mgġKvY (wġġ 8.14) |

mZġivs cġ_vġMviġmi mġ Abjvġġi AwZġrR = $|OP| = r = \sqrt{x^2 + y^2}$ | Zvntġ th tKvġbv tKvġYi θ Gi Rb“ wĭ tKvYvgnwZK AbjcvZmgnt nte :

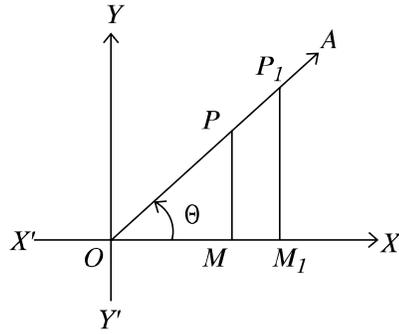
$$\begin{aligned} \sin \theta &= \frac{j^{\wedge}}{\text{AwZġrR}} = \frac{y}{r} \\ \cos \theta &= \frac{fġg}{\text{AwZġrR}} = \frac{x}{r} \\ \tan \theta &= \frac{j^{\wedge}}{fġg} = \frac{y}{x} \quad [x \neq 0] \\ \sec \theta &= \frac{\text{AwZġrR}}{fġg} = \frac{r}{x} \quad [x \neq 0] \\ \operatorname{cosec} \theta &= \frac{\text{AwZġrR}}{j^{\wedge}} = \frac{r}{y} \quad [y \neq 0] \\ \cot \theta &= \frac{fġg}{j^{\wedge}} = \frac{x}{y} \quad [y \neq 0] \end{aligned}$$



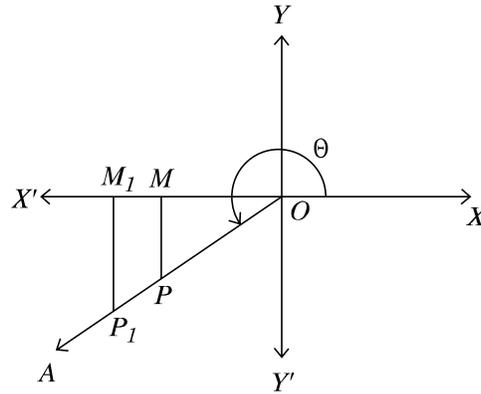
wġġ : 8.14

j ġġYxq 1 | P Ges O wv`ywfbaenl qvq $r = |OP| > 0$ Ges $\sin \theta$ | $\cos \theta$ memġqB A_ġn | OA cġġK evġ x-Aġġġi Dci _vKġj $y = 0$ nq etġ Giġc tKvġYi Rb“ $\operatorname{cosec} \theta$ | $\cot \theta$ msÁwqZ bq | Abjġcġvġte, OA cġġK evġ y-Aġġġi Dci _vKġj $x = 0$ nq Ges Giġc tKvġYi Rb“ $\sec \theta$ | $\tan \theta$ msÁwqZ nq bv |

j ¶|bxq 2| cŃŠK evŃ OA Gi Dci $P(x, y)$ we`y wFbœAb` tKvŃbv we`y $P_1(x_1, y_1)$ wB (wPŃ 8.15(K) | wPŃ 8.15 (L))| $P(x, y)$ | $P_1(x_1, y_1)$ we`Ńq t_ŃK x AŃ¶i Dci PM | P_1M_1 j œ^AwwK | Zvntj ΔOPM | ΔOP_1M_1 m`k|



wPŃ 8.15(K)



wPŃ 8.15(L)

$$A_{\text{w}} \frac{|x|}{|x_1|} = \frac{|y|}{|y_1|} = \frac{|OP|}{|OP_1|} = \frac{r}{r_1}$$

GLvŃb, $OP = r, OP_1 = r_1, x$ | x_1 Ges y | y_1 GKB wPŃyŃ |

$$\therefore \frac{x}{x_1} = \frac{y}{y_1} = \frac{r}{r_1} \text{ A}_{\text{w}}, \frac{x}{r} = \frac{y}{r} \text{ Ges } \frac{x_1}{r_1} = \frac{y_1}{r_1}$$

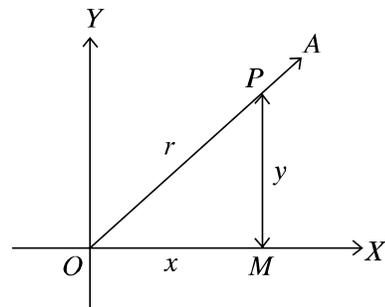
$$\text{mZivs } \sin\theta = \frac{y}{r} = \frac{y_1}{r_1}$$

$$\cos\theta = \frac{x}{r} = \frac{x_1}{r_1}$$

$$\tan\theta = \frac{y}{x} = \frac{y_1}{x_1} \text{ BZ`w` |}$$

wmœvš:- wŃ tKvYwZK AbœvZmgŃni gvb cŃŠK i wKŃ OA Gi Dci wbeŃPZ we`y P Gi Dci wbfŃ KŃi bv|

j ¶|Yxq 3| θ mœŃKvY ntj cŃgZ ev Av`kœAe`vŃb Gi cŃŠK evŃ OA cŃg PZfŃM vŃK Ges $\theta = \angle XOA$ nq (wPŃ 8.16)| OA evŃtZ tŃKvŃbv we`y $P(x, y)$ wbtq Ges P



t_ŃK OX Gi Dci PM j œ^tUŃb t`Lv hvq th, $OM = x, PM = y$ Ges $OP = r$ aŃi ceœZŃ AvŃj vPbvi (K) | (L) t_ŃK θ tKvŃYi AbœvZ_Ńj vi GKB gvb cvl qv hvq|

(M) wĭ tKvYwgwZK AbjcvZ, tĭ vi cvi ūwi K mαúK©

wĭ tKvYwgwZK AbjcvZmgñi msÁv t_tK Avgiv j ħ Kwi th,

$$\sin\theta = \frac{j^{\alpha^{\wedge}}}{\text{AwZfR}} \operatorname{cosec}\theta = \frac{\text{AwZfR}}{j^{\alpha^{\wedge}}} = \frac{1}{\frac{j^{\alpha^{\wedge}}}{\text{AwZfR}}} = \frac{1}{\sin\theta}$$

$$\therefore \sin\theta = \frac{1}{\operatorname{cosec}\theta} \quad \text{Ges } \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

Abjcfvte, $\cos\theta = \frac{\text{fĭg}}{\text{AwZfR}}$, $\sec\theta = \frac{\text{AwZfR}}{\text{fĭg}} = \frac{1}{\frac{\text{fĭg}}{\text{AwZfR}}} = \frac{1}{\cos\theta}$

$$\text{A}_\text{f} \cos\theta = \frac{1}{\sec\theta} \quad \text{Ges } \sec\theta = \frac{1}{\cos\theta}$$

GKBfvte, $\tan\theta = \frac{1}{\cot\theta}$ Ges $\cot\theta = \frac{1}{\tan\theta}$

8.8 wĭ tKvYwgwZK AbjcvZ msμivš-KwZcq mnR A_t` vej x (**Identifics**)

(i) $\sin^2\theta + \cos^2\theta = 1$

cġvY : cvtki wPĭ t_tK Avgiv t` wL tq,

$$\cos\theta = \frac{\text{fĭg}}{\text{AwZfR}} = \frac{x}{r}$$

$$\sin\theta = \frac{j^{\alpha^{\wedge}}}{\text{AwZfR}} = \frac{y}{r}$$

$$\text{Ges } r^2 = x^2 + y^2$$

$$\therefore \sin^2\theta + \cos^2\theta = \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{x^2 + y^2}{r^2} = \frac{r^2}{r^2} = 1$$

$$\therefore \sin^2\theta + \cos^2\theta = 1 \quad (\text{cġwvYZ})$$

(i) bs dj vdj t_tK Avgiv cvB, $\sin^2\theta = 1 - \cos^2\theta$ ev, $\cos^2\theta = 1 - \sin^2\theta$

Abjcfvte cġvY Kiv hvq th, ..

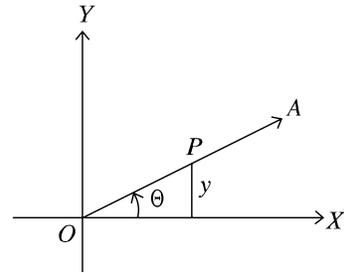
(ii) $1 + \tan^2\theta = \sec^2\theta$ ev, $\sec^2\theta - 1 = \tan^2\theta$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ev, $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

KvR : cġvY Ki th, (wPĭ i mrvtĥ) :

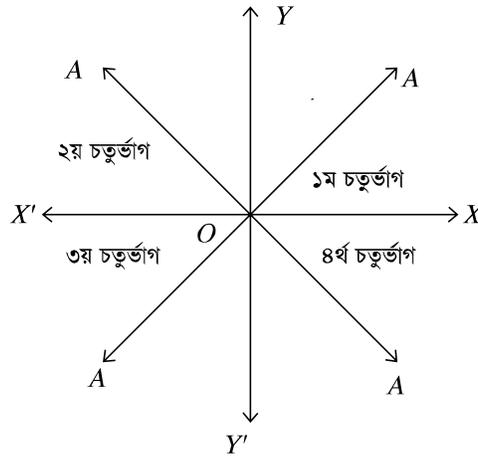
(i) $\sec^2\theta - \tan^2\theta = 1$

(ii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$



8.9 weifbafPZffM wî tkvYwuzK AbcvZmgñi wPy

bxtPi wPî (wPî 8.18) KvZxq Zj tk X'OX Ges Y'OY Aq0q 0viv Pviw PZffM (Quadrant) h_vµtg XOY (1g PZffM), YOX' (2q PZffM) X'OY' (3q PZffM) Ges Y'OX (4_PZffM) wef³ Kiv ntq0



wPî 8.18

Aw` Ae`vb OX t_tK GKw iukf OA, Nvoi Kuvvi weci xZ w` tk NY0bi dtj OA Gi c0SK Ae`vtbi Dci wbfP Kti weifbafKiv Drcbante | NY0gvb iukf OA Gi Dci thtKvfbv we`y P(x, y) wb | Zvntj | OP | = r | c0SK iukf OA Ges P we`j weifbafPZffM Ae`vtbi m½ m½ x | y Gi wPy cwieZ0 nte wKŠ` r memgq abvZK_vKte |

OA iukf hLb c0g PZffM_vtK, ZLb x | y Gi gvb abvZK | ZvB c0g PZffM mKj wî tgvYwuzK AbcvZ abvZK OA iukf hLb w0Zxq PZffM_vtK ZLb P we`j fR x FYvZK Ges tkwU y abvZK |

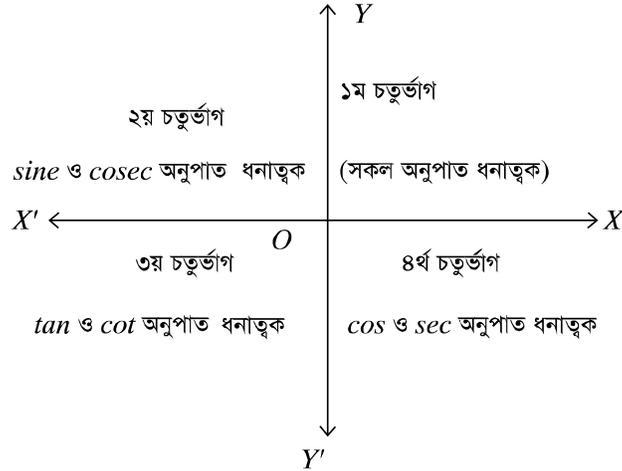
GRb` w0Zxq PZffM $\sin\left(\sin\theta = \frac{y}{r}\right)$ Ges $\operatorname{cosec}\left(\operatorname{cosec}\theta = \frac{r}{y}\right)$ AbcvZ `Bw abvZK Ab`me AbcvZ FYvZK | GKBFvte ZZxq PZffM P we`j fR x | tkwU y DfqB FYvZK Ges $\tan\left(\tan\theta = \frac{-y}{-x} = \frac{y}{x}\right)$ | $\cot\left(\cot\theta = \frac{-y}{-x} = \frac{y}{x}\right)$ abvZK | Ab` AbcvZmgñ FYvZK | PZL`PZffM

OA iukf Dci P we`j fR x abvZK Ges tkwU y FYvZK etj $\cos\left(\cos\theta = \frac{x}{r}\right)$ Ges $\sec\left(\sec\theta = \frac{r}{x}\right)$ abvZK Ges Ab`me wî tkvYwuzK AbcvZ FYvZK |

Avei, x-Aqñi thtKvfbv Ae`vtb y Gi gvb kb` etj $\operatorname{cosec}\left(\operatorname{cosec}\theta = \frac{r}{y}\right)$ Ges $\cot\left(\cot\theta = \frac{x}{y}\right)$

AbcvZ `Bw msÁmqZ bq |

Abjfcvte, y -A π thi th π Kv π bv Ae π v π b x Gi gvb kb π | ZvB y -A π thi Dci $\sec\left(\sec\theta = \frac{r}{x}\right)$ Ges
 $\tan\left(\tan\theta = \frac{y}{x}\right)$ ms π v π qZ bq | $\sin\left(\sin\theta = \frac{y}{r}\right)$ Ges $\cos\left(\cos\theta = \frac{x}{r}\right)$ AbjcvZ π B π U P we π |
 th π Kv π bv Ae π v π bB ms π v π qZ Ges ev π e gvb Av π Q |
 Dc π v π Av π j vPbvi mvi vsk w π tq π w π \pi π (w π \pi π 8.19) t π Lv π bv ntj v D π w π \pi π i m π v π th π th π Kv π bv tKv π Yi
 c π Sk i w π k π i Ae π v π b π Dci w π f π K π i D π tKv π Yi mKj w π \pi π K π vY π g π wZK Abjcv π Zi w π \pi π w π Y π m π R n π e |



8.10 | Av π k π K π vY π g π ni w π \pi π K π vY π g π wZK AbjcvZ : GB Astk e π j e π e π Z tKvY π g π ni $\left(0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right)$

w π \pi π K π vY π g π wZK AbjcvZm π g π ni gvb w π tq Av π g π v GK π w tU π ej π Zwi Kie, hv π Z w π k π v π MY m π t π RB tKvY π g π ni
 w π \pi π K π vY π g π wZK AbjcvZ w π Y π q m π \pi π g nq | gva π gK R π v π g π wZi π v π k Aa π v π tq Dwj wLZ Av π k π tKvY π g π ni
 w π \pi π K π vY π g π wZK AbjcvZ w π Y π q π tK π Skj we π w π i Zfv π te Av π j vPbv Kiv ntq π Q | ZvB we π w π i Z Av π j vPbvq bv w π tq
 Av π g π v c π Z π \pi π AbjcvZm π g π ni gvb w π Y π K π i Zv GK π w tU π ej ev PvU π Av π K π ti c π K π v Kie |

(K) $\frac{\pi}{6}$ (30°) tKv π Yi w π \pi π K π vY π g π wZK AbjcvZm π g π

cv π ki w π \pi π $r = 2a$ ntj

$$y = a \text{ Ges } x = \sqrt{3a} \text{ Ges } \angle POB = \frac{\pi}{6}$$

$$\therefore \sin \frac{\pi}{6} = \frac{y}{r} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{x}{r} = \frac{\sqrt{3a}}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \frac{y}{x} = \frac{a}{\sqrt{3a}} = \frac{1}{\sqrt{3}}$$

$$\cot \frac{\pi}{6} = \frac{x}{y} = \frac{\sqrt{3a}}{a} = \sqrt{3}$$

$$\sec \frac{\pi}{6} = \frac{r}{x} = \frac{2a}{\sqrt{3a}} = \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} \frac{\pi}{6} = \frac{r}{y} = \frac{2a}{a} = 2$$

(L) $\frac{\pi}{4}$ (45°) tKv†Yi w††Kv†Yw†ZK AbjcvZmgñ

cv†ki w††† $r = \sqrt{2a}$, $x = a$

$y = a$ Ges $\angle POB = \frac{\pi}{4}$

$$\therefore \sin \frac{\pi}{4} = \frac{y}{r} = \frac{a}{\sqrt{2a}} = \frac{1}{\sqrt{2}}$$

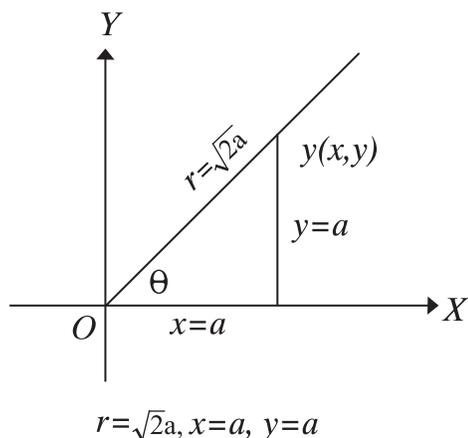
$$\cos \frac{\pi}{4} = \frac{x}{r} = \frac{a}{\sqrt{2a}} = \frac{1}{\sqrt{2}}$$

$$\tan \frac{\pi}{4} = \frac{y}{x} = \frac{a}{a} = 1$$

$$\cot \frac{\pi}{4} = \frac{x}{y} = \frac{a}{a} = 1$$

$$\sec \frac{\pi}{4} = \frac{r}{x} = \frac{\sqrt{2a}}{a} = \sqrt{2}$$

$$\operatorname{cosec} \frac{\pi}{4} = \frac{r}{y} = \frac{\sqrt{2a}}{a} = \sqrt{2}$$



(M) $\frac{\pi}{3}$ (60°) tKv†Yi w††Kv†Yw†ZK AbjcvZmgñ

cv†ki w††† t_†K cvB, $x = a$, $y = \sqrt{3a}$, $r = 2a$

Ges $\angle POB = \frac{\pi}{3}$

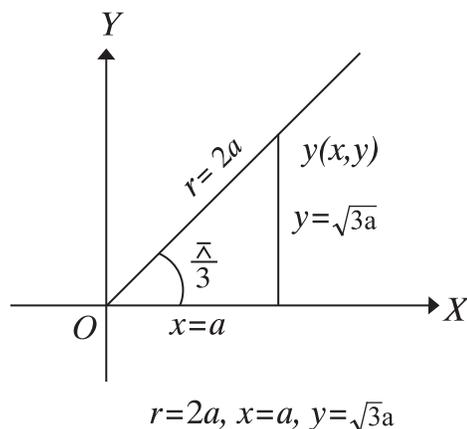
$$\therefore \sin \frac{\pi}{3} = \frac{y}{r} = \frac{\sqrt{3a}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{x}{r} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3a}}{a} = \sqrt{3}$$

$$\cot \frac{\pi}{3} = \frac{x}{y} = \frac{a}{\sqrt{3a}} = \frac{1}{\sqrt{3}}$$

$$\sec \frac{\pi}{3} = \frac{r}{x} = \frac{2a}{a} = 2$$



$$\operatorname{cosec} \frac{\pi}{3} = \frac{r}{y} = \frac{2a}{\sqrt{3a}} = \frac{2}{\sqrt{3}}$$

$\frac{\pi}{2}$ (90°) Ges (0°) tKvYi wî tKvYigwZK AbjcvZmgñi gvb wbYqj Rb° Avgiv wî tKvYigwZK AbjcvZi msÁv e°envi Kie| GLvþb Dþj L° th, kb° Øviv tKvb wKQþKB fvm Kiv hvq bv ev kb° Øviv fvm MðYþhvM° bq (Divisin by zero is not allowed) A_ev kb° Øviv fvm AmsÁwqZ (undefined)|

(N) $\frac{\pi}{2}$ (90°) tKvYi wî tKvYigwZK AbjcvZmgñ : Gþññ cðŠK iñkñ OA Gi Ae°vb avZK y Añ eivei ev OY Gi Dci _vþK| dþj OY Gi Dci th tKvþv wë`y P Gi °vbstK fñ 0 l tKwU y nþe|

awi, $y = a$ Zvntj, $r = a$ nþe|

$$\therefore \sin \frac{\pi}{2} = \frac{y}{r} = \frac{a}{a} = 1$$

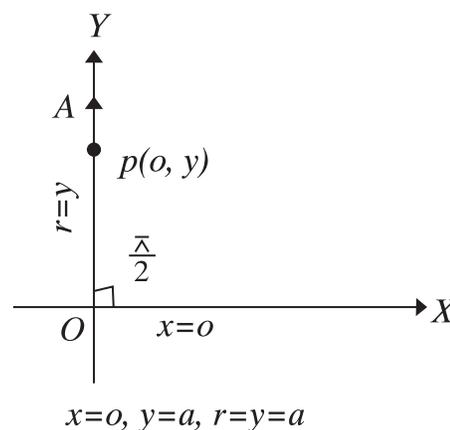
$$\cos \frac{\pi}{2} = \frac{x}{r} = \frac{0}{a} = 0$$

$$\tan \frac{\pi}{2} = \frac{y}{x} = \frac{a}{0} \text{ AmsÁwqZ A_ñ } \tan \frac{\pi}{2} \text{ AmsÁwqZ |}$$

$$\cot \frac{\pi}{2} = \frac{x}{y} = \frac{0}{a} = 0$$

$$\sec \frac{\pi}{2} = \frac{r}{x} = \frac{a}{0} \text{ AmsÁwqZ A_ñ } \sec \frac{\pi}{2} \text{ AmsÁwqZ |}$$

$$\operatorname{cosec} \frac{\pi}{2} = \frac{r}{y} = \frac{a}{a} = 1$$



(O) 0 tiwqvb (0°) tKvYi wî tKvYigwZK AbjcvZmgñ

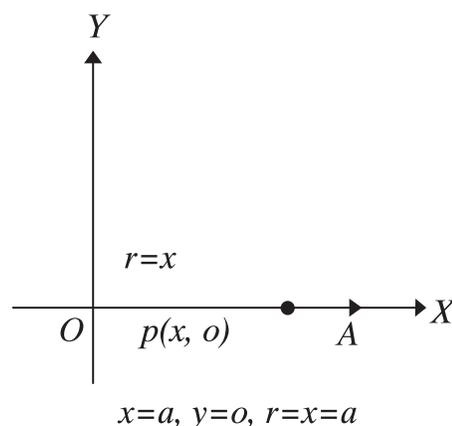
Gþññ cðŠK iñkñ OA AñK| iñkñ OX Gi Dci _vþe| dþj OA iñkñ Dci th tKvþv P wë`j °vbstñi fñ $x = a$ tKwU $y = 0$ Ges $r = OP = a$ nþe|

$$\therefore \sin 0 = \frac{y}{r} = \frac{0}{a} = 0$$

$$\cos 0 = \frac{x}{r} = \frac{a}{a} = 1$$

$$\tan 0 = \frac{y}{x} = \frac{0}{a} = 0$$

$$\cot 0 = \frac{x}{y} \left(= \frac{a}{0} \right) \text{ AmsÁwqZ A_ñ } \cot 0 \text{ AmsÁwqZ |}$$



$$\sec 0 = \frac{r}{x} = \frac{a}{a} = 1$$

$$\operatorname{cosec} 0 = \frac{r}{y} \left(= \frac{a}{0} \right) \text{ AmsÁwqZ A_} \operatorname{cosec} 0 \text{ AmsÁwqZ}$$

we. : i agvĭ eSvĭbvi Rb" $\left(\frac{a}{0}\right)$ tj Lv ntqĭQ| GwU tj Lv wK bq| wkĭv_ĭv mi vmi AmsÁwqZ

wj Ltē|

kg tkĭYi mvavi Y R'wqWzi ĭv`k Aa'vtq wĭtkvYwqWZK AbjcvZmgĭni $\left(0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ Zwj Kv ĩ`l qv

AvĭQ| wkĭv_ĭv i mjeavĭ_ĈZwj KwU GLvĭbI mstĭhwRZ ntjv :

ĭKvY	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	AmsÁwqZ
cot	AmsÁwqZ	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	AmsÁwqZ
cosec	AmsÁwqZ	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

8.11 wĭtkvYwqWZK AbjcvZmgĭni meĭogel mĭePp gvb ev gvĭbi cwi wa

wĭĭ 8.20 j ĭ Kwĭ OA i wkĭNwoi Kuĭvi weci xZ w ĭK AveZĭbi dtj θ ĭKvY DrcbantqĭQ| thĭKvĭbv θ ĭKvĭYi cĭgZ ev Av`kAe`vb OA i wkĭ (OA i wkĭ thĭKvĭbv PZrĭM_vKĭZ cvĭi) Dci P we`j ĭvbsK $P(x, y)$ ntj Avgiv cvB, $r = \sqrt{x^2 + y^2}$ [POQ mgĭKvYx wĭ fR Ges $OP = r$ AwZfR]

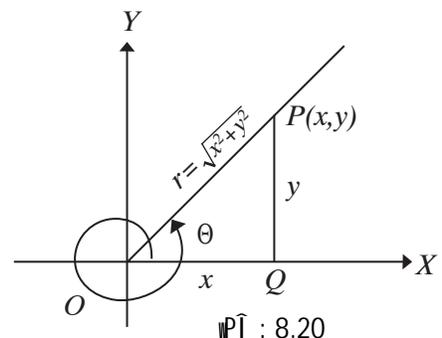
ev, $r^2 = x^2 + y^2$

$\therefore x^2 \leq r^2$ Ges $y^2 \leq r^2$

ev, $|x| \leq r$ Ges $|y| \leq r$

ev, $-r \leq x \leq r$ Ges $-r \leq y \leq r$

ev, $-1 \leq \frac{x}{r} \leq 1$ Ges $-1 \leq \frac{y}{r} \leq 1$ (1)



wĭĭ : 8.20

GLb POQ mgtkvYx wî fRi tñtî

$$\sin\theta = \frac{x}{r}, \cos\theta = \frac{y}{r} \dots\dots\dots (2)$$

$$\operatorname{cosec}\theta = \frac{r}{x}, \sec\theta = \frac{r}{y} \dots\dots\dots (3)$$

$$\tan\theta = \frac{y}{x}, \cot\theta = \frac{x}{y} \dots\dots\dots (4)$$

Zvntj (1) l (2) bs mgxkiY ntZ cvl qv hvq $-1 \leq \sin\theta \leq 1$ Ges $-1 \leq \cos\theta \leq 1$
 mZivs $\sin\theta$ l $\cos\theta$ Gi gvb -1 Afcñv tQvU Ges $+1$ Afcñv eo bq|

Avevi, (1) l (3) bs mgxkiY t_tK cvB, $\operatorname{cosec}\theta \geq 1$ ev, $\operatorname{cosec}\theta \leq -1$
 Ges $\sec\theta \geq 1$ ev, $\sec\theta \leq -1$

mZivs $\sec\theta$ Ges $\operatorname{cosec}\theta$ Ges gvb -1 , Ges -1 Afcñv tQvU Ges $+1$, Ges $+1$ Afcñv eo|

thñZl $\tan\theta = \frac{y}{x}$ Ges $\cot\theta = \frac{x}{y}$

∴ fR $x=0$ ntj $\tan\theta$ AmsÁvqZ Ges tKvU $y=0$ ntj $\cot\theta$ AmsÁvqZ | AmsÁvqZ Gi
 avi Yvfk Amxg (∞) vPy ðviv cKvk Ki tj Avgiv ej tZ cwi |
 $-\infty < \tan\theta < +\infty$ Ges $-\infty < \cot\theta < +\infty$.

D`vniY 1 | θ m²tkvY $\left(0 < \theta < \frac{\pi}{2}\right)$ Ges $\cos\theta = \frac{4}{5}$ ntj, Ab`me wî tkvYvqZK AbjcvZmgñi gvb
 vbYq Ki |

mgvavb : Avgiv Rvb, $\cos\theta = \frac{\text{fig}}{\text{AvZfR}} = \frac{4}{5}$ [t` l qv AvtQ]

cvtki vPñt mgtkvYx wî fR POQ t_tK cvB,

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{5^2 - 4^2}$$

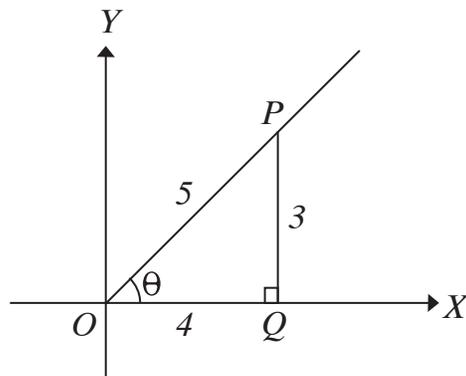
$$= \sqrt{25 - 16} = \sqrt{9} = 3 \text{ GKK}$$

$$\therefore \sin\theta = \frac{\text{j}^{\wedge}}{\text{AvZfR}} = \frac{PQ}{OP} = \frac{3}{5}$$

$$\tan\theta = \frac{\text{j}^{\wedge}}{\text{fig}} = \frac{PQ}{OQ} = \frac{3}{4}$$

$$\sec\theta = \frac{\text{AvZfR}}{\text{fig}} = \frac{Op}{OQ} = \frac{5}{4}$$

$$\operatorname{cosec}\theta = \frac{\text{AvZfR}}{\text{j}^{\wedge}} = \frac{Op}{PQ} = \frac{5}{3}$$



$$\cot\theta = \frac{OQ}{PQ} = \frac{4}{3}$$

weKí : wí ðKvYwZK Aðf` (Identities) e`envi Kti |

Avgi v Rvwb, $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned} \text{ev, } \sin^2\theta &= 1 - \cos^2\theta = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} \\ &= \frac{25 - 16}{25} = \frac{9}{25} \end{aligned}$$

$$\therefore \sin\theta = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

thðnZið mððKvY, ZvB ð cðg PZfðM Aew`Z Ges mKj wí ðKvYwZK AbjvZ avZK |

$$\therefore \sin\theta = \frac{3}{5}$$

$$\text{GLb, } \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

GLb POQ mgðKvYx wí fR ð_ðK cvB,

$$\begin{aligned} \tan\theta &= \frac{j^{\wedge}}{f\wedge} = \frac{j^{\wedge}/A\wedge Z fR}{f\wedge/A\wedge Z fR} = \frac{PQ/OP}{OQ/OP} \\ &= \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \cot\theta &= \frac{f\wedge}{j^{\wedge}} = \frac{f\wedge/A\wedge Z fR}{j^{\wedge}/A\wedge Z fR} = \frac{OQ/OP}{PQ/OP} \\ &= \frac{\cos\theta}{\sin\theta} = \frac{4/5}{3/5} = \frac{4}{3} \end{aligned}$$

$$\text{we.}^{\wedge}: \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$$

wí ðKvYwZK Aðf` i mwnvth, $\sec^2\theta - \tan^2\theta = 1$

$$\text{ev, } \tan^2\theta = \sec^2\theta - 1 = \left(\frac{5}{4}\right)^2 - 1 = \frac{25}{16} - 1 = \frac{9}{16}$$

$$\therefore \tan\theta = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\text{Avevi, } \operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\text{ev, } \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 = \left(\frac{5}{3}\right)^2 - 1 = \frac{25}{9} - 1 = \frac{16}{9}$$

$$\therefore \cot \theta = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

KvR : θ j tKvY $\left(\frac{\pi}{2} < \theta < \pi\right)$ Ges $\tan \theta = -\frac{1}{2}$ ntj , Aci wĭ tKvYugwZK AbjcvZmgñ mg tKvYx wĭ fR Ges wĭ tKvYugwZK A t f` Gi mnv t h` w b Y q Ki |

$$\text{D`niY 2 | } \cos A = \frac{4}{5}, \sin B = \frac{12}{13} \text{ Ges } A \text{ l } B \text{ Dfqb m}^2 \text{ tKvY ntj } \frac{\tan B - \tan A}{1 + \tan B \cdot \tan A} \text{ Gi gvb w b Y q Ki |}$$

$$\text{mgvavb : t` l qv Av t Q, } \cos A = \frac{4}{5}$$

$$\text{Avgiv Rwb, } \sin^2 A + \cos^2 A = 1$$

$$\text{ev, } \sin^2 A = 1 - \cos^2 A$$

$$= 1 - \frac{16}{25} = \frac{9}{25}$$

$$\therefore \sin A = \sqrt{\frac{9}{25}} = \frac{3}{5} \text{ [A m}^2 \text{ tKvY]}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\text{Averi, } \sin B = \frac{12}{13}$$

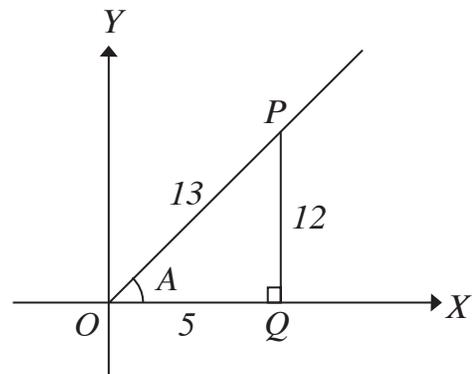
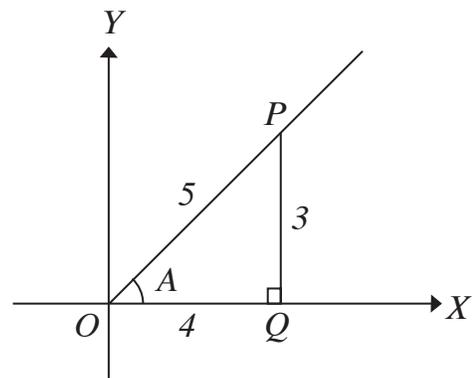
$$\therefore \cos B = \sqrt{1 - \sin^2 B} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}}$$

$$\therefore \cos B = \frac{5}{13}$$

$$\therefore \tan B = \frac{\sin B}{\cos B} = \frac{12/13}{5/13} = \frac{12}{5}$$

$$\begin{aligned} \text{GLb, } \frac{\tan B - \tan A}{1 + \tan B \cdot \tan A} &= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5} \cdot \frac{3}{4}} \\ &= \frac{\frac{48 - 15}{20}}{1 + \frac{36}{20}} = \frac{\frac{33}{20}}{\frac{20 + 36}{20}} = \frac{33}{56} \end{aligned}$$

$$\therefore \frac{\tan B - \tan A}{1 + \tan B \cdot \tan A} = \frac{33}{56}$$



$$D`ni Y 3 | gvb wBY@ Ki : \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{4} + \tan^2 \frac{\pi}{3} + \cot^2 \frac{\pi}{2}$$

$$\text{mgvavb : Avgi v Rwb, } \sin^1 \frac{\pi}{6} = \frac{1}{2}, \cos^1 \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \tan^1 \frac{\pi}{3} = \sqrt{3} \quad \text{Ges } \cot^1 \frac{\pi}{2} = 0$$

$$\therefore \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{4} + \tan^2 \frac{\pi}{3} + \cot^2 \frac{\pi}{2}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 + (0)^2$$

$$= \frac{1}{4} + \frac{1}{2} + 3 = 3\frac{3}{4}$$

$$\text{KvR : 1 | } \sin^2 \frac{\pi}{4} \cos^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{6} \sec^2 \frac{\pi}{3} + \cot^2 \frac{\pi}{3} \operatorname{cosec}^2 \frac{\pi}{4} \text{ Gi gvb wBY@ Ki |}$$

$$2 | \text{ mij Ki : } \frac{\sin^2 \frac{\pi}{3} + \sin \frac{\pi}{3} \cos \frac{\pi}{3} + \cos^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} + \cos \frac{\pi}{3}} - \frac{\sin^2 \frac{\pi}{3} - \sin \frac{\pi}{3} \cos \frac{\pi}{3} + \cos^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} - \cos \frac{\pi}{3}}$$

$$D`ni Y 4 | 7 \sin^2 \theta - 3 \cos^2 \theta = 4 \text{ ntj c@wY Ki th, } \tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\text{mgvavb : } \uparrow \text{ l qv Avt0, } 7 \sin^2 \theta - 3 \cos^2 \theta = 4$$

$$\text{ev, } 7 \sin^2 \theta + 3(1 - \sin^2 \theta) = 4 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\text{ev, } 7 \sin^2 \theta + 3 - 3 \sin^2 \theta = 4$$

$$\text{ev, } 4 \sin^2 \theta = 1$$

$$\text{ev, } \sin^2 \theta = \frac{1}{4}$$

$$\text{Avevi, } \cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\text{ev, } \tan^2 \theta = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$\therefore \tan^2 \theta = \frac{1}{3}$$

$$\therefore \tan \theta = \pm \frac{1}{\sqrt{3}} \text{ (c@wYZ) |}$$

D`vniY 5 | $15 \cos^2 \theta + 2 \sin \theta = 7$ Ges $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ nřj, $\cot \theta$ Gi gvb wBYq Ki |

mgvavb : ř l qv Avř0, $15 \cos^2 \theta + 2 \sin \theta = 7$

ev, $15 \cos^2 \theta + 2 \sin \theta = 7$ [∴ $\sin^2 \theta + \cos^2 \theta = 1$]

ev, $15 - 15 \sin^2 \theta + 2 \sin \theta = 7$

ev, $15 \sin^2 \theta - 2 \sin \theta - 8 = 0$

ev, $15 \sin^2 \theta - 12 \sin \theta + 10 \sin \theta - 8 = 0$

ev, $(3 \sin \theta + 2)(5 \sin \theta - 4) = 0$

∴ $\sin \theta = -\frac{2}{3}$ ev, $\sin \theta = \frac{4}{5}$

$\sin \theta$ Gi Dřq gvb MřYřhvMř | řKbbv $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sin \theta = -\frac{2}{3}$ nřj $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

$\sin \theta = \frac{4}{5}$ nřj $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

∴ $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2}$ [hLb $\sin \theta = -\frac{2}{3}$]

Ges $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$ [hLb $\sin \theta = \frac{4}{5}$]

Ans : $\frac{\sqrt{5}}{2}$ ev, $\frac{3}{4}$

D`vniY 6 | $A = \frac{\pi}{3}$ l $B = \frac{\pi}{6}$ nřj cřvY Ki th,

(i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

cřvY : (i) evgcř = $\sin(A + B) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin \frac{\pi}{2} = 1$

Wbcř = $\sin A \cos B + \cos A \sin B = \sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3} \sin \frac{\pi}{6}$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

$$\therefore \operatorname{evgc} \angle = \operatorname{Wbc} \angle (\operatorname{c} \angle \text{mYZ})$$

$$\begin{aligned} \operatorname{c} \angle \text{vY} : (ii) \operatorname{evgc} \angle &= \tan(A - B) = \tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right) \\ &= \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \operatorname{Wbc} \angle &= \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{6}} \\ &= \frac{\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3}}{1 + \sqrt{3} \cdot \frac{\sqrt{3}}{3}} = \frac{\frac{\sqrt{3}}{3} - \sqrt{3}}{\frac{3}{2}} = \frac{2\sqrt{3}}{3} \times \frac{1}{2} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\therefore \operatorname{evgc} \angle = \operatorname{Wbc} \angle (\operatorname{c} \angle \text{mYZ})$$

KvR : $A = \frac{\pi}{3}$ | $B = \frac{\pi}{6}$ Gi Rb° wbtg° Atf° mgn c°vY Ki :

(i) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(ii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(iii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(iv) $\tan(2B) = \frac{2 \tan B}{1 - \tan^2 B} \sin A \cos B + \cos A \sin B$

Abkxj bx 8·2

1| K°vj K°tj Ui e°envi bv K°ti gvb wby° Ki :

(i) $\frac{\cos \frac{\pi}{4}}{\cos \frac{\pi}{6} + \sin \frac{\pi}{3}}$ (ii) $\tan \frac{\pi}{4} + \tan \frac{\pi}{6} \cdot \tan \frac{\pi}{3}$

2| $\cos \theta = -\frac{4}{5}$ Ges $\pi < \theta < \frac{3\pi}{2}$ n°tj $\tan \theta$ Ges $\sin \theta$ Gi gvb wby° Ki |

3| $\sin A = \frac{2}{\sqrt{5}}$ Ges $\frac{\pi}{2} < A < \pi$ Gi t°t°t° $\cos A$ Ges $\tan A$ Gi gvb KZ ?

4| f` l qv AvfQ, $\cos A = \frac{1}{2}$ Ges $\cos A$ l $\sin A$ GKB wPýwekkó | $\sin A$ Ges $\tan A$ Gi gvb KZ ?

5| f` l qv AvfQ, $\tan A = -\frac{5}{12}$ Ges $\tan A$ l $\cos A$ weciX wPýwekkó | $\sin A$ Ges $\cos A$ Gi gvb wBYq Ki |

6| wbgwj wLZ Af` mgn cõvY Ki :

(i) $\tan A + \cot A = \sec A \operatorname{cosec} A$

(ii) $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \operatorname{cosec} \theta + \cot \theta = \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}}$

(iii) $\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$

(iv) $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

(v) $(\sec \theta - \cos \theta)(\operatorname{cosec} \theta - \sin \theta)(\tan \theta + \cot \theta) = 1$

(vi) $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec + 1} = \tan \theta + \sec \theta$

7| hw` $\operatorname{cosec} A = \frac{a}{b}$ nq, thLvfb $a > b > 0$, Zte cõvY Ki th, $\tan A = \frac{\pm b}{a^2 - b^2}$

8| hw` $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ nq, Zte f` Lvl th, $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

9| $\tan \theta = \frac{x}{y}$ ($x \neq y$) ntj, $\frac{x \sin \theta + y \cos \theta}{x \sin \theta - y \cos \theta}$ Gi gvb wBYq Ki |

10| $\tan \theta + \sec \theta = x$ ntj, f` Lvl th, $\sin \theta = \frac{x^2 - 1}{x^2 + 1}$

11| $a \cos \theta - b \sin \theta = c$ ntj, cõvY Ki th, $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$

12| gvb wBYq Ki :

(i) $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{4} + \tan^2 \frac{\pi}{3} + \cot^2 \frac{\pi}{6}$

(ii) $3 \tan^2 \frac{\pi}{4} - \sin^2 \frac{\pi}{3} - \frac{1}{2} \cot^2 \frac{\pi}{6} + \frac{1}{3} \sec^2 \frac{\pi}{4}$

(iii) $\tan^2 \frac{\pi}{4} - \sin^2 \frac{\pi}{3} \tan^2 \frac{\pi}{6} \tan^2 \frac{\pi}{3} \cdot \cos^2 \frac{\pi}{4}$

(iv) $\frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}} + \cos \frac{\pi}{3} \cos \frac{\pi}{6} + \sin \frac{\pi}{3} \sin \frac{\pi}{6}$

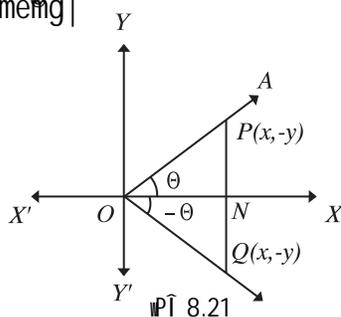
$$13| \text{ mij Ki : } \frac{1 - \sin^2 \frac{\pi}{6}}{1 + \sin^2 \frac{\pi}{4}} \times \frac{\cos^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{6}}{\operatorname{cosec}^2 \frac{\pi}{2} - \cot^2 \frac{\pi}{2}} \div \left(\sin \frac{\pi}{3} \tan \frac{\pi}{6} \right) + \left(\sec^2 \frac{\pi}{6} - \tan^2 \frac{\pi}{6} \right)$$

8.12 wî tKvYvYvZK Avtj vPbvi wZxq Astk Avgiv m² tKvYi $\left(0 < \theta < \frac{\pi}{2} \right)$ AbjcvZmgñ wBYqñi tKškj Avtj vPbv Kñi wQ | AbjcvZmgñi cvi -úwi K m²úK Ges GZ`msµvš-KtqKwU mnR Atf` çõvY Kiv ntqtQ | wwfbc PZfM AbjcvZmgñi wPý wbañY, Av`k tKvYmgñi wî tKvYvYvZK AbjcvZ Ges AbjcvZmgñi meñgñel mtePp gvñbi aviYvI t`lqv ntqtQ | Avtj vPbvi GB Astk cõtq FYvZK tKvY $(-\theta)$ Gi AbjcvZmgñ wBYqñi Kiv nte | Gi Dci wñE Kñi avivewñKfvte $\frac{\pi}{2} - \theta, \frac{\pi}{2} + \theta, \pi + \theta, \pi - \theta, \frac{3\pi}{2} + \theta, \frac{3\pi}{2} - \theta, 2\pi + \theta, 2\pi - \theta$ Ges $n \times \frac{\pi}{2} + \theta$ | $n \times \frac{\pi}{2} - \theta$ [thLvñ n abvZK cYmsL`v Ges $0 < \theta < \frac{\pi}{2}$] tKvYmgñi wî tKvYvYvZK Avtj vPbv AšfP`_vKte |

8.12 (K) $(-\theta)$ tKvYi wî tKvYvYvZK AbjcvZmgñ $\left(0 < \theta < \frac{\pi}{2} \right)$ |

gñb Kwi NYqgvb iñkñ OA Gi Av` Ae`vb OX t`tK Nwoi Kuvvi wñcixZ w`tK Nñi cõtq PZfM $\angle XOA = \theta$ Ges Nwoi Kuvvi w`tK GKB`ñi PZL`PZfM $\angle XOA' = -\theta$ tKvY Drcbañkñi (wPñ : 8.21) | OA iñkñ Dci thñKvñv w`y P(x, y) wñB | GLb P(x, y) w`y t`tK OX Gi Dci PN j` AñK Ges PN tK ewñZ Kivq Zv OA' tK Q w`y t`tQ` Kñi | Zvñtj QN tiLv OX Gi Dci j` tññZl P(x, y) w`y Ae`vb cõtq PZfM tññZl $x > 0, y > 0$ Ges $ON = x, PN = y$.

GLb $\triangle OPN$ | $\triangle OQN$ mgñKvYx wî fRñtqi $\angle PON = \angle QON, \angle ONP = \angle ONQ$ Ges ON Dfq wî fñRi mñaviY evú | mZivs wî fRñq meñg |



$\therefore PN = QN$ Ges $OP = OQ$.

Q w`y Ae`vb PZL`PZfM nIqvq Gi tKwU FYvZK | mZivs Q w`y j` vñvñ Q(x, -y) | $\triangle OQN$ mgñKvYx wî fñRi tñññ $ON = fñg, QN = j$ Ges $OQ = AñZfR = r$ (awi) |

Zvñtj ceZP Avtj vPbv t`tK Avgiv cvB,

$$\sin(-\theta) \frac{j}{AñZfR} = \frac{QN}{OQ} = \frac{-y}{r} = -\sin\theta$$

$$\cos(-\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{ON}{OQ} = \frac{x}{r} = \cos\theta$$

$$\tan(-\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{QN}{ON} = \frac{-y}{r} = -\tan\theta$$

GKBfvte, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$, $\sec(-\theta) = \sec\theta$, $\cot(-\theta) = -\cot\theta$.

D`vniY 8 | $\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$, $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4}$, $\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4}$,

$$\operatorname{cosec}\left(-\frac{\pi}{3}\right) = -\operatorname{cosec}\frac{\pi}{3}$$
, $\sec\left(-\frac{\pi}{3}\right) = \sec\frac{\pi}{3}$, $\cot\left(-\frac{\pi}{6}\right) = -\cot\frac{\pi}{6}$.

8.13 (K) | $\left(\frac{\pi}{2} - \theta\right)$ tkvY ev ci K tkvYi w tkvYigwZK AbcYvZmgñ $\left(0 < \theta < \frac{\pi}{2}\right)$.

awi, tkvYbv NYvqybv iwk OA Zvi Aw` Ae`vb OX t`tk Nmoi Kuvvi wecixZ w`tk Nfi c`g PZfM $\angle XOA = \theta$ tkvY DrcbaKti | Avevi Aci GKw iwk OA' Aw` Ae`vb OX t`tk GKBw`tk Nfi $\angle XOY = \frac{\pi}{2}$ tkvY DrcbaKivi ci OY Ae`vb t`tk Nmoi Kuvvi w`tk Nfi $\angle YOA' = -\theta$ tkvY DrcbaKti (wP` : 8.22) |

Zvntj, $\angle XOA' = \frac{\pi}{2} + (-\theta) = \frac{\pi}{2} - \theta$

OP Ges OQ mgvb `tZiati P I Q we`j t`tk OX Gi Dci PM I QN j`dq Awk |

awi, $OP = OQ = r$ Ges P we`j `vbv¼ P(x, y).

GLb $\triangle POM$ I $\triangle QON$ mgtkvYx w`fRtqi $\angle OMP = \angle ONQ$, $\angle POM = \angle OQN$ Ges $OP = OQ$.

∴ w`fRtq meñg |

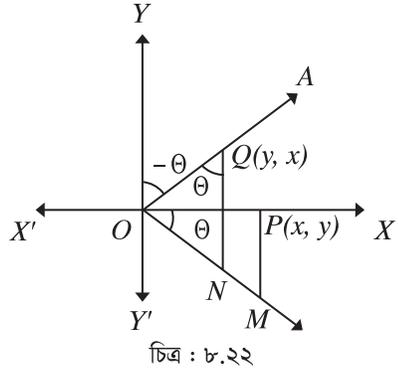
∴ $ON = PM = y$ Ges Q we`j `vbv¼ Q(y, x) .

Zvntj $\triangle NOQ$ Gi t`t` Avgiv cvB,

$$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{QN}{OQ} = \frac{y}{r} = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{ON}{OQ} = \frac{y}{r} = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{QN}{ON} = \frac{x}{y} = \cot\theta$$



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GKBFvte, $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$, $\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$

Ges $\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$.

D`niY 9| $\sin\left(\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}$

$\tan\left(\frac{\pi}{6}\right)=\tan\left(\frac{\pi}{2}-\frac{\pi}{3}\right)=\cot\frac{\pi}{3}$

$\sec\left(\frac{\pi}{4}\right)=\sec\left(\frac{\pi}{2}-\frac{\pi}{4}\right)=\operatorname{cosec}\frac{\pi}{4}$

j 9|Yxq : θ Ges $\left(\frac{\pi}{2}-\theta\right)$ tKvY `Bm ci`ui cK (*Complement Angle*) | Gt`i GKw sine AciwUi cosine, GKwUi tangent AciwUi cotangent Ges GKwUi secant AciwUi cosecant Gi mgvb | wk9|v`9|v mel qmU wetk| fvte j 9| i vLte |

8.13 (L) | $\left(\frac{\pi}{2}+\theta\right)$ tKvYi wI tKvYmgwZK AbcvZmgA $\left(0 < \theta < \frac{\pi}{2}\right)$ |

awi, NY9|gvb iwk9| OA Gi Aw` Ae`vb OX t`tK Nmoi KuUvi wecixZ w`tK N9| c0g PZf9|M $\angle XOA = \theta$ Ges GKB w`tK Avil N9| $\angle AOA' = \frac{\pi}{2}$ tKvY Drcb9| Kti (wP 8.23) | Zvntj,

$\angle XOA = \angle YOA' = \theta$ Ges $\angle XOA' = \frac{\pi}{2} + \theta$ |

gtb Kwi, OA iwk9| Dci $P(x, y)$ thtKvfvb we9| OA' Gi Dci Q we9| Ggbfvte wB thb $OP = OQ$ nq | P | Q we9|y t`tK x-A9|i Dci PM | QN j 9|Uwb |

$\therefore \angle POM = \angle NQO = \angle YOQ = \theta$.

GLb mg9|KvYx wI fR POM | QON Gi g9|a` $\angle POM = \angle NQO$
 $\angle PMO = \angle QNO$

Ges $OP = OQ = r$ (awi) |

$\therefore \triangle POM \cong \triangle QON$ me9|g |

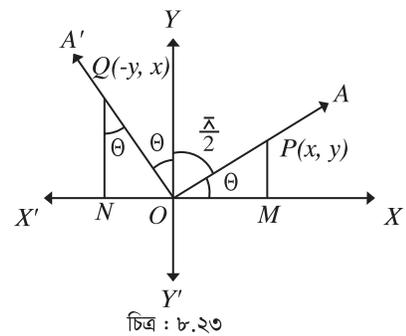
$\therefore |QN| = |OM| = x$

Ges $|ON| = |PM| = y$

A`9|, $ON = -y$, $QN = x$

$\therefore Q$ we9| j `v9|9| $Q(-y, x)$

\therefore Av9|v cvB,



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$$\sin\left(\frac{\pi}{2} + \theta\right) = \frac{x}{r} = \cos\theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = \frac{-y}{r} = -\sin\theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = \frac{x}{-y} = -\cot\theta$$

GKBFvte,

$$\operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) = \sec\theta, \quad \sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec}\theta$$

$$\operatorname{Gescot}\left(\frac{\pi}{2} + \theta\right) = -\tan\theta.$$

D`niY 10 | $\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$$\cos\left(\frac{3\pi}{4}\right) = \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\tan\left(\frac{5\pi}{6}\right) = \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = -\cot\frac{\pi}{3} = -\frac{1}{\sqrt{3}}$$

KvR : $\sec\left(\frac{3\pi}{4}\right), \operatorname{cosec}\left(\frac{5\pi}{6}\right)$ Ges $\cot\left(\frac{2\pi}{3}\right)$ Gi gvb wbyq Ki |

8.14 (K) | $(\pi + \theta)$ tkvtYi wtkvYigwZK AbcivZmgn $\left(0 < \theta < \frac{\pi}{2}\right)$

awi NYqgvb iwkf OA Aw` Ae`vb OX t`tk Nmoi Kulvi wecixZ w`tk Nfi c`g PZfPM $\angle XOA = \theta$
Ges GKb w`tk Avi l Nfi ZZxq PZfPM $\angle AOA' = \pi$ tkvY DrcbaKti (wP` : 8.24) |

Zvntj $\angle XOA' = (\pi + \theta)$.

GLb OA iwkfi Dci thtkvfbv we`y $P(x, y)$ wB | OA' Gi Dci Q we`y wJ Ggbfvte wB thb,
 $OP = OQ = r$ nq |

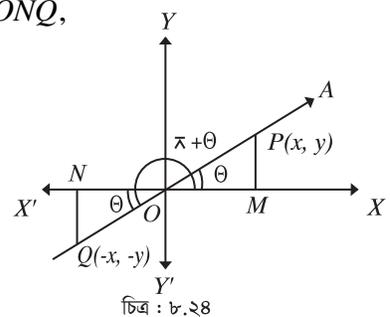
P l Q ntZ x-Af`i Dci PM l QN j wUwb |

$\triangle POM$ l $\triangle QON$ mgtkvYx w`fR0tqi gta` $\angle OMP = \angle ONQ,$
 $\angle POM = \angle QON$ Ges $OP = OQ = r$ | mZi vs w`fR0q mefng |

$$\therefore |PM| = |QN| \quad \text{Ges} \quad |OM| = |ON|$$

thtnZi P we`j `vbv` (x, y) tmtnZi Q we`j `vbv` (-x, -y) .

\therefore Avgiv cvB,



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$$\sin(\pi + \theta) = \frac{-y}{r} = -\sin\theta$$

$$\cos(\pi + \theta) = \frac{-x}{r} = -\cos\theta$$

$$\tan(\pi + \theta) = \frac{-y}{-x} = \tan\theta$$

Abjfcfvte,

$$\operatorname{cosec}(\pi + \theta) = -\operatorname{cosec}\theta, \quad \sec(\pi + \theta) = -\sec\theta \quad \text{Ges } \cot(\pi + \theta) = \cot\theta.$$

$$\text{D'vni Y 11} \quad \sin\left(\frac{4\pi}{3}\right) = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{5\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\tan\left(\frac{7\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\text{KvR : } \sec\left(\frac{4\pi}{3}\right), \operatorname{cosec}\left(\frac{5\pi}{4}\right) \text{ Ges } \cot\left(\frac{7\pi}{6}\right) \text{ Gi gvb wbyq Ki |}$$

8.14 (L) | $(\pi - \theta)$ tkvtYi wî tkvYigwZK AbjcvZmgñ $\left(0 < \theta < \frac{\pi}{2}\right)$ |

awi, NYqgvb iwkf OA Awî Ae⁻vb OX t₋tk Nmoi KuUvi mecixZ wî tk Nñi $\angle XO A = \theta$ tkvY Drcbæ Kti | iwkW GKB wî tk Avil Nñi $\angle AO X' = \pi$ tkvY DrcbæKivi ci OX' t₋tk Nmoi KuUvi wî tk Nñi $\angle X'OA' = -\theta$ tkvY DrcbæKti (wPî : 8.25) |

Zvntj $\angle XO A' = \pi + (-\theta) = \pi - \theta$.

OA iwkfi Dci $P(x, y)$ thtkvtbv we⁻ywb Ges awi $OP = r$ |

OA' iwkfi Dci Q thtkvtbv we⁻ywb thb $OP = OQ = r$ nq |

GLb $\triangle OMP$ | $\triangle ONQ$ mgtkvYx wî fRØtqi gta⁻ $\angle OMP = \angle ONQ, \angle POM = \angle QON$

Ges $OP = OQ = r$

mZi vs wî fRØq m⁻k |

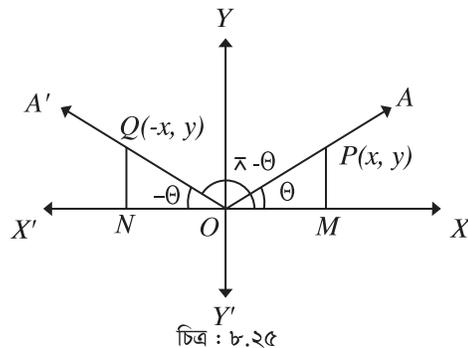
$$\therefore |OM| = |ON| = x \quad \text{Ges } |PM| = |QN| = y$$

$$\therefore Q \text{ we⁻j } \text{vbn} \frac{1}{4} Q(-x, y).$$

Zvntj, Avgiv ciB,

$$\sin(\pi - \theta) = \frac{y}{r} = \sin\theta$$

$$\cos(\pi - \theta) = \frac{-x}{r} = -\cos\theta$$



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$$\tan(\pi - \theta) = \frac{y}{-x} = -\tan\theta$$

Abjfcvte,

$$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta, \quad \sec(\pi - \theta) = -\sec\theta \quad \text{Ges } \cot(\pi - \theta) = -\cot\theta.$$

$$\text{D'vni Y 12} \quad \sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{3\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\tan\left(\frac{5\pi}{6}\right) = \tan\left(\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

$$\text{KvR : } \operatorname{cosec}\left(\frac{3\pi}{4}\right), \sec\left(\frac{5\pi}{6}\right) \text{ Ges } \cot\left(\frac{2\pi}{3}\right) \text{ Gi gvb vbYq Ki |}$$

j P'Yxq : θ Ges $(\pi - \theta)$ tKvY `BvU ci'úi m'útk | m'útk tKvYi sine | cosecant mgvb |
GKB P'Yvewkó | KŠ'cosine, secant, tan gent | cotangent mgvb | veciXZ P'Yvewkó |

$$8.15 (K) \quad \left(\frac{3\pi}{2} - \theta\right) \text{ tKvYi } \widehat{\text{v}} \text{ tKvYvgnZ AbjcvZmgv } \left(0 < \theta < \frac{\pi}{2}\right).$$

ceZP'Avtj vPbvi 8.13 (K) | 8.14 (L) Gi gva'tg Avgiv cvB,

$$\sin\left(\frac{3\pi}{2} - \theta\right) = \sin\left\{\pi + \left(\frac{\pi}{2} - \theta\right)\right\} = -\sin\left(\frac{\pi}{2} - \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = \cos\left\{\pi + \left(\frac{\pi}{2} - \theta\right)\right\} = -\cos\left(\frac{\pi}{2} - \theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = \tan\left\{\pi + \left(\frac{\pi}{2} - \theta\right)\right\} = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

Abjfcvte,

$$\operatorname{cosec}\left(\frac{3\pi}{2} - \theta\right) = -\sec\theta, \quad \sec\left(\frac{4\pi}{2} - \theta\right) = -\operatorname{cosec}\theta$$

$$\text{Ges } \cot\left(\frac{3\pi}{2} - \theta\right) = \tan\theta.$$

$$8.15 (L) \quad \left(\frac{3\pi}{2} + \theta\right) \text{ tKvYi } \widehat{\text{v}} \text{ tKvYvgnZK AbjcvZmgv } \left(0 < \theta < \frac{\pi}{2}\right) |$$

ceZP'Avtj vPbvi 8.13 (L) | 8.14 (K) e'envi Kti cvl qv hvq

$$\sin\left(\frac{3\pi}{2} + \theta\right) = \sin\left\{\pi + \left(\frac{\pi}{2} + \theta\right)\right\} = -\sin\left(\frac{\pi}{2} + \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = \cos\left\{\pi + \left(\frac{\pi}{2} + \theta\right)\right\} = -\cos\left(\frac{\pi}{2} + \theta\right) = -(-\sin\theta) = \sin\theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = \tan\left\{\pi + \left(\frac{\pi}{2} + \theta\right)\right\} = \tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$$

Abjfcfvte,

$$\operatorname{cosec}\left(\frac{3\pi}{2} + \theta\right) = -\sec\theta$$

$$\sec\left(\frac{3\pi}{2} + \theta\right) = \operatorname{cosec}\theta$$

$$\cot\left(\frac{3\pi}{2} + \theta\right) = -\tan\theta.$$

8.16 (K) | $(2\pi - \theta)$ tkvfyi wlf tkvfyiwzk AbjcvZmgñ $\left(0 < \theta < \frac{\pi}{2}\right)$

cögZ ev Av`kAe`vfb $(2\pi - \theta)$ tkvfyi Ae`vb PZL`PZL`fvM`vfk Ges $(-\theta)$ tkvfyi mvf` wgtj hvq| ZvB $(-\theta)$ | $(2\pi - \theta)$ tkvfyi wlf tkvfyiwzk AbjcvZ mgvb |

$$\therefore \sin(2\pi - \theta) = \sin(-\theta) = -\sin\theta$$

$$\cos(2\pi - \theta) = \cos(-\theta) = \cos\theta$$

$$\tan(2\pi - \theta) = \tan(-\theta) = -\tan\theta$$

$$\operatorname{cosec}(2\pi - \theta) = \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$$

$$\sec(2\pi - \theta) = \sec(-\theta) = \sec\theta$$

$$\text{Ges } \cot(2\pi - \theta) = \cot(-\theta) = -\cot\theta$$

8.16 (L) | $(2\pi + \theta)$ tkvfyi wlf tkvfyiwzk AbjcvZmgñ $\left(0 < \theta < \frac{\pi}{2}\right)$

cögZ ev Av`kAe`vfb $(2\pi + \theta)$ tkvfyi Ae`vb cög PZL`fvM`vfkvq θ tkvfyi | $(2\pi + \theta)$ tkvfyi AbjcvZmgñ GKB nte|

mZivs

$$\sin(2\pi + \theta) = \sin\theta, \cos(2\pi + \theta) = \cos\theta$$

$$\tan(2\pi + \theta) = \tan\theta, \operatorname{cosec}(2\pi + \theta) = \operatorname{cosec}\theta$$

$$\sec(2\pi + \theta) = \sec\theta, \cot(2\pi + \theta) = \cot\theta.$$

8.17 | thtkvfbv tkvfyi A`f, $\left(n \times \frac{\pi}{2} \pm \theta\right)$ tkvfyi wlf tkvfyiwzk AbjcvZmgñ wbyqj c`wZ $\left(0 < \theta < \frac{\pi}{2}\right)$ |

wbtg` c`wZtZ thtkvfbv wlf tkvfyiwzk tkvfyi AbjcvZmgñ wbyq Kiv hvq|

avc 1 : (K) cđtg cđĚ tKvYtK `βfvM fvM KiZ nte hvi GKW Ask $\frac{\pi}{2}$ ev $\frac{\pi}{2}$ Gi n ,YZK Ges

AcivW mZtKvY | A_ŕ cđĚ tKvYtK $\left(n \times \frac{\pi}{2} \pm \theta\right)$ AvKvti cKvk KiZ nte |

avc 2 : n tRvo mSL'v ntj AbjvZi aiY GKB _vKte A_ŕ sine AbjvZ sine _vKte, cosine AbjvZ cosine _vKte BZ'w |

n wetrvo ntj sine, tangent | secant AbjvZ ,tjv cosine, cotangent | cosecant G cwi ewZ' nte | GKBfvte, cosine, cotangent | cosecant h_vutg sine, tangent | secant G cwi ewZ' nte |

avc 3 : $\left(n \times \frac{\pi}{2} \pm \theta\right)$ tKvYi Ae'vb tKv PZfM tmUv Rvbi ci H PZfM cđĚ AbjvZi th wPy tmB wPy avc-2 t_tK wbi fucZ AbjvZi cte'emvZ nte |

we.' : 8.17 t_tK ewZ' c'wZi mrvvth' thtKvbtv wĭ tKvYwZK AbjvZ wYq m'ē etj wkv'v_ŕ i GB c'wZtZ wĭ tKvYwZK AbjvZ wYqi Rb' Dct' k t' l qv ntj v |

D`vniY 13 | $\sin\left(\frac{9\pi}{2} + \theta\right)$ tKvYi t'ŕt' $n=9$ GKW wetrvo mSL'v | ZvB sin cwi ewZ' ntq cos nte |

Avevi , $\left(9 \cdot \frac{\pi}{2} + \theta\right)$ `kg ev wZxq PZfM _vtK dtj sin Gi wPy avvZK |

$$\therefore \sin\left(\frac{9\pi}{2} + \theta\right) = \cos \theta .$$

$\sin\left(\frac{9\pi}{2} - \theta\right)$ Gi t'ŕt' $n=9$ wetrvo Ges $\left(\frac{9\pi}{2} - \theta\right)$ beg ev cđg PZfM _vKvq sin Gi wPy avvZK |

$$\therefore \sin\left(\frac{9\pi}{2} - \theta\right) = \cos \theta .$$

$\tan\left(\frac{9\pi}{2} + \theta\right)$ Gi t'ŕt' $n=9$ wetrvo etj tan nte cot Ges $\left(\frac{9\pi}{2} + \theta\right)$ `kg ev wZxq PZfM _vKvq tan Gi wPy FYvZK |

$$\therefore \tan\left(\frac{9\pi}{2} + \theta\right) = -\cot \theta .$$

GKBfvte, $\tan\left(\frac{9\pi}{2} - \theta\right) = \cot \theta$

$$\text{KvR} : \sin\left(\frac{11\pi}{2} \pm \theta\right), \cos(11\pi \pm \theta), \tan\left(17\frac{\pi}{2} \pm \theta\right), \cot(18\pi \pm \theta), \sec\left(\frac{19\pi}{2} \pm \theta\right),$$

Ges $\operatorname{cosec}(8\pi \pm \theta)$ AbrcvZmgntK θ tKvtYi AbrcvtZ cKvk Ki |

8.18 | KwZcq D`vni Y :

D`vni Y 14 | (i) $\sin(10\pi + \theta)$, (ii) $\cos\left(\frac{19\pi}{3}\right)$

(iii) $\tan\left(\frac{11\pi}{6}\right)$, (iv) $\cot\left(\theta - \frac{9\pi}{2}\right)$ |

(v) $\sec\left(-\frac{17\pi}{2}\right)$ Gi gvb wbyq Ki |

mgvavb : (i) $\sin(10\pi + \theta) = \sin\left(20 \times \frac{\pi}{2} + \theta\right)$

GLvtb, $n = 20$ Ges $\sin\left(20 \times \frac{\pi}{2} + \theta\right)$

tKvYwU 21Zg ev cDg PZfM AewZ |

$$\therefore \sin(10\pi + \theta) = \sin \theta .$$

$$\begin{aligned} \text{(ii) } \cos\left(\frac{19\pi}{3}\right) &= \cos\left(6\pi + \frac{\pi}{3}\right) \\ &= \cos\left(12 \times \frac{\pi}{2} + \frac{\pi}{3}\right) \end{aligned}$$

GLvtb $n = 12$ Ges $\frac{19\pi}{3}$ cDg PZfM AewZ |

$$\therefore \cos\left(\frac{19\pi}{3}\right) = \cos \frac{\pi}{3} = \frac{1}{2} .$$

$$\begin{aligned} \text{(iii) } \tan\left(\frac{11\pi}{6}\right) &= \tan\left(2\pi - \frac{\pi}{6}\right) \\ &= \tan\left(4 \times \frac{\pi}{2} - \frac{\pi}{6}\right) \\ &= -\tan \frac{\pi}{6} \quad [n = 4 \text{ | PZL } \Phi ZfM] \\ &= -\frac{1}{\sqrt{3}} . \end{aligned}$$

$$\text{(iv) } \cot\left(\theta - \frac{9\pi}{2}\right) = \cot\left\{-\left(\frac{9\pi}{2} - \theta\right)\right\}$$

$$\begin{aligned}
&= -\cot\left(\frac{9\pi}{2} - \theta\right) \\
&= -\cot\left(9 \times \frac{\pi}{2} - \theta\right) \\
&= -(-\tan\theta) \\
&= \tan\theta \quad [n=9, \frac{9\pi}{2} - \theta \text{ Gi Ae } \bar{\text{v}}\text{b } 4_ \text{PZfAM}]
\end{aligned}$$

$$\begin{aligned}
(v) \sec\left(-\frac{17\pi}{2}\right) &= \sec\left(\frac{17\pi}{2}\right) \quad [\because \sec(-\theta) = \sec\theta] \\
&= \sec\left(17 \times \frac{\pi}{2} + 0\right) \\
&= \sec 0 \quad [n=17, \frac{17\pi}{2} \text{ y A}\bar{\text{t}}\bar{\text{r}} \text{ Dci}] \quad (\text{Ams}\bar{\text{A}}\text{wqZ})
\end{aligned}$$

D`vni Y 15 | gvb wbyq Ki :

$$(i) \sin \frac{11}{90}\pi + \cos \frac{5}{36}\pi + \sin \frac{101}{90}\pi + \cos \frac{31}{30}\pi + \cos \frac{5}{3}\pi$$

$$(ii) \cos^2 \frac{\pi}{15} + \cos^2 \frac{13\pi}{30} + \cos^2 \frac{16\pi}{15} + \cos^2 \frac{47\pi}{30}$$

$$\begin{aligned}
\text{mgvavb : } (i) \sin \frac{11}{90}\pi + \cos \frac{5}{36}\pi + \sin \frac{101}{90}\pi + \cos \frac{31}{30}\pi + \cos \frac{5}{3}\pi \\
&= \sin \frac{22\pi}{180} + \cos \frac{25}{180}\pi + \sin \frac{202}{180}\pi + \cos \frac{186}{180}\pi + \cos \frac{300}{180}\pi \\
&= \sin \frac{22}{180}\pi + \cos \frac{25}{180}\pi + \sin\left(\pi + \frac{22}{180}\pi\right) + \cos\left(\pi - \frac{25}{180}\pi\right) + \cos\left(2\pi - \frac{60}{180}\pi\right) \\
&= \sin \frac{22}{180}\pi + \cos \frac{25}{180}\pi - \sin \frac{22}{180}\pi - \cos \frac{25}{180}\pi + \cos \frac{60}{180}\pi \\
&= \cos \frac{\pi}{3} \\
&= \frac{1}{2} .
\end{aligned}$$

$$\begin{aligned}
(ii) \cos^2 \frac{\pi}{15} + \cos^2 \frac{13\pi}{30} + \cos^2 \frac{16\pi}{15} + \cos^2 \frac{47\pi}{30} \\
= \cos^2 \frac{2\pi}{30} + \cos^2 \frac{13\pi}{30} + \cos^2 \frac{32\pi}{30} + \cos^2 \frac{47\pi}{30}
\end{aligned}$$

$$\begin{aligned}
&= \cos^2 \frac{2\pi}{30} + \cos^2 \frac{13\pi}{30} + \left\{ \cos \left(3 \cdot \frac{\pi}{2} - \frac{13\pi}{30} \right) \right\}^2 + \left\{ \cos \left(3 \cdot \frac{\pi}{2} + \frac{2}{30} \pi \right) \right\}^2 \\
&= \cos^2 \frac{2\pi}{30} + \cos^2 \frac{13\pi}{30} + \left(-\sin \frac{13\pi}{30} \right)^2 + \left(-\sin \frac{2\pi}{30} \right)^2 \\
&= \cos^2 \frac{2\pi}{30} + \cos^2 \frac{13\pi}{30} + \sin^2 \frac{13\pi}{30} + \sin^2 \frac{2\pi}{30} \\
&= \left(\cos^2 \frac{2\pi}{30} + \sin^2 \frac{2\pi}{30} \right) + \left(\cos^2 \frac{13\pi}{30} + \sin^2 \frac{13\pi}{30} \right) = 1 + 1 = 2
\end{aligned}$$

D`vniY 16 | $\tan \theta = \frac{5}{12}$ Ges $\cos \theta$ FYvZK ntj, cõvY Ki th, $\frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta} = \frac{51}{26}$

cõvY : $\tan \theta = \frac{5}{12}$ Ges $\cos \theta$ FYvZK nl qvq θ tKvYi Ae`vb ZZxq PZfM

A`f, $\tan \theta = \frac{5}{12} = \frac{-5}{-12} = \frac{y}{x}$

$\therefore x = -12, y = -5$

$\therefore r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144 + 25}$
 $= \sqrt{169} = 13$

$\therefore \sin \theta = \frac{-y}{r} = -\frac{5}{13}$

$\cos \theta = \frac{-x}{r} = -\frac{-12}{13}$ Ges $\sec \theta = \frac{1}{\cos \theta} = -\frac{13}{12}$

$\therefore \frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta} = \frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta}$ [$\because \cos(-\theta) = \cos \theta, \sec(-\theta) = \sec \theta$]

$= \frac{-\frac{5}{13} - \frac{12}{13}}{-\frac{12}{13} - \frac{5}{13}} = \frac{-\frac{17}{13}}{-\frac{17}{13}} = \frac{17}{13} \times \frac{12}{8} = \frac{51}{26}$ [cõvYZ]

D`vniY 17 | $\theta = \frac{\pi}{6}$ ntj vbtg³ Aft`i (m`i) mZ`Zv hvPvB Ki |

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

cõvY : t`l qv AvfQ, $\theta = \frac{\pi}{6}$

$\therefore \cos 2\theta = \cos \left(2 \cdot \frac{\pi}{6} \right) = \cos \frac{\pi}{3} = \frac{1}{2}$ [$\because \cos \frac{\pi}{3} = \frac{1}{2}$]

$$\begin{aligned}\cos^2 \theta - \sin^2 \theta &= \cos^2 \frac{\pi}{6} - \sin^2 \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 \quad \left[\because \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \sin \frac{\pi}{6} = \frac{1}{2} \right] \\ &= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.\end{aligned}$$

$$\begin{aligned}2\cos^2 \theta - 1 &= 2\cos^2 \frac{\pi}{6} - 1 = 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \\ &= 2 \cdot \frac{3}{4} - 1 = \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}\end{aligned}$$

$$\begin{aligned}1 - 2\sin^2 \theta &= 1 - 2 \cdot \sin^2 \frac{\pi}{6} = 1 - 2 \cdot \left(\frac{1}{2}\right)^2 \\ &= 1 - 2 \cdot \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} &= \frac{1 - \tan^2 \frac{\pi}{6}}{1 + \tan^2 \frac{\pi}{6}} \\ &= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \quad \left[\because \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \right] \\ &= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} \\ &= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}\end{aligned}$$

$$\therefore \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \quad [\text{hvPvBKZ}]$$

[ve. 1: Dctiv³ Atf` mgrn θ Gi th tKv†Yi gv†bi Rb` mZ") cieZPch†q Atf` mgrn c†vY Kiv nte]

D`vniY 15 | $\tan \theta = -\sqrt{3}$, $\frac{\pi}{2} < \theta < 2\pi$ n†j θ Gi gv†b KZ ?

mgv†vb : $\tan \theta$ Gi FYvZK n† qv†q θ Gi Ae`vb †Zxq ev PZL`PZf†M _vK†e |

$$\begin{aligned}\text{†Zxq PZf†M } \tan \theta &= -\sqrt{3} = \tan\left(\pi - \frac{\pi}{3}\right) \\ &= \tan \frac{2\pi}{3}\end{aligned}$$

$$\therefore \theta = \frac{2\pi}{3}$$

GwJ MhYthvM' gvb | Kvi Y $\frac{\pi}{2} < \theta < 2\pi$.

$$\begin{aligned} \text{Avevi, PZL } \theta \text{ fM } \tan \theta &= -\sqrt{3} = \tan\left(2\pi - \frac{\pi}{3}\right) \\ &= \tan \frac{5\pi}{3} \end{aligned}$$

$$\therefore \theta = \frac{5\pi}{3} \text{ hv } \frac{\pi}{2} < \theta < 2\pi \text{ kZ}^{\text{Cvj b Kti}} |$$

$$\therefore \theta \text{ Gi } \text{wbY}^{\text{q}} \text{ gvb, } \frac{2\pi}{3} \text{ | } \frac{5\pi}{3} .$$

D`vni Y 16 | mgvavb Ki $\left(0 < \theta < \frac{\pi}{2}\right)$:

$$(i) \sin \theta + \cos \theta = \sqrt{2}$$

$$(ii) \sec \theta + \tan \theta = \sqrt{3}$$

mgvavb : (i) $\sin \theta + \cos \theta = \sqrt{2}$

$$\text{ev, } \sin \theta = \sqrt{2} - \cos \theta$$

$$\text{ev, } \sin^2 \theta = 2 - 2\sqrt{2} \cos \theta + \cos^2 \theta$$

$$\text{ev, } 1 - \cos^2 \theta = 2 - 2\sqrt{2} \cos \theta + \cos^2 \theta$$

$$\text{ev, } 2\cos^2 \theta - 2\sqrt{2} \cos \theta + 1 = 0$$

$$\text{ev, } (\sqrt{2} \cos \theta - 1)^2 = 0$$

$$\text{ev, } \sqrt{2} \cos \theta - 1 = 0$$

$$\text{ev, } \cos \theta = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4} .$$

$$\therefore \text{wbY}^{\text{q}} \text{ mgvavb } \theta = \frac{\pi}{4} .$$

$$(ii) \sec \theta + \tan \theta = \sqrt{3}$$

$$\text{ev, } \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\text{ev, } \frac{1 + \sin \theta}{\cos \theta} = \sqrt{3}$$

$$\text{ev, } (1 + \sin \theta)^2 = (\sqrt{3} \cos \theta)^2$$

$$\text{ev, } 1 + 2\sin \theta + \sin^2 \theta = 3\cos^2 \theta$$

$$\text{eV, } 1 + 2\sin\theta + 2\sin^2\theta = 3(1 - \sin^2\theta)$$

$$\text{eV, } 1 + 2\sin\theta + \sin^2\theta + 3\sin^2\theta = 3$$

$$\text{eV, } 4\sin^2\theta + 2\sin\theta = 2$$

$$\text{eV, } 2\sin^2\theta + \sin\theta - 1 = 0$$

$$\text{eV, } 2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$$

$$\text{eV, } (2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\therefore 2\sin\theta - 1 = 0 \quad \text{eV, } \sin\theta + 1 = 0$$

$$\text{A}_\text{f}, \sin\theta = \frac{1}{2} \quad \text{eV, } \sin\theta = -1$$

$$0 < \theta < \frac{\pi}{2} \quad \text{Gi Rb} \quad \sin\theta = -1 \quad \text{n}\ddot{\text{t}}\text{Z c}\ddot{\text{v}}\text{i b}\ddot{\text{v}} \text{e}\ddot{\text{t}}\text{j GiU M}\ddot{\text{h}}\text{Y}\ddot{\text{t}}\text{h}\text{v}\text{M} \quad \text{bq}$$

$$\therefore \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6} \quad \text{w}\ddot{\text{b}}\ddot{\text{t}}\text{Y}\ddot{\text{q}} \text{ mgvavb}$$

$$\text{D}\ddot{\text{E}}\text{i} : \theta = \frac{\pi}{6}$$

$$\text{w}\ddot{\text{e}}.\ddot{\text{a}} : \text{h}\ddot{\text{w}} \quad 0 < \theta < 2\pi \quad \text{nZ, Zvntj} \quad \sin\theta = \frac{1}{2} \quad \text{Ges} \quad \sin\theta = -1 \quad \text{Df}\ddot{\text{q}} \text{ gvb M}\ddot{\text{h}}\text{Y}\ddot{\text{t}}\text{h}\text{v}\text{M} \quad \text{nZ} \quad \text{t}\ddot{\text{m}}\ddot{\text{t}}\ddot{\text{q}}\ddot{\text{t}}\ddot{\text{t}}$$

$$\text{mgvavb nZ} \quad \theta = \frac{\pi}{6} \quad \text{A}_\text{e}\text{V} \quad \theta = \frac{3\pi}{2}$$

$$\text{D} \quad \text{vni Y 17} \quad | \quad 0 < \theta < 2\pi \quad \text{n}\ddot{\text{t}}\text{j}, \text{w}\ddot{\text{b}}\ddot{\text{t}}\text{g}\ddot{\text{u}}\ddot{\text{e}}\ddot{\text{e}} \text{ mgxKi Ymg}\ddot{\text{t}}\text{ni mgvavb w}\ddot{\text{b}}\text{Y}\ddot{\text{q}} \text{ Ki} :$$

$$(i) \sin^2\theta - \cos^2\theta = \cos\theta$$

$$(ii) 2(\sin\theta \cos\theta + \sqrt{3}) = \sqrt{3} \cos\theta + 4\sin\theta$$

$$\text{mgvavb} : (i) \sin^2\theta - \cos^2\theta = \cos\theta$$

$$\text{eV, } 1 - \cos^2\theta - \cos^2\theta = \cos\theta$$

$$\text{eV, } 1 - 2\cos^2\theta - \cos\theta = 0$$

$$\text{eV, } 2\cos^2\theta + \cos\theta - 1 = 0$$

$$\text{eV, } (2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\therefore 2\cos\theta - 1 = 0 \quad \text{A}_\text{e}\text{V} \quad \cos\theta + 1 = 0$$

$$\text{A}_\text{f}, \cos\theta = \frac{1}{2} \quad \text{A}_\text{e}\text{V} \quad \cos\theta = -1$$

$$\text{A}_\text{f}, \cos\theta = \cos\frac{\pi}{3} \quad \text{A}_\text{e}\text{V} \quad \cos\theta = \cos\pi$$

$$\therefore \theta = \frac{\pi}{3}, \pi.$$

thñnZi $0 < \theta < 2\pi$ thñnZi Dfç gvb MhYthvM |

wbñYq mgvavb : $\theta = \frac{\pi}{3}, \pi$.

$$(ii) 2(\sin\theta \cos\theta + \sqrt{3}) = \sqrt{3} \cos\theta + 4\sin\theta$$

$$\text{ev, } 4(\sin^2\theta \cos^2\theta + 2\sqrt{3} \sin\theta \cos\theta + 3) = 3\cos^2\theta + 8\sqrt{3} \cos\theta \sin\theta + 16\sin^2\theta$$

$$\text{ev, } 4\sin^2\theta \cos^2\theta + 8\sqrt{3} \sin\theta \cos\theta + 12 = 3\cos^2\theta + 8\sqrt{3} \cos\theta \sin\theta + 16\sin^2\theta$$

$$\text{ev, } 4\sin^2\theta \cos^2\theta + 12 = 3\cos^2\theta + 16\sin^2\theta$$

$$\text{ev, } 4\sin^2\theta(1 - \sin^2\theta) + 12 = 3(1 - \sin^2\theta) + 16\sin^2\theta$$

$$\text{ev, } 4\sin^2\theta - 4\sin^4\theta + 12 = 3 - 3\sin^2\theta + 16\sin^2\theta$$

$$\text{ev, } 4\sin^4\theta + 9\sin^2\theta - 9 = 0$$

$$\text{ev, } 4\sin^4\theta + 12\sin^2\theta - 3\sin^2\theta - 9 = 0$$

$$\text{ev, } (4\sin^2\theta - 3)(\sin^2\theta + 3) = 0$$

$$\therefore 4\sin^2\theta - 3 = 0 \quad \text{ev, } \sin^2\theta + 3 = 0$$

$$\text{A}_\text{fr}, 4\sin^2\theta = 3 \quad \text{ev} \quad \sin^2\theta = -3$$

$$\therefore \sin^2\theta = \frac{3}{4} \quad [\sin^2\theta = -3 \text{ nñZ cvñi bv etj MhYthvM bq, Kvi } -1 \leq \sin\theta \leq 1]$$

$$\text{ev, } \sin\theta = \pm \frac{\sqrt{3}}{2}$$

$$\therefore \sin\theta = \frac{\sqrt{3}}{2} \quad \text{Ges} \quad \sin\theta = -\frac{\sqrt{3}}{2}$$

$$\text{A}_\text{fr}, \sin\theta = \sin\frac{\pi}{3} \quad \text{Ges} \quad \sin\theta = \sin\left(\pi + \frac{\pi}{3}\right)$$

$$\therefore \theta = \frac{\pi}{3} \quad \text{Ges} \quad \theta = \frac{4\pi}{3}$$

$$\therefore \text{wbñYq mgvavb } \theta = \frac{\pi}{3}, \frac{4\pi}{3}$$

Abñkj bx 8.3

1 | $\sin A = \frac{1}{\sqrt{2}}$ nñj $\sin 2A$ Gi gvb KZ ?

$$\text{K. } \frac{1}{\sqrt{2}}$$

$$\text{L. } \frac{1}{2}$$

$$\text{M. } 1$$

$$\text{N. } \sqrt{2}$$

2| -300° tKvYwU tKvb&PZL_¶v¶M_vKte ?

K. cŃg

M. ZZxq

L. wŃZxq

N. PZL_©

3| $\sin\theta + \cos\theta = 1$ ntj $\sin\theta$ Gi gvb nteŃ

i 0°

ii 30°

iii 90°

wb¶Pi tKvbU mWVK ?

K. i

M. i l ii

L. ii

N. i l ii

4. cv¶ki wPŃ Abynv¶i

(i) $\tan\theta = \frac{4}{3}$

(ii) $\sin\theta = \frac{5}{3}$

(iii) $\cos^2\theta = \frac{9}{25}$

wb¶Pi tKvbU mWVK?

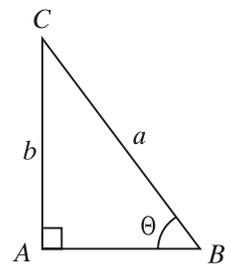
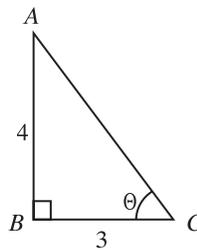
K) i l i i

(L) i l i i i

(M) i i l i i i

(N) i , i i l i i i

wb¶Pi wPŃŃ i Av¶j v¶K 5 bs l 6 bs cŃkŃ DEi `vl :



5| $\sin B + \cos C = KZ$?

K. $\frac{2b}{a}$

M. $\frac{a^2 + b^2}{ab}$

L. $\frac{2a}{b}$

N. $\frac{ab}{a^2 + b^2}$

6| $\tan B$ Gi gvb tKvbU ?

K. $\frac{a}{a^2 - b^2}$

M. $\frac{a}{\sqrt{a^2 - b^2}}$

L. $\frac{b}{a^2 - b^2}$

N. $\frac{b}{\sqrt{a^2 - b^2}}$

7| gvb wYŃ Ki :

(i) $\sin 7\pi$

(ii) $\cos \frac{11\pi}{2}$

(iii) $\cot 1\pi$

(iv) $\tan\left(-\frac{23\pi}{6}\right)$

$$(v) \operatorname{cosec} \frac{19\pi}{3} \quad (vi) \sec\left(-\frac{25\pi}{2}\right) \quad (vii) \sin \frac{31\pi}{6} \quad (viii) \cos\left(-\frac{25\pi}{6}\right)$$

8| cōvY Ki th,

$$(i) \cos \frac{17\pi}{10} + \cos \frac{13\pi}{10} + \cos \frac{9\pi}{10} + \cos \frac{\pi}{10} = 0$$

$$(ii) \tan \frac{\pi}{12} \tan \frac{5\pi}{12} \tan \frac{7\pi}{12} \tan \frac{11\pi}{12} = 1$$

$$(iii) \sin^2 \frac{\pi}{7} + \sin^2 \frac{5\pi}{14} + \sin^2 \frac{8\pi}{7} + \sin^2 \frac{9\pi}{14} = 2$$

$$(iv) \sin \frac{7\pi}{3} \cos \frac{13\pi}{6} - \cos \frac{5\pi}{3} \sin \frac{11\pi}{6} = 1$$

$$(v) \sin \frac{13\pi}{3} \cos \frac{13\pi}{6} - \sin \frac{11\pi}{6} \cos\left(-\frac{5\pi}{3}\right) = 1$$

$$(vi) \tan \theta = \frac{3}{4} \text{ Ges } \sin \theta \text{ FvYZK ntj t`Lvl th, } \frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta} = \frac{14}{5}.$$

9| gvb wYq Ki :

$$(i) \cos \frac{9\pi}{4} + \cos \frac{5\pi}{4} + \sin \frac{31\pi}{36} - \sin \frac{5\pi}{36}$$

$$(ii) \cot \frac{\pi}{15} \cot \frac{3\pi}{20} \cot \frac{5\pi}{20} \cot \frac{7\pi}{20} \cot \frac{9\pi}{20}$$

$$(iii) \sin^2 \frac{\pi}{4} + \sin^2 \frac{3\pi}{4} + \sin^2 \frac{5\pi}{4} + \sin^2 \frac{7\pi}{4}$$

$$(iv) \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8}$$

$$(v) \sin^2 \frac{17\pi}{18} + \sin^2 \frac{5\pi}{18} + \cos^2 \frac{37\pi}{18} + \cos^2 \frac{5\pi}{8}$$

10| $\theta = \frac{\pi}{3}$ ntj wYq Af` mgn cōvY Ki :

$$(i) \sin \theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$(ii) \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$(iii) \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$(iv) \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

11| cō È kZCtY Kti α (Avj dv) Gi gvb wYq Ki :

$$(i) \cot \alpha = -\sqrt{3}; \frac{3\pi}{\alpha} < \alpha < 2\pi$$

(ii) $\cos\alpha = -\frac{1}{2}; \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

(iii) $\sin\alpha = -\frac{\sqrt{3}}{2}; \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

(iv) $\cot\alpha = -1; \pi < \alpha < 2\pi$

12 | mgvavb Ki : (hLb $0 < \theta < \frac{\pi}{2}$)

(i) $2\cos^2\theta = 1 + 2\sin^2\theta$

(ii) $2\sin^2\theta - 3\cos\theta = 0$

(iii) $6\sin^2\theta - 11\sin\theta + 4 = 0$

(iv) $\tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$

(v) $2\sin^2\theta + 3\cos\theta = 3$

13 | mgvavb Ki : (hLb $0 < \theta < 2\pi$)

(i) $2\sin^2\theta + 3\cos\theta = 0$

(ii) $4(\cos^2\theta + \sin\theta) = 5$

(iii) $\cot^2\theta + \operatorname{cosec}^2\theta = 3$

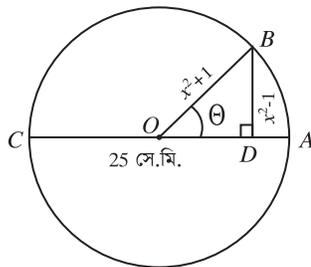
(iv) $\tan^2\theta + \cot^2\theta = 2$

(v) $\sec^2\theta + \tan^2\theta = \frac{5}{3}$

(vi) $5\operatorname{cosec}^2\theta - 7\cot\theta \operatorname{cosec}\theta - 2 = 0$

(vii) $2\sin x \cos x = \sin x (0 \leq x \leq 2\pi)$.

14 |



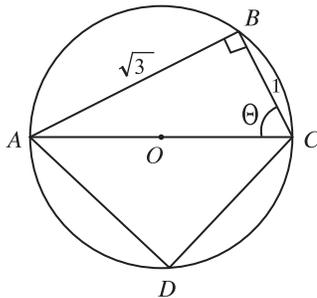
K. ஸ்திரீ ABC GKஹு eஃvKvi Pvk Ges Pvkஹுi AB Pvcி ^N°25 tm.ஹ. nஃ θ = KZ ?

Pvkஹு 1 evi Nஃi KZ ஹுvi `ஃZ; AஹZஹு Ki ஃe ?

L. ABC Pvkஹு cஃZ tmஃKஃU 5 evi AveஹZஃ nஃ Pvkஹுi MஹZஃM Nஃvq KZ nஃe ?

M. ஸ்திரீ ΔBOD nஃ sinθ Gi gv b e`envi Kஃi cஃvY Ki th, tanθ + secθ = x

15 |



K. ஸ்திரீ O, eஃi tk`nஃ ∠B Gi eஃxqgvb Ges AC ஹYஃ Ki |

L. cஃvY Ki th, tanA + tanB + tanC + tanD = 0

M. Secθ + Cosθ = P nஃ P Gi gv b ஹYஃ Ki Ges mgxKi Yஹு mgvavb Ki |

beg Aa'vq mPKxq I j Mwii`gxq dvskb (Exponential & Logarithmic Functions)

mgmvgwqK ev`eZvq mPK I j Mwii`gxq dvsktbi AfbK c0qvM weavq Gi PPP Ae`vnZ itqtQ| thgb RbmsL`v ewx, Pµewx gbdv BZ`w` tZ Dfq dvsktbi c0qvM we`gvb|

Aa'vq tkfI wk¶v_¶v

- gj` mPK I Agj` mPK e`vL`v Ki tZ cvi te|
- gj` I Agj` mPtKi Rb` weifbomf` c0vY I c0qvM Ki tZ cvi te|
- mPK I j Mwii`tgi cvi`úwii K m`úK`e`vL`v Ki tZ cvi te|
- j Mwii`tgi weifbomf` c0vY I c0qvM Ki tZ cvi te|
- j Mwii`tgi wfwE` cwi eZ0 Ki tZ cvi te|
- mPKxq, j Mwii`gxq I ciggvb dvsktbi aviYv e`vL`v Ki tZ cvi te Ges Mwii`ZK mgm`v mgvavb Ki tZ cvi te|
- dvskbmgfni tj LwP` AsKtb AvM0x nte|
- mPKxq, j Mwii`gxq I ciggvb dvskbmgfntK tj LwP` i mrvth` Dc`vcb Ki tZ cvi te|
- K`vj Ktj Utj i mrvth` j M I c0Zj M wby0 Ki tZ cvi te|

9.1 gj` I Agj` mPK : gva`wgK exRMwYtZ Avtj wPZ wKQzweI q hv G Aa`vtqi Avtj vPbvi`¶_D`tj L Kiv ntjv :

- R mKj ev`e msL`vi tmU
- N mKj`wfwK msL`vi tmU
- Z mKj cY`msL`vi tmU
- Q mKj gj` msL`vi tmU wbt`R Kti |

awi a GKwU ALÉ msL`v ev fMusk hv abvZK ev FYvZK ntZ cvi i Ges n GKwU abvZK ALÉ msL`v| Zvntj a tK n evi`_Y Ki tj`_Ydj wJtK wj Lv nq $a^n = a \cdot a \cdot a \dots$ (n evi) a

Ges a^n tK ejv nq a Gi n NvZ| Gi jc t¶t` a tK ejv nq wvavb ev wfwE` (base) Ges n tK ejv nq a Gi NtZi mPK (exponent) A`ev a Gi mPK|

mZivs 3^4 Gi t¶t` wfwE` 3 Ges mPK 4

Aevi, $\left(\frac{2}{3}\right)^4$ Gi t¶t` wfwE` $\frac{2}{3}$ Gi mPK 4 |

msÁv : mKj $a \in R$ Gi Rb`

(1) $a^1 = a$

(2) $a^n = a \cdot a \cdot a \dots a$ (n msL"K Drcv`K), thLvfb, $n \in N, n > 1$

Agj` mPK :

Agj` mPtKi Rb" $a^x (a > 0)$ Gi gvb Ggbfvte wlv`θ Kiv nq th, x Gi gj` Avmbægvb p Gi Rb" a^p Gi gvb a^x Gi gvfb Avmbæq| D`vniY`†fc, $3^{\sqrt{5}}$ msL"vWJ wePbv Kwi | Avgiv Rvb, $\sqrt{5}$ GKwU Agj` msL"v Ges $\sqrt{5} = 2.236067977\dots$ (GB gvb K"vj Ktj Ui e`envi Kti cvl qv wMtqtQ Ges`kugK we`vi th Abš-Zv $\sqrt{5}$ Øviv wbt`R Kiv ntqtQ) | $\sqrt{5}$ Gi Avmbægvb wntmte

$p_1 = 2 \cdot 23$	$p_2 = 2 \cdot 236$	$p_3 = 2 \cdot 2360$
$p_4 = 2 \cdot 236067$	$p_5 = 2 \cdot 2360679$	$p_6 = 2 \cdot 23606797$

wePbv Kti $3^{\sqrt{5}}$ Gi Avmbægvb wntmte

$q_1 = 3^{2 \cdot 23} = 11.5872505\dots$
$q_2 = 3^{2 \cdot 236} = 11.6638822\dots$
$q_3 = 3^{2 \cdot 2360} = 11.6638822\dots$
$q_4 = 3^{2 \cdot 236067} = 11.6647407\dots$
$q_5 = 3^{2 \cdot 2360679} = 11.6647523\dots$
$q_6 = 3^{2 \cdot 23606797} = 11.6647532\dots$

cvl qv hvq (GB gvb ,tj v l K"vj Ktj Ui e`envi Kti cvl qv wMtqtQ)

ev`weK c†¶, $3^{\sqrt{5}} = 11.6647533\dots$

9.2 mPK m"úwKZ m† :

m† 1 : $a \in R$ Ges $n \in N$ ntj , $a^1 = a, a^{n+1} = a^n \cdot a.$

cØvY : msÁvbhvqx $a^1 = a$ Ges $n \in N$ Gi Rb" $a^{n+1} = \underbrace{a \cdot a \cdot a \dots a \cdot a}_{n \text{ msL"K}} = a^n \cdot a$

`ðe" : N mKj `†fweK msL"vi tmU

m† 2 : $n \in R$ Ges $m, n \in N$ ntj $a^m \cdot a^n = a^{m+n}$

cØvY : th†Kvfbv $m \in N$ wlv`θ Kti Ges n tK Pj K a†i tLvj v evK" $a^m \cdot a^n = a^{m+n} \dots (1)$ wePbv Kwi |

(1) G $n=1$ ewmtq cvB,

evgc¶ $a^m \cdot a^1 = a^m \cdot a = a^{m+1}$ Wwbc¶ [m† 1]

$\therefore n=1$ Gi Rb" (1) mZ" |

GLb awi , $n = k$ Gi Rb" (1) mZ" | A_ŕ, $a^m \cdot a^k = a^{m+k} \dots \dots$ (2)

Zvntj , $a^m \cdot a^{k+1} = a^m(a^k \cdot a)$ [mŕ 1]

$$= (a^m \cdot a^k) \cdot a \text{ [,†Yi mn†hvRb]}$$

$$= a^{m+k} \cdot a \text{ [Av†i vn Kí bv]}$$

$$= a^{m+k+1} \text{ [mŕ 1]}$$

A_ŕ, $n = k+1$, Gi Rb" (1) mZ" |

mZi vs MvYvZK Av†i vn c×wZ Abjvqx mKj $n \in N$ Gi Rb" (1) mZ" |

\therefore th †Kv†bv $m, n \in N$ Gi Rb" $a^m \cdot a^n = a^{m+n}$

$$\boxed{\therefore a^m \cdot a^n = a^{m+n}}$$

ewYZ mŕ w†K mP†Ki tgšwj K mŕ ej v nq |

$$\text{mŕ 3} \mid a \in R, a \neq 0 \text{ Ges } m, n \in N \text{ n†j } \frac{a^m}{a^n} = \begin{cases} a^{m-n} \text{ hLb } m > n \\ \frac{1}{a^{n-m}} \text{ hLb } m < n \end{cases}$$

cŕvY : (1) g†b Kwi , $m > n$ Zvntj $m - n \in N$

$$\therefore a^{m-n} \cdot a^n = a^{(m-n)+n} = a^m \text{ [mŕ 2]}$$

$$\therefore \frac{a^m}{a^n} = a^{m-n} \text{ [fv†Mi msÁv]}$$

(2) g†b Kwi , $m < n$ Zvntj $n - m \in N$

$$\therefore a^{n-m} \cdot a^m = a^{(n-m)+m} = a^n \text{ [mŕ 2]}$$

$$\therefore \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \text{ [fv†Mi msÁv]}$$

ŕbe" : mŕ w† MvYvZK Av†i vn c×wZ†Z cŕvY Ki [mŕ 2 Gi Abjfc]

mŕ 4 : $a \in R$ Ges $m, n \in N$ n†j , $(a^m)^n = a^{mn}$

mŕ 5 : $a, b \in R$ Ges $n \in N$ n†j , $(a \cdot b)^n = a^n \cdot b^n$

[mŕ 0q Av†i vn c×wZ†Z cŕvY Ki]

$$\boxed{\text{k b" I FYvZ†K cY†mvsuL"K mPK}}$$

msÁv : $a \in R, a \neq 0$ n†j ,

$$(3) a^0 = 1$$

$$(4) a^{-n} = \frac{1}{a^n}, \text{ thLv†b } n \in N$$

gše" : mPtki aviYv mæcñvi†Yi mgq j ¶" ivLv nq, thb mPtki tgšwj K mĤ $a^m \cdot a^n = a^{m+n}$ mKj
†¶††B Ĥea _v†K|

mĤwJ hw` $m=0$ Gi Rb" mZ" nq, Zte $a^0 \cdot a^n = a^{0+n}$ A_ŕ, $a^0 = \frac{a^n}{a^n} = 1$ n†Z n†e|

GKBfvte, mĤwJ hw` $m=-n$ ($n \in N$) Gi Rb" mZ" n†Z nq,

Zte $a^{-n} \cdot a^n = a^{-n+n} = a^0 = 1$ A_ŕ, $a^{-n} = \frac{1}{a^n}$ n†Z n†e| Gw†K j ¶" ti†LB Dc†i msÁv eYŒv Kiv
n†q†Q|

D`vni Y 1 | $2^5 \cdot 2^6 = 2^{5+6} = 2^{11}$

$$\frac{3^5}{3^3} = 3^{5-3} = 3^2$$

$$\frac{3^3}{3^5} = \frac{1}{3^{5-3}} = \frac{1}{3^2}$$

$$\left(\frac{5}{4}\right)^3 = \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{5 \times 5 \times 5}{4 \times 4 \times 4} = \frac{5^3}{4^3}$$

$$(4^2)^7 = 4^{2 \times 7} = 4^{14}$$

$$(a^2 b^3)^5 = (a^2)^5 \cdot (b^3)^5 = a^{2 \times 5} \cdot b^{3 \times 5} = a^{10} b^{15}$$

D`vni Y 2 | $6^0 = 1, (-6)^0 = 1, 7^{-1} = \frac{1}{7}$.

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}, 10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

D`vni Y 3 | $m, n \in N$ n†j $(a^m)^n = a^{mn}$ mĤwJi mZ"Zv ĤKvi K†i w†q †`LvI th, $(a^m)^n = a^{mn}$
thLv†b $a \neq 0$ Ges $m \in N$ Ges $n \in Z$

mgvavb : (1) GLv†b, $(a^m)^n = a^{mn}$(1)

thLv†b, $a \neq 0$ Ges $m \in N$ | $n \in Z$

cĬtg g†b Kwi, $n > 0$ G†¶†† (1) Gi mZ"Zv ĤKvi K†i t†l qv n†q†Q|

(2) GLb g†b Kwi, $n = 0$ G†¶†† $(a^m)^n = (a^m)^0 = a^0 = 1$

∴ (1) mZ" |

(3) me†k†l g†b Kwi, $n < 0$ Ges $n = -k$, thLv†b $k \in N$

$$G\text{t}\text{q}\text{t}\text{f}\text{f} \quad (a^m)^n = (a^m)^{-k} = \frac{1}{(a^m)^k} = \frac{1}{a^{mk}} = a^{-mk} = a^{m(-k)} = a^{mn}.$$

$$D`vniY 4 | \text{t`Lvl th, mKj } m, n \in N \text{ Gi Rb` } \frac{a^m}{a^n} = a^{m-n} \text{ thLv\text{t}b } a \neq 0$$

$$\text{mgvavb : } m > n \text{ ntj , } \frac{a^m}{a^n} = a^{m-n} \text{ [m\text{f} 3]}$$

$$m < n \text{ ntj , } \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \text{ [m\text{f} 3]}$$

$$\therefore \frac{a^m}{a^n} = a^{-(n-m)} \text{ [m\text{f} 4]} \\ = a^{m-n}$$

$$m = n \text{ ntj , } \frac{a^m}{a^n} = \frac{a^n}{a^n} = 1 = a^0 \text{ [ms\text{A}v 3]} \\ = a^{m-m} = a^{m-n}$$

`be` : Dcti eWYZ mPtKi msAv,tjv t_tK th\text{t}Kv\text{t}bv $m \in Z$ Gi Rb` a^m Gi e`vL`v cvl qv hvq, thLv\text{t}b $a \neq 0$, mPK abvZ\text{K} A_ev kb` A_ev FYvZ\text{K} at\text{i} mvaviYfv\text{t}e mKj cY\text{m}vsuL`K mPtKi Rb` v\text{b}t\text{g}e\text{3} m\text{f}uJ c\text{g}vY Kiv hvq |

m\text{f} 6 : $a \neq 0, b \neq 0$ Ges $m, n \in Z$ ntj ,

$$(K) a^m \cdot a^n = a^{m+n} \quad (L) \frac{a^m}{a^n} = a^{m-n}$$

$$(M) (a^m)^n = a^{mn} \quad (N) (ab)^n = a^n b^n$$

$$(O) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

KvR :

1 | MmYwZK Av\text{t}ivn c\text{x}wZ\text{t}Z t`Lvl th, $(a^m)^n = a^{mn}$ thLv\text{t}b $a \in R$ Ges $n \in N$

2 | MmYwZK Av\text{t}ivn c\text{x}wZ\text{t}Z t`Lvl th, $(a \cdot b)^n = a^n b^n$ thLv\text{t}b $a, b \in R$ Ges $n \in N$

3 | MmYwZK Av\text{t}ivn c\text{x}wZ\text{t}Z t`Lvl th, $\left(\frac{1}{a}\right)^n = \frac{1}{a^n}$, thLv\text{t}b $a > 0$ Ges $n \in N$ |

AZtci $(ab)^n = a^n b^n$ m\text{f} e`envi K\text{t}i t`Lvl th, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ thLv\text{t}b, $a, b \in R, b > 0$, Ges $n \in N$ |

4 | g\text{t}b Ki, $a \neq 0$, Ges $m, n \in Z$ abvZ\text{K} cY\text{m}vsuL`K mPtKi Rb` $a^m \cdot a^n = a^{m+n}$, m\text{f}uJi mZ`Zv`Kvi K\text{t}i t`Lvl th, $a^m \cdot a^n = a^{m+n}$ hLb (i) $m > 0$ Ges $n < 0$, (ii) $m < 0$ Ges $n < 0$ |

9.3 gj Gi e"vL"v

msÁv : $n \in \mathbb{N}, n > 1$ Ges $a \in \mathbb{R}$ ntj , hw` Ggb $x \in \mathbb{R}$ _vtK thb $x^n = a$ nq, Zte tmB x tK a Gi GKwJ n Zg gj ej v nq | 2 Zg gj tK eMgJ Ges 3 Zg gj tK Nbgj ej v nq |

D`vniY 5 | (i) 2 Ges -2 DfqB 16-Gi 4 Zg gj , KviY $(2)^4 = 16$ Ges $(-2)^4 = 16$

(ii) -27 Gi Nbgj -3 , KviY $(-3)^3 = -27$

(ii) 0 Gi n Zg gj 0, KviY mKj $0^n = 0$

(ii) -9 Gi tKv"bv eMgJ tbB, KviY thtKv"bv ev `e msL"vi eMgJ AFYvZK |

GLv"tb, D'tj L" th,

(K) hw` $a > 0$ Ges $n \in \mathbb{N}, n > 1$ nq, Zte a -Gi GKwJ Abb" abvZK n Zg gj AvtQ | GB abvZK gj tK $\sqrt[n]{a}$ Øviv mPZ Kiv nq ($\sqrt[n]{a}$ Gi `tj $\sqrt[n]{a}$ tj Lv nq) Ges GtK a Gi n Zg gj ej v nq | n tRvo msL"v ntj Gi fc a -Gi Aci GKwJ n Zg gj AvtQ Ges Zv ntj v $\sqrt[n]{a}$ |

(L) hw` $a < 0$ Ges $n \in \mathbb{N}, n > 1$ we"tRvo msL"v nq, Zte a -Gi GKwJ gv" n Zg gj AvtQ hv FYZK | GB gj tK $\sqrt[n]{a}$ Øviv mPZ Kiv nq | n tRvo ntj Ges a FYvZK ntj a -Gi tKvb n Zg gj tbB |

(M) 0 Gi n Zg gj " $\sqrt[n]{0} = 0$

`be" : (1) $a > 0$ ntj $\sqrt[n]{a} > 0$

(2) $a < 0$ Ges n we"tRvo ntj ,

$\sqrt[n]{a} = -\sqrt[n]{|a|} < 0$ [thLv"tb $|a|$ nt"Q a Gi ci ggvb] |

D`vniY 6 | $\sqrt{4} = 2, (\sqrt{4} \neq -2) \sqrt[3]{-8} = -2 = -\sqrt[3]{8}, \sqrt{a^2} = |a| = \begin{cases} a, \text{ hLb } a > 0 \\ -a, \text{ hLb } a < 0 \end{cases}$

m"t 7 : $a < 0$ Ges $n \in \mathbb{N}, n > 1, n$ we"tRvo ntj t`Lvl th, $\sqrt[n]{a} = -\sqrt[n]{|a|}$

cgvY : gtb Kwi , $\sqrt[n]{|a|} = x$

Zvntj , $x^n = |a|$ [g'tj i msÁv]

ev, $x^n = -a$ [|a| Gi msÁv]

ev, $-x^n = a$

ev, $(-x)^n = a$ [∴ n we"tRvo]

∴ $\sqrt[n]{a} = -x$ [g'tj i msÁv]

mZivs $\sqrt[n]{a} = -\sqrt[n]{|a|}$ tKbbv a Gi n Zg gj Abb" |

D`vniY 7 | $-\sqrt[3]{27}$

$$\text{mgvavb : } -\sqrt[3]{27} = -\sqrt[3]{(3)^3} = -3$$

mĤ 8 : $a > 0, m \in \mathbb{Z}$ Ges $n \in \mathbb{N}, n > 1$ ntj , $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

cġvY : gġb Kwġ , $\sqrt[n]{a} = x$ Ges $\sqrt[n]{a^m} = y$

$$\text{Zvntj , } x^n = a \text{ Ges } y^n = a^m$$

$$\therefore y^n = a^m = (x^n)^m = (x^m)^n$$

thġnZl $y > 0, x^m > 0$, mġi vs gġ n Zg

$$\text{gj weġPbv Kġi cvB, } y = x^m$$

$$\text{ev, } \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$\text{A_ġ, } (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

mĤ 9 : hw` $a > 0$ Ges $\frac{m}{n} = \frac{p}{q}$ nq, thLvġb $m, p \in \mathbb{Z}$ Ges $n, q \in \mathbb{N}, n > 1, q > 1$

$$\text{Zġe, } \sqrt[n]{a^m} = \sqrt[q]{a^p}$$

cġvY : GLvġb $qm = pn$.

gġb Kwġ , $\sqrt[n]{a^m} = x$ Zvntj , $x^n = a^m$

$$\therefore (x^n)^q = (a^m)^q$$

$$\therefore x^{nq} = a^{mq} = a^{pm}$$

$$\text{ev, } (x^q)^n = (a^p)^n$$

$$\therefore x^q = a^p \text{ [gġ n Zg gj weġPbv Kġi]}$$

$$\therefore x = \sqrt[q]{a^p}$$

$$\therefore \sqrt[n]{a^m} = \sqrt[q]{a^p}$$

Abymxvš-: hw` $a > 0$ Ges $n, k \in \mathbb{N}, n > 1$ nq,

$$\text{Zġe, } \sqrt[n]{a} = \sqrt[nk]{a^k}$$

9.4 gj` fMusk mPK :

msÁv : $a \in \mathbb{R}$ Ges $n \in \mathbb{N}, n > 1$ ntj , (5) $a^{\frac{1}{n}} = \sqrt[n]{a}$ hLb $a > 0$ A_ev $a < 0$ Ges weġRvo |

gše` 1 : mPK wġqg $(a^m)^n = a^{mn}$ [mĤ 6`ġe`]

hw` mKj tġġġ mZ` ntZ nq, Zġe $\left(a^{\frac{1}{n}}\right)^n = a^{\frac{n}{n}} = a^1 = a$ ntZ nġe, A_ġ, $a^{\frac{1}{n}}$ Gi n Zg gj ntZ nġe |

G Rb` GKwġK gġj i tġġġ 0`_Zv cwi nvġi i j tġġ` Dġġi i msÁv eYġv Kiv ntġġQ |

gšë 2 : $a < 0$ Ges $n \in \mathbb{N}, n > 1$ wëfRvo ntj mî 7 t_#K t`Lv hvq

th, $a^{\frac{1}{n}} = \sqrt[n]{a} = -\sqrt[n]{|a|} = -|a|^{\frac{1}{n}}$

Gi fc t¶¶tî GB mî i mvrth`B $a^{\frac{1}{n}}$ Gi gvb wbY@ Kiv nq|

gšë 3 : a gj` msL`v ntj | Awakvsk t¶¶tî $a^{\frac{1}{n}}$ Agj` msL`v nq| Gi fc t¶¶tî $a^{\frac{1}{n}}$ Gi Avmbægyb e`envi Kiv nq|

msÁv : $a > 0, m \in \mathbb{Z}$ Ges $n \in \mathbb{N}, n > 1$ ntj , (6) $a^{\frac{m}{n}} = a^{\left(\frac{1}{n}\right)^m}$

`bë 1 : msÁv (5) | (6) Ges mî 8 t_#K t`Lv hvq th, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

thLvfb, $a > 0, m \in \mathbb{Z}$ Ges $n \in \mathbb{N}, n > 1$

mÿivs $p \in \mathbb{Z}$ Ges $q \in \mathbb{Z}, n > 1$ hw` Ggb nq th, $\frac{m}{n} = \frac{p}{q}$ nq, Zte mî 9 t_#K t`Lv hvq th,

$$a^{\frac{m}{n}} = a^{\frac{p}{q}}$$

`bë 2 : cY¶vswL`K mPK gj` fMusk mP¶Ki msÁv t_#K a^r Gi e`vL`v cvl qv hvq, thLvfb $a > 0$ Ges $r \in \mathbb{Q}$ | Dcti i Avtj vPbv t_#K t`Lv hvq th, $a > 0$ ntj , r tK wewfbæmgZj fMusk AvKv¶i cKvk Kiv ntj | a^r Gi gv¶bi tKv¶bv Zvi Zg` nq bv|

`bë 3 : mî 6 G ewY@ mPK wbcg_s tj v mvavi Yfvte th¶Kv¶bv mP¶Ki Rb` mZ` nq|

mî 10 | $a > 0, b > 0$ Ges $r, s \in \mathbb{Q}$ ntj

(K) $a^r \cdot a^s = a^{r+s}$

(L) $\frac{a^r}{a^s} = a^{r-s}$

(M) $(a^r)^s = a^{rs}$

(N) $(ab)^r = a^r b^r$

(O) $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

(K) | (N) Gi cp:cp c¶qv¶Mi gva`tg t`Lv hvq th,

Abymxvš=: (1) $a > 0$ Ges $r_1, r_2, \dots, r_k \in \mathbb{Q}$ ntj ,

$$a^{r_1} \cdot a^{r_2} \cdot a^{r_3} \dots a^{r_k} = a^{r_1+r_2+r_3+\dots+r_k}$$

(2) $a_1 > 0, a_2 > 0, \dots, a_n > 0$ Ges $r \in \mathbb{Q}$ ntj , $(a_1 \cdot a_2 \dots a_n)^r = a_1^r \cdot a_2^r \dots a_n^r$.

D`vni Y 7 | t`Lvl th, $a^{\frac{m}{n}} \cdot a^{\frac{p}{q}} = a^{\frac{m+p}{n \cdot q}}$

thLvfb, $a > 0; m, p \in \mathbb{Z}; n, q \in \mathbb{N}, n > 1, q > 1$.

mgvavb : $\frac{m}{n} \mid \frac{p}{q}$ tK mgni ~~weikó fMusk~~ cwí YZ Kti t`Lv hvq th,

$$\begin{aligned} a^{\frac{m}{n}} \cdot a^{\frac{p}{q}} &= a^{\frac{mq}{nq}} \cdot a^{\frac{np}{nq}} = \left(a^{\frac{1}{nq}} \right)^{mq} \left(a^{\frac{1}{nq}} \right)^{np} \quad [\text{msÁv 6 e'envi Kti}] \\ &= \left(a^{\frac{1}{nq}} \right)^{mq+np} \quad [\text{mĤ 6}] \\ &= a^{\frac{mq+np}{nq}} \quad [\text{msÁv 6}] \\ &= a^{\frac{mq}{nq} + \frac{np}{nq}} \\ &= a^{\frac{m}{n} + \frac{p}{q}} \end{aligned}$$

KtqKw cġqvRbxq Z_ :

- (i) hw` $a^x = 1$ nq, thLvfb $a > 0$ Ges $a \neq 1$ Zvntj $x = 0$
- (ii) hw` $a^x = 1$ nq, thLvfb $a > 0$ Ges $x \neq 0$ Zvntj $a = 1$
- (iii) hw` $a^x = a^y$ nq, thLvfb $a > 0$ Ges $a \neq 1$ Zvntj $x = y$
- (iv) hw` $a^x = b^x$ nq, thLvfb $\frac{a}{b} > 0$ Ges Ges $x \neq 0$ Zvntj $a = b$

D`vniY 8 | mij Ki :

$$\text{hw` } a^x = b, b^y = c \text{ Ges } c^z = a \text{ nq, Zte t`Lvl th, } xyz = 1.$$

mgvavb : (i) cġ Ē kZġntZ, $b = a^x$, $c = b^y$ Ges $a = c^z$

$$\text{GLb, } b = a^x = (c^z)^x = c^{zx} = (b^y)^{zx} = b^{xyz}$$

$$\Rightarrow b = b^{xyz} \Rightarrow b^1 = b^{xyz}$$

$$\therefore xyz = 1. \text{ (cġwYZ) |}$$

D`vniY 9 | hw` $a^b = b^a$ nq, Zte t`Lvl th, $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$ Ges G t`tK cġvY Ki th,

$$a = 2b \text{ ntj, } b = 2$$

mgvavb : t`l qv AvtQ $a^b = b^a$

$$\therefore b = (a^b)^{\frac{1}{a}} = a^{\frac{b}{a}}$$

$$\begin{aligned} \text{evgc}\uparrow &= \left(\frac{a}{b}\right)^{\frac{a}{b}} = \left(\frac{a}{a^{\frac{b}{a}}}\right)^{\frac{a}{b}} = \left(a^1 \cdot a^{-\frac{b}{a}}\right)^{\frac{a}{b}} \\ &= a^{\frac{a}{b}} \cdot a^{-1} = a^{\frac{a}{b}-1} \text{ Wbc}\uparrow \text{ (c}\hat{\text{g}}\text{wYZ)} | \end{aligned}$$

cpivq, $a = 2b$ ntj

$$\left(\frac{2b}{b}\right)^{\frac{2b}{b}} = (2b)^{\frac{2b}{b}-1} \Rightarrow (2)^2 = (2b)^{2-1}$$

$$\Rightarrow 4 = 2b \quad \therefore b = 2 \text{ (c}\hat{\text{g}}\text{wYZ)} |$$

D`vniY 10 | hw` $x^{x\sqrt{x}} = (x\sqrt{x})^x$ nq Zte x Gi gvb wbyq Ki |

mgvavb : t` l qv AvtQ $x^{x\sqrt{x}} = (x\sqrt{x})^x$

$$\Rightarrow (x^x)^{\sqrt{x}} = \left(x \cdot x^{\frac{1}{2}}\right)^x = \left(x^{1+\frac{1}{2}}\right)^x$$

$$= \left(x^{\frac{3}{2}}\right)^x = (x^x)^{\frac{3}{2}}$$

$$\therefore (x^x)^{\sqrt{x}} = (x^x)^{\frac{3}{2}}$$

$$\Rightarrow \sqrt{x} = \frac{3}{2} \quad \therefore \Rightarrow x = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

D`vniY 11 | hw` $a^x = b^y = c^z$ Ges $b^2 = ac$ nq, Zte cgvY Ki th, $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

mgvavb : thtnZl $a^x = b^y$

$$\text{ev, } a = b^{\frac{y}{x}}$$

Avevi, $c^z = b^y \quad \therefore c = b^{\frac{y}{z}}$

GLb $b^2 = ac$

$$\therefore b^2 = b^{\frac{y}{x}} \cdot b^{\frac{y}{z}} = b^{\frac{y+y}{x+z}}$$

$$\Rightarrow 2 = \frac{y}{x} + \frac{y}{z} \Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{2}{y} \text{ (c}\hat{\text{g}}\text{wYZ)} |$$

D`vniY 12 | cgvY Ki th, $\left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \times \left(\frac{x^a}{x^b}\right)^{a+b} = 1$

$$\text{mgvavb : evgc}\uparrow = \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \times \left(\frac{x^a}{x^b}\right)^{a+b}$$

$$\begin{aligned}
&= (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \times (x^{a-b})^{a+b} \\
&= x^{b^2-c^2} \times x^{c^2-a^2} \times x^{a^2-b^2} \\
&= x^{b^2-c^2+c^2-a^2+a^2-b^2} \\
&= x^0 \\
&= 1 = \text{Wwbc} \blacksquare
\end{aligned}$$

D`vni Y 13 | hw` $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ Ges $abc = 1$ nq, Zte t` Lvl th, $x + y + z = 0$

mgvavb : awi, $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$.

Zvntj cvB, $a = k^x, b = k^y, c = k^z$

$$\therefore abc = k^x k^y k^z = k^{x+y+z}$$

t` l qv AvtQ, $abc = 1$

$$\therefore k^{x+y+z} = k^0$$

$$\therefore x + y + z = 0$$

D`vni Y 14 | mij Ki : $\frac{1}{1+a^{y-z}+a^{y-x}} + \frac{1}{1+z^{z-x}+a^{z-y}} + \frac{1}{1+a^{x-y}+a^{x-z}}$

$$\text{GLvtb, } \frac{1}{1+a^{y-z}+a^{y-x}} = \frac{a^{-y}}{a^{-y}(1+a^{y-z}+a^{y-x})} = \frac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}}$$

$$\text{GKBfite, } \frac{1}{1+z^{z-x}+a^{z-y}} = \frac{a^{-z}}{a^{-z}(1+a^{z-x}+a^{z-y})} = \frac{a^{-z}}{a^{-z}+a^{-x}+a^{-y}}$$

$$\text{Ges } \frac{1}{1+a^{x-y}+a^{x-z}} = \frac{a^{-x}}{a^{-x}+a^{-y}+a^{-z}}$$

$$\begin{aligned}
\text{m} \ddot{\text{Z}} \text{ i vs c} \ddot{\text{O}} \ddot{\text{E}} \text{ i w} \ddot{\text{K}} & \frac{1}{1+a^{y-z}+a^{y-x}} + \frac{1}{1+z^{z-x}+a^{z-y}} + \frac{1}{1+a^{x-y}+a^{x-z}} \\
&= \frac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}} + \frac{a^{-z}}{a^{-z}+a^{-x}+a^{-y}} + \frac{a^{-x}}{a^{-x}+a^{-y}+a^{-z}} \\
&= \frac{a^{-x}+a^{-y}+a^{-z}}{a^{-x}+a^{-y}+a^{-z}} = 1
\end{aligned}$$

D`vni Y 15 | hw` $a = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$ nq, Zte t` Lvl th, $a^3 - 6a^2 + 6a - 2 = 0$.

mgvavb : t` l qv AvtQ, $a = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$

$$\therefore a - 2 = 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$$

$$\text{ev, } (a - 2)^3 = \left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} \right)^3$$

$$= 2^2 + 2 + 3 \cdot 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} \left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} \right)$$

$$= 6 + 6(a-2) \left[\because 2^{\frac{2}{3}} + 2^{\frac{1}{3}} = a-2 \right]$$

$$\text{ev, } a^3 - 3a^2 \cdot 2 + 3a \cdot 2^2 - 2^3 = 6 + 6a - 12$$

$$\text{ev, } a^3 - 6a^2 + 12a - 8 = 6 + 6a - 12$$

$$\text{ev, } a^3 - 6a^2 + 6a - 2 = 0$$

$$\text{D`vni Y 16 | mgvavb Ki : } 4^x - 3 \cdot 2^{x+2} \cdot 2^5 = 0$$

$$\text{mgvavb : } 4^x - 3 \cdot 2^{x+2} + 2^5 = 0$$

$$\Rightarrow (2^2)^x - 3 \cdot 2^x \cdot 2^2 + 2^5 = 0$$

$$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 = 0$$

$$\Rightarrow y^2 - 12y + 32 = 0 \quad [\text{gtb Kwi } 2^x = y]$$

$$\Rightarrow y^2 - 4y - 8y + 32 = 0$$

$$\Rightarrow y(y-4) - 8(y-4) = 0$$

$$\Rightarrow (y-4)(y-8) = 0$$

$$\therefore y-4=0$$

$$\text{or } y-8=0$$

$$\Rightarrow 2^x - 4 = 0 \quad [\because 2^x = y]$$

$$\Rightarrow 2^x - 8 = 0 \quad [\because 2^x = y]$$

$$\Rightarrow 2^x = 4 = 2^2$$

$$\Rightarrow 2^x = 8 = 2^3$$

$$\therefore x=2$$

$$\therefore x=3$$

$$\therefore \text{wb tY} \text{ mgvavb } x=2,3$$

KvR :

1 | gvb wbY} Ki :

$$(i) \frac{5^{n+2} + 35 \times 5^{n-1}}{4 \times 5^n}$$

$$(ii) \frac{3^4 \cdot 3^8}{3^{14}}$$

$$2 | \text{f`Lvl th, } \left(\frac{p^a}{p^b} \right)^{a^2+ab+b^2} \times \left(\frac{p^b}{p^c} \right)^{b^2+bc+c^2} \times \left(\frac{p^c}{p^a} \right)^{c^2+ca+a^2} = 1$$

$$3 | \text{hw` } a = xy^{p-1}, b = xy^{q-1} \text{ Ges } c = xy^{r-1} \text{ nq, Zte f`Lvl th, } a^{q-r} b^{r-p} c^{p-q} = 1$$

$$4 | \text{mgvavb Ki : (i) } 4^x - 3^{\frac{x-1}{2}} = 3^{\frac{x+1}{2}} - 2^{2x-1}$$

$$(ii) 9^{2x} = 3^{x+1}$$

$$(iii) 2^{x+3} + 2^{x+1} = 320$$

$$5 | \text{ mij Ki : (i) } \sqrt[12]{(a^8)} \sqrt{(a^6)} \sqrt{a^4} .$$

$$(ii) \left[1 - 1 \left\{ 1 - (1 - x^3)^{-1} \right\}^{-1} \right]^{-1} .$$

$$6 | \text{ hw } \sqrt[3]{a} = \sqrt[3]{b} = \sqrt[3]{c} \text{ Ges } abc = 1 \text{ nq, Zte c} \ddot{\text{g}}\text{vY Ki } x + y + z = 0 .$$

$$7 | \text{ hw } a^m \cdot a^n = (a^m)^n \text{ nq, Zte c} \ddot{\text{g}}\text{vY Ki th, } m(n-2) + n(m-2) = 0 .$$

Abkxj bx 9.1

$$1 | \text{ c} \ddot{\text{g}}\text{vY Ki th, } \left(a^{\frac{m}{n}} \right)^p = a^{\frac{mp}{n}} \text{ thLv} \ddot{\text{t}}\text{b } m, p \in Z \text{ Ges } n \in N .$$

$$2 | \text{ c} \ddot{\text{g}}\text{vY Ki th, } \left(a^{\frac{1}{m}} \right)^n = a^{\frac{1}{mn}} \text{ thLv} \ddot{\text{t}}\text{b } m, n \in Z$$

$$3 | \text{ c} \ddot{\text{g}}\text{vY Ki th, } (ab)^{\frac{m}{n}} = a^{\frac{m}{n}} b^{\frac{m}{n}}, \text{ thLv} \ddot{\text{t}}\text{b } m \in Z, n \in N$$

$$4 | \text{ t}^{\text{h}} \text{ Lvl th, (K) } \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} \right) \left(a^{\frac{2}{3}} + a^{\frac{1}{3}} b^{\frac{1}{3}} + b^{\frac{2}{3}} \right) = a - b$$

$$(L) \frac{a^{\frac{3}{2}} + a^{-\frac{3}{2}} + 1}{a^{\frac{3}{2}} + a^{-\frac{3}{2}} + 1} = \left(a^{\frac{3}{2}} + a^{-\frac{3}{2}} - 1 \right)$$

5 | mij Ki :

$$(K) \left\{ \left(x^{\frac{1}{a}} \right)^{\frac{a^2 - b^2}{a - b}} \right\}^{\frac{a}{a + b}} \quad (L) \frac{a^{\frac{3}{2}} + ab}{ab - b^3} - \frac{\sqrt{a}}{\sqrt{a - b}}$$

$$(M) \frac{\left(\frac{a+b}{b} \right)^{\frac{a}{a-b}} \times \left(\frac{a-b}{a} \right)^{\frac{a}{a-b}}}{\left(\frac{a+b}{b} \right)^{\frac{b}{a-b}} \times \left(\frac{a-b}{a} \right)^{\frac{b}{a-b}}}$$

$$(N) \frac{1}{1 + a^{-m} b^n + a^{-m} c^p} + \frac{1}{1 + b^{-n} c^p + b^{-n} a^m} + \frac{1}{1 + c^{-p} a^m + c^{-p} b^n}$$

$$(O) \sqrt[bc]{\frac{b}{x^c}} \times \sqrt[ca]{\frac{c}{x^a}} \times \sqrt[ab]{\frac{a}{x^b}} \quad (P) \frac{(a^2 - b^2)^a (a - b^{-1})^{b-a}}{(b^2 - a^{-2})^b (b + a^{-1})^{a-b}}$$

6| t`Lvl th,

(K) hw` $x = a^{q+r} b^p, y = a^{r+p} b^q, z = a^{p+q} b^r$ nq, Zte $x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$.

(L) hw` $a^p = b, b^q = c$ Ges $c^r = a$ nq, Zte $pqr = 1$.

(M) hw` $a^x = p, a^y = q$ Ges $a^z = (p^y q^x)^z$ nq, Zte $xyz = 1$.

7| (K) hw` $x\sqrt[3]{a} + y\sqrt[3]{b} + z\sqrt[3]{c} = 0$ Ges $a^2 = bc$ nq, Zte t`Lvl th, $ax^3 + by^3 + cz^3 = 3axyz$.

(L) hw` $x = (a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}$ Ges $a^2 - b^2 = c^3$ nq, Zte t`Lvl th, $x^3 - 3cx - 2a = 0$

(M) hw` $a = 2^{\frac{1}{3}} + 2^{-\frac{1}{3}}$ nq, Zte t`Lvl th, $2a^3 - 6a = 5$

(N) hw` $a^2 + 2 = 3^{\frac{1}{3}} + 3^{-\frac{2}{3}}$ Ges $a \geq 0$ nq, Zte t`Lvl th, $a^3 + 9a = 8$

(O) hw` $a^2 = b^3$ nq, Zte t`Lvl th, $\left(\frac{a}{b}\right)^{\frac{3}{2}} + \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{2}} + b^{-\frac{1}{3}}$

(P) hw` $b = 1 + 3^{\frac{2}{3}} + 3^{-\frac{1}{3}}$ nq, Zte t`Lvl th, $b^3 - 3b^2 - 6b - 4 = 0$

(Q) hw` $a + b + c = 0$ nq, Zte t`Lvl th,

$$\frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{-a} + 1} + \frac{1}{x^a + x^{-b} + 1} = 1.$$

8| (K) hw` $a^x = b, b^y = c$ Ges $c^z = 1$ nq, Zte $xyz = KZ$?

(L) hw` $x^a = y^b = z^c$ Ges $xyz = 1$ nq, Zte $ab + bc + ca = KZ$?

(M) hw` $9^x = (27)^y$ nq, Zv ntj $\frac{x}{y}$ Gi gvb KZ ?

9| mgvavb Ki :

(K) $3^{2x+2} + 27^{x+1} = 36$

(L) $5^x + 3^y = 8$

$$5^{x-1} + 3^{y-1} = 2$$

(M) $4^{3y-2} = 16^{x+y}$

$$3^{x+2y} = 9^{2x+1}$$

(N) $2^{2x+1} \cdot 2^{3y+1} = 8$

$$2^{x+2} \cdot 2^{y+2} = 16$$

9.6 j Mwi`g (Logarithm)

Logos Ges arithmas bvgK`yU MhK kã ntZ j Mwi g kãwU DrctE | Logos A_@Avtj vPbv Ges arithmas A_@msL`v A_@r, wetkl msL`v vbtq Avtj vPbv |

msÁv : hw` $a^x = b$ nq, thLvþb $a > 0$ Ges $a \neq 1$, Zte x tK ejv nq b Gi a wfvÉK j Mwvi`g, A_ŕ,
 $x = \log_a b$

AZGe, $a^x = b \Rightarrow x = \log_a b$

wecixZµtg, hw` $x = \log_a b \Rightarrow a^x = b$ nte|

GtŕŕtĪ b msL`wUþK wfvÉ a Gi mvþcŕŕ| x Gi cŕZj M (*anti-logarithm*) eŕj

Ges Avgiv wj wL $b = \text{anti log}_a x$

hw` $\log a = n$ nq, Zte a tK n Gi cŕZj M ejv nq A_ŕ, $\log a = n$ nŕj $a = \text{anti log } n$.

D`vni Y 1| $\text{anti log } 2 \cdot 82679 = 674.1042668$

$$\text{anti log}(9 \cdot 82672 - 10) = 0 \cdot 671$$

$$\text{Ges } \text{anti log}(6 \cdot 74429 - 10) = 0 \cdot 000555$$

`be` : ^eÁwþK K`vj Kŕj Ui e`envi Kŕi $\log a$ Gi Avmbægvb wþYŕ Kiv hvq (G mæúþK©gva`wgK
 exRMwYþZ we`wvi Z eYŕv t` l qv AvŕQ)|

msÁvþvŕi, Avgiv cvB,

$$\log_2 64 = 6 \text{ thŕnZi } 2^6 = 64 \text{ Ges } \log_8 64 = 2 \text{ thŕnZi } 8^2 = 64$$

mYzivs, GKB msL`vi j Mwvi`g wfvÉwfvÉwfvÉi tŕŕŕŕZ wfvÉwfvÉwfvÉi | avvZŕK wKŠ' GKŕKi mgvb bq
 Ggb thŕKvþbv msL`vŕK wfvÉ aŕi GKB msL`vi wfvÉwfvÉwfvÉ Mwvi`g wþYŕ Kiv hvq| thŕKvþbv avvZŕK
 msL`vŕK j Mwvi`ŕgi wfvÉ wmvŕte MY` Kiv nq| tKvþbv FYvZŕK msL`vi j Mwvi`g wþYŕ Kiv hvq bv|

Note: $a > 0$ | $a > 1$ Ges $b \neq 0$ nŕj b Gi Abb` a wfvÉK j Mwvi`gŕK $\log_a b$ ŕviv mwPZ Kiv nq|
 mYzivs (K) $\log_a b = x$ hw` l tKej hw` $a^x = b$ nq| (K) t_ŕK t` Lv hvq th,

$$(L) \log_a(a^x) = x \quad (M) a^{\log_a b} = b$$

D`vni Y 1| (1) $4^2 = 16 \Rightarrow \log_4 16 = 2$

$$(2) 5^{-2} = \frac{1}{5^2} = \frac{1}{25} \Rightarrow \log_5\left(\frac{1}{25}\right) = -2$$

$$(3) 10^3 = 1000 \Rightarrow \log_{10}(1000) = 3$$

$$(4) 7^{\log_7 9} \quad [\because a^{\log_a b} = b]$$

$$(5) 18 = \log_2 2^{18} \quad [\because \log_a a^x = x]$$

9.7 j Mwvi`ŕgi mŕvej x : (gva`wgK exRMwYþZ cŕvY t` l qv nŕqŕQ weavq GLvþb i aymŕ t` v t` Lvþbv nŕj v|)

$$1. \log_a a = 1 \text{ Ges } \log_a 1 = 0$$

$$2. \log_a (M \times N) = \log_a M + \log_a N$$

$$3. \log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

$$4. \log_a (M)^N = N \log_a M$$

$$5. \log_a M = \log_b M \times \log_a b$$

$$D^{\text{vni}} Y 2 \mid \log_2 5 + \log_2 7 + \log_2 3 = \log_2 (5 \cdot 7 \cdot 3) = \log_2 105$$

$$D^{\text{vni}} Y 3 \mid \log_3 20 - \log_3 5 = \log_3 \frac{20}{5} = \log_3 4$$

$$D^{\text{vni}} Y 4 \mid \log_5 64 = \log_5 2^6 = 6 \log_5 2$$

Note: (i) h \ddot{w} $x > 0, y > 0$ Ges $a \neq 1$ ZLb $x = y$

h \ddot{w} Ges tKej h \ddot{w} $\log_a x = \log_a y$

(ii) h \ddot{w} $a > 1$ Ges $x > 1$ nq Zte $\log_a x > 0$

(iii) h \ddot{w} $0 < a < 1$ Ges $0 < x < 1$ nq, Zte $\log_a x > 0$

(iv) h \ddot{w} $a > 1$ Ges $0 < x < 1$ nq, Zte $\log_a x < 0$

$D^{\text{vni}} Y 5 \mid x$ Gi gvb wby \ddot{w} Ki hLb

$$(i) \log_{\sqrt{8}} x = 3 \frac{1}{3}$$

$$(ii) h\ddot{w} \log_{10} [98 + \sqrt{x^2 - 12x + 36}] = 2$$

$$\text{mgvavb : } (i) \text{ th\ddot{t}nZl } \log_{\sqrt{8}} x = 3 \frac{1}{3} = \frac{10}{3}$$

$$\Rightarrow x = (\sqrt{8})^{\frac{10}{3}} = \left(\sqrt{2^3} \right)^{\frac{10}{3}}$$

$$\Rightarrow x = \left(2^{\frac{3}{2}} \right)^{\frac{10}{3}} = 2^{\frac{3 \cdot 10}{2 \cdot 3}} = 2^5 = 32$$

$$\therefore x = 32$$

$$(ii) \text{ th\ddot{t}nZl } \log_{10} [98 + \sqrt{x^2 - 12x + 36}] = 2$$

$$\Rightarrow 98 + \sqrt{x^2 - 12x + 36} = 10^2 = 100$$

$$\Rightarrow \sqrt{x^2 - 12x + 36} = 2$$

$$\Rightarrow x^2 - 12x + 36 = 4$$

$$\Rightarrow (x-4)(x-8) = 0$$

$$\therefore x = 4 \quad \text{ev} \quad x = 8.$$

D`vniY 6 | f`Lvl th, $a^{\log_k b - \log_k c} \times b^{\log_k c - \log_k a} \times c^{\log_k a - \log_k b} = 1.$

mgvavb : awi, $P = a^{\log_k b - \log_k c} \times b^{\log_k c - \log_k a} \times c^{\log_k a - \log_k b}$

Zvntj, $\log_k P = (\log_k b - \log_k c) \log_k a + (\log_k c - \log_k a) \log_k b + (\log_k a - \log_k b) \log_k c.$

$$\Rightarrow \log_k P = 0 \text{ [mij Kti]}$$

$$\Rightarrow P = k^0 = 1$$

D`vniY 7 | f`Lvl th, $x^{\log_a y} = y^{\log_a x}$

cgvY : awi $p = \log_a y, q = \log_a x$

mZivs $a^p = y, a^q = x$

$$\therefore (a^p)^q = y^q \Rightarrow y^q = a^{pq}$$

Ges $(a^q)^p = x^p \Rightarrow x^p = a^{pq}$

$$\therefore x^p = y^q \Rightarrow x \log_a y = y \log_a x$$

D`vniY 8 | f`Lvl th, $\log_a p \times \log_p q \times \log_q r \times \log_r b = \log_a b$

evgc¶ = $\log_a p \times \log_p q \times \log_q r \times \log_r b$

$$= (\log_p q \times \log_a p) \times (\log_r b \times \log_q r)$$

$$= \log_a q \times \log_q b = \log_a b = \text{Wwbc¶}$$

D`vniY 9 | f`Lvl th, $\frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)} = 1$

mgvavb : awi, $\log_a(abc) = x, \log_b(abc) = y, \log_c(abc) = z$

mZivs, $a^x = abc, b^y = abc, c^z = abc$

$$\therefore a = (abc)^{\frac{1}{x}}, b = (abc)^{\frac{1}{y}}, c = (abc)^{\frac{1}{z}}$$

GLb, $(abc)^1 = abc = (abc)^{\frac{1}{x}} (abc)^{\frac{1}{y}} (abc)^{\frac{1}{z}}$

$$= (abc)^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

$$A_{\text{fr}}, \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)} = 1$$

$$D`vni Y 10 | hw` P = \log_a(bc), q = \log_b(ca), r = \log_c(ab) nq$$

$$Zte t`Lvl th, \frac{1}{p+1} + \frac{1}{q+1} + \frac{1}{r+1} = 1.$$

$$mgvavb : 1 + P = 1 + \log_a(bc) = \log_a a + \log_a(bc) = \log_a(abc)$$

$$GKBFvte, 1 + q = \log_b(abc), 1 + r = \log_c(abc)$$

$$D`vni Y (9) G Avgiv cgvY Kti nq, \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)} = 1$$

$$\therefore \frac{1}{p+1} + \frac{1}{q+1} + \frac{1}{r+1} = 1.$$

$$D`vni Y 11 | hw` \frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y} nq, Zte t`Lvl th, a^x b^y c^z = 1$$

$$mgvavb : awi, \frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y} = k$$

$$Zvntj, \log a = k(y-z), \log b = k(z-x), \log c = k(x-y)$$

$$\therefore x \log a + y \log b + z \log c = k(xy - zx + yz - xy + zx - yz) = 0$$

$$ev, \log_a x + \log_a y + \log_a z = 0$$

$$ev, \log(a^x b^y c^z) = 0$$

$$ev, \log(a^x b^y c^z) = \log 1 \quad [\log 1 = 0]$$

$$\therefore a^x b^y c^z = 1$$

KvR :

$$1 | hw` \frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} Zvntj a^a \cdot b^b \cdot c^c Gi gvb wbyq Ki |$$

$$2 | hw` a, b, c cici wZbwU abvZK ALÊ msL'v nq, Zte cgvY Ki th, \log(1+ac) = 2 \log b$$

$$3 | hw` a^2 + b^2 = 7ab nq, Zte t`Lvl th, \log\left(\frac{a+b}{3}\right) = \frac{1}{2} \log(ab) = \frac{1}{2}(\log a + \log b)$$

$$4 | hw` \log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y) Zte t`Lvl th, \frac{x}{y} + \frac{y}{x} = 7$$

$$5 | hw` x = 1 + \log_a bc, y = 1 + \log_b ca \text{ Ges } z = 1 + \log_c ab nq, \\ Zte cgvY Ki th, xyz = xy + yz + zx$$

6| (K) hw` $2\log_8 A = p$, $2\log_2 2A = q$ Ges $q - p = 4$ nq, Zte A Gi gvb wbye Ki |
 (L) hw` $\log x^y = 6$ Ges $\log 14x^{8y} = 3$ nq, Zte x Gi gvb wbye Ki |

7| j M mvi wY (gva`wgK exRMmYZ cy`K`be`) e`envi Kti P Gi Avmbægvb wbye Ki thLvfb,
 (K) $P = (0.087721)^4$
 (L) $P = \sqrt[3]{30 \cdot 00618}$

9.7 mPKxq, j Mmwi`gxq l ciggvb dvskb

cŭg Aa`vtq Avgiv dvskb mæutK`we`wii Z tRtbnQ | GLvfb mPK, j Mmwi`g l ciggvb dvskb mæutK` Avtj vPbv Kiv ntjv :

wbtpi wZbnU tUwej ewY (x, y) µgtRvtoi gvb,tj v j "Kwi :

tUwej 1 :

x	-2	-1	0	1	2	3
y	-4	-2	0	2	4	6

tUwej 2 :

x	0	1	2	3	4	5
y	1	3	9	27	81	243

tUwej 3 :

x	0	1	2	3	4	5	6	7	8	9	10
y	1	2	4	8	16	32	64	128	256	512	1024

tUwej 1 G ewY x Gi wfbæwfbægvfb Rb` y Gi gvb,tj vi Ašt mgvb hv mij`tlvi dvskb ewY ntqtQ |

tUwej 2 G ewY (x, y) µgtRvtoi gvb,tj v wNvZ dvskb ewY ntqtQ |

tUwej 3 G ewY (x, y) µgtRvtoi gvb,tj v $y = 2^x$ Øviv eYv Kiv hvq | GLvfb 2 GKwU wbn`æ abvZK ev`e msl`v Ges x Gi wfbæwfbægvfb Rb` y Gi ewY gvb,tj v cvl qv hvq hv wbgj wLZfvte msÁwqZ Kiv hvq |

mPK dvskb $f(x) = a^x$ mKj ev`e msl`v x Gi Rb` msÁwqZ, thLvfb $a > 0$ Ges $a \neq 1$

thgb $y = 2^x, 10^x, e^x$ BZ`w` mPK dvskb |

KvR :
 wbtpi QtK ewY mPK dvskb tj L :

1	x	-2	-1	0	1	2	2	x	-1	0	1	2	3
	y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4		y	-3	0	3	6	9

3	x	1	2	3	4	5	4	x	-3	-2	-1	0	1
	y	4	16	64	256	1024		y	0	1	2	3	4

5	x	-2	-1	0	1	2	6	x	1	2	3	4	5
	y	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25		y	5	10	15	20	25

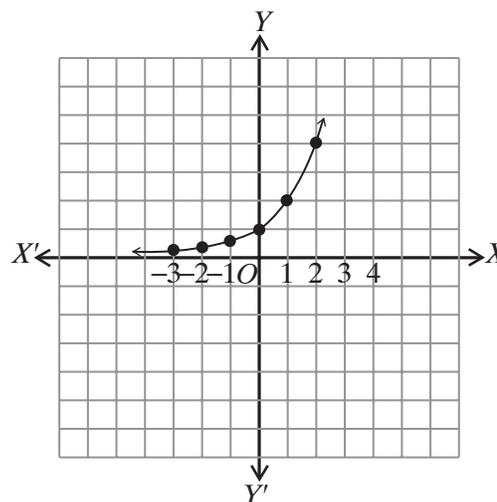
WbP i tKvbwU mPK dVskb WbP` R Kti :

- 7 | $y = -3^x$ 8 | $y = 3x$ 9 | $y = -2x - 3$ 10 | $y = 5 - x$
 11 | $y = x^2 + 1$ 12 | $y = 3x^2$

$f(x) = 2^x$ Gi tj LwPÎ A¼b :

cÔ È dVsktbi tj LwPÎ A¼tbi Rb" x Ges y Gi gvb, tj vi Zwj Kv cÔ Z Kw |

x	-3	-2	-1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



OK KvMfR gvb, tj v`vcb Ki tj Wbgdjc tj LwPÎ cvl qv hvq-

GLvfb tWtgb = $(-\infty, \infty)$

tiÄ = $(0, \infty)$

wPÎ t_tK j ¶ | Ki tj t`Lv hvq, hLb $x = 0$ ZLb $y = 2^0 = 1$ KvRB tiLwU $(0, 1)$ we` Mvgx

Avei, x Gi FYvZK thtKvfbv gvtbi Rb" y Gi gvb tKvfbv mgq 0 (i tb"i) LpB KvQvKwQ tctQvq wKŠ' ktb" (0) nq bv A_ŕ, $x \rightarrow -\infty, y \rightarrow 0+$

GKBFvte, x Gi thtKvfbv abvZK gvtbi Rb" y Gi gvb µgvštq Wwbw` tK (Dctii) ep× tctZ_vKte

A_ŕ, $-\infty$ w` tK aweZ nq | A_ŕ, $x \rightarrow -\infty, y \rightarrow \infty$

mZivs tWtgb (D) = $(-\infty, \infty)$

Ges tiÄ (R) = $(0, \infty)$

KvR : tj LwPÎ A¼b Ki thLvfb $-3 \leq x \leq 3$

1 | $y = 2^{-x}$ 2 | $y = 4^x$ 3 | $y = 2^{\frac{x}{2}}$ 4 | $y = \left(\frac{3}{2}\right)^x$

thtnZmPK dVskb GKwU GK-GK dVskb |

mZivs, Gi weciXZ dVskb AvtQ |

$f(x) = y = a^x$ mPKxq ifc

$f^{-1}(y) = x = a^y$ x Ges y cwi eZB Kti

A_ŕ, x ntjv y Gi a wfvĚK j Mwi`g |

msÁv : j Mwi`wgK dskb $f(x) = \log_a x$ ðvi v msÁwqZ thLvfb $a > 0$ Ges $a \neq 1$

$f(x) = \log_3 x, \ln x, \log_{10} x$ BZ`w` j Mwi`wgK dskb |

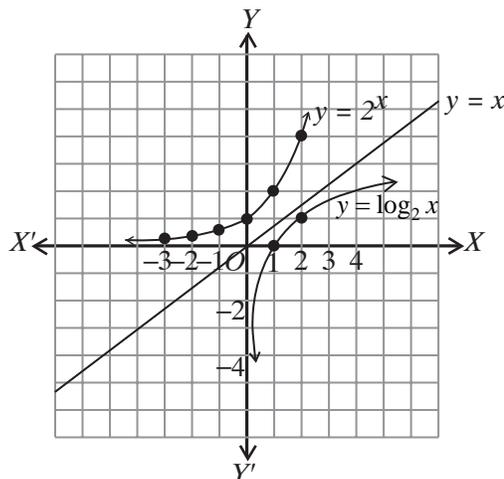
$y = \log_2 x$ tj LwPÎ A¼b :

thþnZly = $\log_2 x$ ntjv $y = 2^x$ Gi wecixZ |

$y = x$ tiLv mvtct¶¶ mPK dsktbi cÛZdj b j Mwi`wgK dskb wby¶¶ Kiv ntqtQ hvnv $y = x$ tiLvi mvtct¶¶ m`k |

GLb tWtgb $R = (0, \infty)$

tiÄ (D) = $(-\infty, \infty)$



KvR : tj LwPÎ A¼b Ki Ges Gt` i wecixZ dskb wby¶¶ |

1 $y = 3x + 2$	2 $y = x^2 + 3$	3 $y = x^3 - 1$	4 $y = \frac{4}{x}$
5 $y = 3x$	6 $y = \frac{2x+1}{x-1}$	7 $y = 2^{-x}$	8 $y = 4^x$

D`vni Y 1 | $f(x) = \frac{x}{|x|}$ dskbwUj tWtgb I tiÄ wby¶¶ Ki |

mgvavb : GLvfb $f(0) = \frac{0}{|0|} = \frac{0}{0}$ hv AmsÁwqZ |

∴ $x = 0$ we`þZ cÛ È dskbwU we`gvb bq |

kb` e`ZxZ x Gi Ab` mKj ev`e gvþbi Rb` cÛ È dskbwU we`gvb

∴ dsktbi tWtgb $D_f = R - \{0\}$

$$\text{Avei, } f(x) = \frac{x}{|x|} = \begin{cases} \frac{x}{x} & \text{hLb } x > 0 \\ \frac{x}{-x} & \text{hLb } x < 0 \end{cases}$$

$$= \begin{cases} 1 & \text{hLb } x > 0 \\ -1 & \text{hLb } x < 0 \end{cases}$$

∴ cÛ È dsktbi tiÄ $R_f = \{-1, 1\}$

D`vni Y 2 | $y = f(x) = \ln \frac{a+x}{a-x}, a > 0$ dskbwUj tWtgb I tiÄ wby¶¶ Ki |

mgvavb : thþnZlj Mwi`g i agvÎ avZ¶K ev`e msL`vi Rb` msÁwqZ nq |

∴ $\frac{a+x}{a-x} > 0$ hw` (i) $a+x > 0$ Ges $a-x > 0$ nq

A_ev (ii) $a+x < 0$ Ges $a-x < 0$ nq |



(i) $\Rightarrow x > -a$ Ges $a > x$

$\Rightarrow -a < x$ Ges $x < a$

\therefore tWtgb = $\{x : -a < x\} \cap \{x : x > a\}$

= $(-a, \infty) \cap (-\infty, a) = (-a, a)$

(ii) $\Rightarrow x < -a$ Ges $a < x$

$\Rightarrow x < -a$ Ges $x > a$

\therefore tWtgb $\{x : x < -a\} \cap \{x : x > a\} = \Phi$.

\therefore c0 E dvrktbi tWtgb

$\therefore D_f = (i) \cup (ii)$ tWtgb c0B tWtgb msthM $(-a, a) \cup \Phi = (-a, a)$

ti A : $y = f(x) = \ln \frac{a+x}{a-x} \Rightarrow e^y = \frac{a+x}{a-x}$

$\Rightarrow a+x = ae^y - xe^y$

$\Rightarrow (1+ae^y)x = a(xe^y - 1)$

$\Rightarrow x = \frac{a(e^y - 1)}{e^y + 1}$

y Gi mKj ev e gvtbi Rb x Gi gvb ev e nq

\therefore c0 E dvrktbi ti A $R_f = R$

KvR :

vbtpi dvrktbi tWtgb I ti A vbY Ki :

1 | $y = \ln \frac{2+x}{2-x}$

2 | $y = \ln \frac{3+x}{3-x}$

3 | $y = \ln \frac{4+x}{4-x}$

4 | $y = \ln \frac{5+x}{5-x}$

ci ggvb

gva wK exRMwYZ G m u u K Z we w i Z e Y v Kiv ntqtQ | GLvtb i ayci ggvbi msAv t` I qv ntj v :

thtKvfbv ev e msl v x Gi gvb kb, avZK ev FYvZK | wKš x Gi ci ggvb memgqB kb ev avZK |

x Gi ci ggvbtK | x | Oviv cKvk Kiv nq Ges ci ggvb vbgvj wLZfvte msAvwqZ Kiv nq |

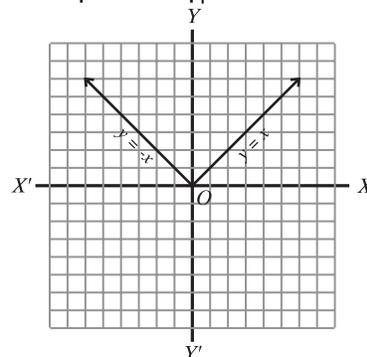
$$|x| = \begin{cases} x & \text{hLb } x > 0 \\ -x & \text{hLb } x < 0 \end{cases}$$

thgb : $|0| = 0, |3| = 3, |-3| = -(-3) = 3$

ci ggvb dvrkb (Absosute Value Fuction)

hw x $\in R$ nq, ZteN

$$y = f(x) = |x| = \begin{cases} x & \text{hLb } x \geq 0 \\ -x & \text{hLb } x < 0 \end{cases}$$



tK ciggvb dvskb ejv nq|

∴ tWtgb = R Ges tiÄ $R_f = [0, \infty]$

D`vniY 3 | $f(x) = e^{\frac{|x|}{2}}$ hLb $-1 < x < 0$ Gi tWtgb I tiÄ wBY@ Ki :

mgvavb : $f(x) = e^{-\frac{|x|}{2}}$, $-1 < x < 0$

x Gi gvb thtnZwv`@ -1 t_tK 0 Gi gta`

mZivs tWtgb $D_f = (-1, 0)$

Averi, $-1 < x < 0$ e`evatZ $f(x) \in \left(e^{-\frac{1}{2}}, 1 \right)$

mZivs tiÄ $f = \left(e^{-\frac{1}{2}}, 1 \right)$

9.8 dvsktbi tj LwPÎ

tKv`bv mgZtj tKv`bv dvskbtK R`wgvZKfvte Dc`vcb Kiv tMtj H dvskbtK tPbv hvq| dvsktbi R`wgvZKfvte GB Dc`vcbtK dvsktbi tj LwPÎ A¼b Kiv ntqtQ ejv nq| GLv`b mPK, j Mwi`wgK I ciggvb dvsktbi tj LwPÎ i A¼b c×wZ wbtq Avtj vPbv Kiv ntjv|

(I) $y = f(x) = a^x$ Gi tj LwPÎ A¼b Ki :

(i) hLb $a > 1$ Ges x thtKv`bv ev`e msL`v ZLb dvskb $f(x) = a^x$ me`v avvZK|

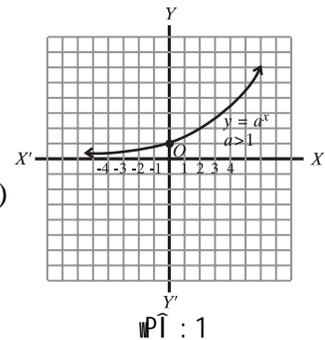
avc 1 : x Gi avvZK gv`bi Rb` x Gi gvb epxi mvt_ mvt_ $f(x)$ Gi gvb epx cvq

avc 2 : hLb $x = 0$ ZLb $y = a^0 = 1$,

mZivs, $(0, 1)$ tiLvi Dci GKwv we`y|

avc 3 : x Gi FYvZK gv`bi Rb` x Gi gvb μgvMZ epxi mvt_ mvt_ $f(x)$

Gi gvb μgvMZ nvm cvte| A_@, $x \rightarrow \infty$ ntj $y \rightarrow 0$ nte|



GLb wPÎ $y = a^x, a > 1$ dvsktbi wPÎ 1 G t`Lv`bv ntjv :

GLv`b $D_f = (-\infty, \infty)$ Ges $R_f = (0, \infty)$

(ii) hLb $0 < a < 1$, x Gi gvb ev`e ZLb $y = f(x) = a^x$ me`vB avvZK|

avc 1 : j @` Kwv, gj we`y Wwv`tK x Gi gvb μgvMZ epx tctZ_vKtj A_@, $x \rightarrow \infty$ ntj $y = 0$ nte|

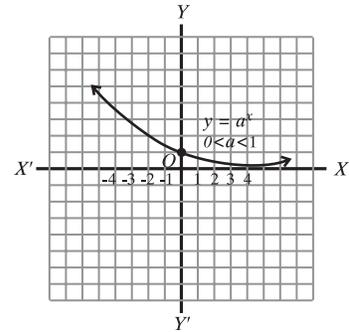
avc 2 : hLb $x = 0$ ZLb $y = a^0 = 1$

mZivs (0, 1) we`yti Lvi Dci cto|

avc 3 : hLb $a < 1$ Ges x Gi FYvZK gv`bi Rb" A_ŕ x Gi gvb gj we`j evgw` tK μgvMZ ew`xi mvt_ mvt_ y Gi gvb μgvMZ ew`x cŕte A_ŕ $y \rightarrow \infty$.

[[awi $a = \frac{1}{2} < 1, x = -2, -3, \dots, n$, ZLb $y = f(x) = a^x = \left(\frac{1}{2}\right)^{-2}$
 $= 2^2, y = 2^3, \dots, y^n = 2^n$. hw` $n \rightarrow \infty$ ZLb $y \rightarrow \infty$]

GLb $y = f(x) = a^x, 0 < a < 1$ Gi tj LwPÎ wPÎ 2 t` Lv`bv ntj v :
 GLv`tb $D_f = (-\infty, \infty)$ Ges $R_f = (0, \infty)$



KvR :

wb`tpi dvskb_ ,tj vi tj LwPÎ A½b Ki Ges tWv`gb l ti Ä wbYŕ Ki :

- (i) $f(x) = 2^x$ (ii) $f(x) = \left(\frac{1}{2}\right)^x$ (iii) $f(x) = e^x, 2 < e < 3$.
- (iv) $f(x) = e^{-x}, 2 < e < 3$. (v) $f(x) = 3^x$

2. $f(x) = a^x$ Gi tj LwPÎ A½b Ki

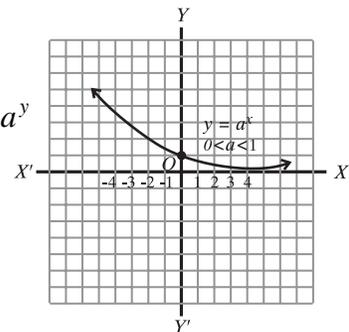
(i) awi , $y = f(x) = \log_a x$ hLb $0 < a < 1$ dvskbWtK tj Lv hvq $x = a^y$

avc 1 : hLb y Gi abvZK gvb μgvMZ ew`x tctZ_v`tk A_ŕ, $y \rightarrow \infty$ nq ZLb x Gi gvb k`b`i w` tK aweZ nq A_ŕ, $x \rightarrow 0$

avc 2 : th`tnZl $a^0 = 1$ Kv`tRB $y = \log_a 1 = 0$,

mZivs ti LwWJ (1, 0) we` Mvgx|

avc 3 : y Gi FYvZK gvb A_ŕ, y Gi gvb gj we`j wb`tpi w` tK μgvMZ ew`x tctZ_v`tk A_ŕ, $y \rightarrow -\infty$ nq Zvntj x Gi gvb μgvMZ ew`x tctZ_v`tk A_ŕ, $x \rightarrow \infty$



GLb wPÎ 3 G $y = \log_a x, 0 < a < 1$ t` Lv`bv ntj v :

(2) $y = \log_a x, a > 1$.

GLv`tb $D_f = (0, \infty)$ Ges $R_f = (-\infty, \infty)$

hLb $y = \log_a x, a > 1$, ZLb

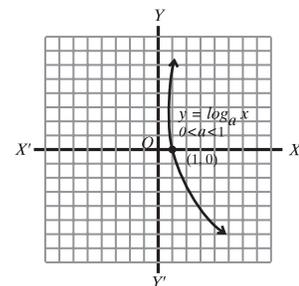
avc 1 : hLb $a > 1$, y Gi mKj gv`bi Rb" x Gi gvb abvZK Ges y

Gi gv`bi μgvMZ ew`xi mvt_ mvt_ x Gi gvb ew`x cŕB nq| A_ŕ, $y \rightarrow \infty$ ntj $x \rightarrow \infty$

avc 2 : th`tnZl $a^0 = 1$ Kv`tRB $y = \log_a 1 = 0$

mZivs, ti LwWJ (1, 0) we` Mvgx|

avc 3 : y Gi FYvZK gv`bi Rb" y Gi gvb μgvMZ nwm tctj A_ŕ, $y = -\infty$ ntj x Gi gvb μgvMZ k`b`i w` tK aweZ nq A_ŕ, $x \rightarrow 0$



GLb $f(x) = \log a^x, a > 1$ Gi tj LwPÎ wPÎ 4 G t` Lvfbv ntjv :

GLvfb $D_f = (-\infty, \infty)$ Ges $R_f = (0, \infty)$

D`vni Y 3 | $f(x) = \log_{10} x$ Gi tj LwPÎ A¼b Ki |

mgvavb : awi $y = f(x) = \log_{10} x$

thþnZl $10^0 = 1$ KvfbRB $y = \log_{10} 1 = 0$ mÿZivs, ti LwU $(1, 0)$ we` Mvgx |

hLb $x \rightarrow 0$ ZLb $y \rightarrow -\infty$ |

$\therefore y = \log_{10} x$ ti LwU ewxcÛB | wþP ti LwUi tj LwPÎ A¼b Kiv ntjv |

GLvfb $D_f = (0, \infty)$ Ges $R_f = (-\infty, \infty)$

D`vni Y 4 | $f(x) = \ln x$ Gi tj LwPÎ A¼b Ki |

mgvavb : awi , $y = f(x) = \ln x$

thþnZl $e^0 = 1$ KvfbRB $y = \ln 1 = 0$. mÿZivs, ti LwU $(1, 0)$ we` Mvgx |

hLb $x \rightarrow 0$ ZLb $y = \ln 0 = -\infty$

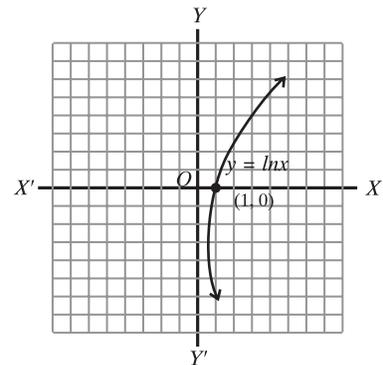
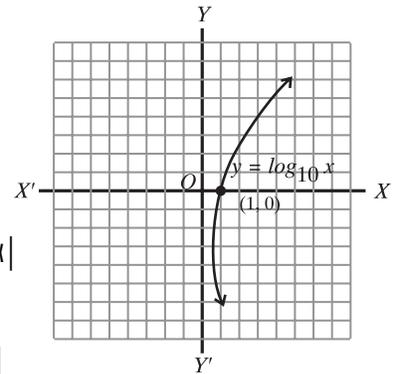
$\therefore y = \ln x$ ti LwU ewxcÛB |

wþP ti LwUi tj LwPÎ A¼b Kiv ntjv :

GLvfb GLvfb $D_f = (0, \infty)$

$R_f = (-\infty, \infty)$

$\therefore f(x) = \ln x$ Gi tj LwPÎ wPÎ 6 G t` Lvfbv ntjv :



KvR :

1 | tUweþj DþjwLZ x | y Gi gvb wþtq $y = \log_{10} x$ Gi tj LwPÎ A¼b Ki |

x	.5	1	2	3	4	5	10	12
y	-.3	0	0.3	0.5	.0	.7	1	1.0

2 | $y = \log_e x$ Gi tj LwPÎ A¼b Rb` 1Gi b`vq x | y Gi gvb wþtq tUweþ `Zwi Ki Ges tj LwPÎ AwK |

Abþxj bx 9.2

1 | $\left\{ \left(x^{\frac{1}{a}} \right)^{\frac{a^2-b^2}{a+b}} \right\}^{\frac{a}{a-b}}$ Gi mij gvb tKvbwU ?

(K) 0 (L) 1 (M) a (N) x

2 | hw` $a, b, p > 0$ Ges $a \neq 1, b \neq 1$ nq, Zþe

i. $\log_a P = \log_b P \times \log_a b$

ii. $\log_a \sqrt{a} \times \log_b \sqrt{b} \times \log_c \sqrt{c}$ Gi gvb 2

iii. $x^{\log_a y} = y^{\log_a x}$

Dcti i Zt`i Avtj vtK vbtPi tKvbiU mVVK ?

(K) i l ii (L) ii l iii (M) i l iii (N) i, ii l iii

3-5 bs c0k0 D0i `vl hLb $x, y, z \neq 0$ Ges $a^x = b^y = c^z$

3| tKvbiU mVVK ?

(K) $a = b^{\frac{y}{z}}$ (L) $c^{\frac{z}{y}}$ (M) $a = c^{\frac{z}{x}}$ (N) $a \neq \frac{b^2}{c}$

4| vbtPi tKvbiU ac Gi mgvb|

(K) $b^{\frac{y}{x}} \cdot b^{\frac{y}{z}}$ (L) $b^{\frac{y}{x}} \cdot b^{\frac{z}{y}}$ (M) $b^{\frac{y+z}{x}}$ (N) $b^{\frac{x+y}{z}}$

5| $b^2 = ac$ ntj vbtPi tKvbiU mVVK ?

(K) $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$ (L) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$ (M) $\frac{1}{y} + \frac{1}{z} = \frac{2}{x}$ (N) $\frac{1}{x} + \frac{1}{y} = \frac{z}{2}$

6| t`Lvl th,

(K) $\log_k \left(\frac{a^n}{b^n} \right) + \log_k \left(\frac{b^n}{c^n} \right) + \log_k \left(\frac{c^n}{a^n} \right) = 0$

(L) $\log_k(ab) \log_k \left(\frac{a}{b} \right) + \log_k(bc) \log_k \left(\frac{b}{c} \right) + \log_k(ca) \log_k \left(\frac{c}{a} \right) = 0$

(M) $\log_{\sqrt{a}} b \times \log_{\sqrt{b}} c \times \log_{\sqrt{c}} a = 8$

(N) $\log_a \log_a \log_a \left(a^{a^a b} \right) = b$

7| (K) hw $\frac{\log_k a}{b-c} = \frac{\log_k b}{c-a} = \frac{\log_k c}{a-b}$ nq, Zte t`Lvl th, $a^a b^b c^c = 1$

(L) hw $\frac{\log_k a}{y-z} = \frac{\log_k b}{z-x} = \frac{\log_k c}{x-y}$ nq, Zte t`Lvl th,

(1) $a^{y+z} b^{z+x} c^{x+y} = 1$

(2) $a^{y^2 + yz + z^2} \cdot b^{z^2 + zx + x^2} \cdot c^{x^2 + xy + y^2} = 1.$

(M) hw $\frac{\log_k(1+x)}{\log_k x} = 2$ nq, Zte t`Lvl th, $x = \frac{1+\sqrt{5}}{2}$

(N) t`Lvl th, $\log_k = \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}} = 2 \log_k \left(x - \sqrt{x^2 - 1} \right)$

(O) hw $a^{3-x} b^{5x} = a^{5+x} b^{3x}$ nq, Zte t`Lvl th, $x \log_k \left(\frac{b}{a} \right) = \log_k a$

(P) hw $xy^{a-1} = P, xy^{b-1} = q, xy^{c-1} = r$ nq,

Zte t` Lvl th, $(b-c)\log_k p + (c-a)\log_k q + (a-b)\log_k r = 0$

(Q) hñ $\frac{ab\log_k(ab)}{a+b} = \frac{bc\log_k(bc)}{b+c} = \frac{ca\log_k(ca)}{c+a}$ nq, Zte t` Lvl th, $a^a = b^b = c^c$

(R) hñ $\frac{x(y+z-x)}{\log_k x} = \frac{y(z+x-y)}{\log_k y} = \frac{z(x+y-z)}{\log_k z}$ nq,

Zte t` Lvl th, $x^y y^z = y^z z^y = z^x x^z$

8| Őj M mvi wY (gva`wgK exRMmYZ `be`) e`envi Kti P Gi Avmbægvb wbyq Ki thLvfb,

(K) $P = 2\pi \sqrt{\frac{l}{g}}$ thLvfb $\pi \approx 3.1416, g = 981$ Ges $l = 25 \cdot 5$

(L) $P = 10000 \times e^{0.05t}$ thLvfb $e = 1.718$ Ges $t = 13.86$

9| $\ln P \approx 2 \cdot 3026 \times \log P$ mñ e`envi Kti $\ln P$ Gi Avmbægvb wbyq Ki, hLb-

(K) $P = 10000$ (L) $P = .001e^2$ (M) $P = 10^{100} \times \sqrt{e}$

10| tj LmPÎ A¼b Ki :

(K) $y = 3^x$ (L) $y = -3^x$ (M) $y = 3^{x+1}$ (N) $y = -3^{x+1}$ (O) $y = 3^{-x+1}$ (P) $y = 3^{x-1}$

11| wbtPi dvisktbi weci xZ dviskb wj L Ges tj LmPÎ A¼b Kti tWtgb I tiÄ wbyq Ki |

(K) $y = 1 - 2^{-x}$

(L) $y = \log_{10} x$

(M) $y = x^2, x > 0$

12| $f(x) = \ln(x-2)$ dviskbwji D_f I R_f wbyq Ki :

13| $f(x) = \ln \frac{1-x}{1+x}$ dviskbwji tWtgb Ges tiÄ wbyq Ki |

14| tWtgb, tiÄ Dti, Lmn tj LmPÎ A¼b Ki |

(K) $f(x) = |x|$ hLb $-5 \leq x \leq 5$

(L) $f(x) = x + |x|$ hLb $-2 \leq x \leq 2$

(M) $f(x) = \begin{cases} |x| & \text{hLb } x \neq 0 \\ x & \text{hLb } x = 0 \end{cases}$

(N) $f(x) = \frac{x}{|x|}$

(O) $f(x) = \log \frac{5+x}{5-x}, -5 < x < 5$

15| t` I qv AvtQ :

$$2^{2x} \cdot 2^{y-1} = 64 \dots \dots \dots (i)$$

$$\text{Ges } 6x \cdot \frac{6^{y-2}}{3} = 72 \dots \dots \dots (ii)$$

K. (i) I (ii) tK x I y Pj Kwekó mi j mgxKi tY cwi YZ Ki |

L. mgxKi Y0q mgvavb Kti i xZv hvPvB Ki |

M. x I y gvb hw` tKv tbn PZ fRi mibnZ evú i `N`nq thLv tbn evú0tqi Aš fP tKvY 90° Zte PZ fRi AvqZ bv eM©D tL Ki Ges Gi t t d j I K t Y P `N`nbY0 Ki |

16| t` I qv AvtQ,

$$\frac{\log(1+x)}{\log x} = 2$$

K. c0 È mgxKi Yw tK x Pj Kmsewj Z GKwU w0NvZ mgxKi tY cwi YZ Ki |

L. c0B mgxKi Yw tK mgvavb Ki Ges t` Lv l th, x Gi tKej GKwU exR mgxKi Yw tK wmx Kti |

M. c0vY Ki th, gj 0tqi c0ZwUi eM©Zvi `xq gvb A t c tlv 1(GK) tewk Ges Zv t` i tj Lv P t ci `ui mgvŠ t v j |

17| t` I qv AvtQ,

$$y = 2^x$$

K. c0 È dvskbwUi tWv tgb Ges ti Ä wby0 Ki |

L. dvskbwUi tj Lv P t A ¼ b Ki Ges Gi `ewkó` wj wj L |

M. dvskbwUi weci xZ dvskb wby0 Kti GwU GK-GK wKbv Zv wbaP Y Ki Ges weci xZ dvskbwUi tj Lv P t Avk |

`kg Aa`vq w0c`x we`wZ

exRMwYZxq i vki (GKc`x, w0c`x, euc`x) thvM, wctqvM, ,Y, fvM, eM©Ges Nb msµvš-Avtj vPbv ce©ZP tkWtZ Kiv ntqtQ| w0c`x i vki ev euc`x i vki NvZ ev kw³ wZb Gi tewk ntj tmB mg`-tqtT gvb wbyq ht_ó ktmva` I mgqmtc¶| ntq cto| GB Aa`vtq w0c`x i vki NvZ ev kw³ wZb Gi tewk ntj Kx cµµqvq KvRwJ m`ubæKiv hvq, Zv Dc`vcb Kiv nte| mvari Yfvte NvZ ev kw³ n Gi Rb` mF c0Zcv`b Kiv nte| hvi gva`tg thKvfbv AFbvZK cYmsLk NvtZi w0c`x i vki gvb wbyq Kiv mæe nte| Zte GB ch¶q n Gi gvb GKwJ wv` 0 mxgv n ≤ 8 AwZµg Ki te bv| wclquJ hvZ wk¶v_¶v mntR eStZ I e`envi Kitz cvti tm Rb` GKwJ wF fR e`envi Kiv nte thwJ 0c`vmtKtj i wF fR0 (Pascals triangle) etj cwipZ| w0c`x i vki NvZ avvZK ev FyvZK cYmsL`v ev fMsk ntZ cvti | wKš` eZgvb Avtj vPbvq Avgiv iagvT avvZK cYmsL`vi NvtZi gta` mxgve× _vKe| cieZP tkWtZ mg`-Avtj vPbv Ašfj` _vKte|

Aa`q tktl wk¶v_¶v –

- w0c`x we`wZ eY0v Kitz cvite|
- c`vmtKj wF fR eY0v Kitz cvite|
- mvari Y NvtZi Rb` w0c`x we`wZ eY0v Kitz cvite|
- n! "C, Gi gvb wbyq Kitz cvite|
- w0c`x we`wZ e`envi Kti MvYwZK mgm`v mgvavb Kitz cvite|

10.1 w0c`x (1 + y)ⁿ Gi we`wZ

`BwJ ct` i mgstq MwZ exRMwYZxq i vktK w0c`x (Binomials) i vki ej v nq|

a + b, x - y, 1 + x, 1 - x², a² - b² BZ`w` w0c`x i vki | Avgiv c0tgB GKwJ w0c`x i vki (1 + y) wPšv Kw | GLb (1 + y) tK hw` (1 + y) 0viv µgvMZ ,Y Kitz _wk Zvntj cve

(1 + y)², (1 + y)³, (1 + y)⁴, (1 + y)⁵..... BZ`w` |

Avgiv Rwb,

$$(1 + y)^2 = (1 + y)(1 + y) = 1 + 2y + y^2$$

$$(1 + y)^3 = (1 + y)(1 + y)^2 = (1 + y)(1 + 2y + y^2) = 1 + 3y + 3y^2 + y^3$$

Abjfcfvte `xN©, Yb cKwqvi gva`tg (1 + y)⁴, (1 + y)⁵..... BZ`w` ,Ydj wbyq mæe| wKš` (1 + y) Gi NvZ ev kw³ hZ evotZ _vKte ,Ydj ZZ `xN© mgqmtc¶| nte| ZvB Ggb GKwJ mnR cxwZ tei Kitz nte, hvZ (1 + y) Gi thKvfbv NvZ (awi n) ev kw³ i Rb` (1 + y)ⁿ Gi we`wZ mntRB wbyq Kiv mæe nte| n Gi gvb 0, 1, 2, 3, 4, A_¶ AFYvZK gvfbv Rb` GB Astk Avtj vPbv mxgve× _vKte| GLb cµµqvW Avgiv fij fvte j ¶ Kw |

n Gi gvb	$(1+y)^n =$	c"vmtKj wlfR	c`msLv
$n=0$	$(1+y)^0 =$	1	1
$n=1$	$(1+y)^1 =$	$1+y$	2
$n=2$	$(1+y)^2 =$	$1+2y+y^2$	3
$n=3$	$(1+y)^3 =$	$1+3y+3y^2+y^3$	4
$n=4$	$(1+y)^4 =$	$1+4y+6y^2+4y^3+y^4$	5
$n=5$	$(1+y)^5 =$	$1+5y+10y^2+10y^3+5y^4+y^5$	6

Dcti i we`wZmgntK wfvE Kti Avgiv $(1+y)^n$ Gi we`wZ mautKbtgw³ vmxvš-AvmtZ cwi |

vmxvš-:

(a) $(1+y)^n$ Gi we`wZtZ $(n+1)$ msL`K c` AvtQ | A`fr NvZ ev kw³ i tPtq c`msLv GKwJ tenk |

(b) y Gi NvZ 0 (kb) t`tk i i" ntq 1, 2, 3,....., n chS-ewx tctqtQ | A`fr y Gi NvZ μ gvštq ewx tctq n chS-tctqtQ |

wc`x mnM : Dcti i c0Z`K wc`x we`wZtZ y Gi we`wZtZi mnM (Coefficient) tK wc`x mnM ej v nq | 1 tK y Gi mnM we`wZtZ Ki tZ nte | Dcti i we`wZi mnM, tj vtK mvrvtj Avgiv cvB,

$n=0$						1
$n=1$					1	1
$n=2$				1	2	1
$n=3$			1	3	3	1
$n=4$		1	4	6	4	1
$n=5$	1	5	10	10	5	1

j ¶ Ki tj t`Le mnM, tj v GKwJ wlfRi AvKvi aviY Kti tQ | wc`x we`wZi mnM wbyqi GKwJ tKškj 'Blaise Pascal' c0g e`envi Ktib | ZvB GB wlfRtK c"vmtKtj i wlfR (Pascal's Triangle) ej v nq | c"vmtKtj i wlfRi mrvth" Avgiv mntRB wc`x i wki we`wZtZ mnMmgn wbyq Ki tZ cwi |

c"vmtKtj i wlfRi e`envi

c"vmtKtj i wlfR t`tk Avgiv t`Ltz cvB Gi evg | Wvb w`tk AvtQ '1' | wlfRi gvSLvtbi msLv, tj vi c0Z`KwJ wK Dcti i `Bw msL`vi thvMdj |

wbtgE D`vni YuU j ¶ Ki tj we`wZtZ Lye mntRB eSv hute |

$n=5$ Gi Rb" wc`x mnM ntjv : 1 5 10 10 5 1

$n = 6$ Gi Rb" mnM₅ tj v nte wbgie/c :

$$\begin{matrix} n=5 & & 1 & 5 & 10 & 10 & 5 & 1 \\ n=6 & 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{matrix}$$

$\therefore (1 + y)^5 = 1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$

$\therefore (1 + y)^6 = 1 + 6y + 15y^2 + 20y^3 + 15y^4 + 6y^5 + y^6$

Ges $(1 + y)^7 = 1 + 7y + 21y^2 + 35y^3 + 35y^4 + 21y^5 + 7y^6 + y^7$

KvR : wbtge³ we⁻wZmgⁿ wby^q Ki : (Dc^ti i we⁻wZmgⁿi mⁿv^h" bvl) :

$(1 + y)^8 =$

$(1 + y)^9 =$

$(1 + y)^{10} =$

Avgiv hw` fvj fvti tLqvj Kwi Zvntj eStZ cvie GB c^xwZi GKwU w^etkl `e^fZv AvtQ | thgb Avgiv hw` $(1 + y)^5$ Gi we⁻wZ RvbtZ Piv Zvntj $(1 + y)^4$ Gi we⁻wZ Rvbn `i Kvi | Avevi th^tKv^tbn w⁰c`x mnM Rvbi Rb" Zvi wK Dc^ti ce⁰Z^p `BwU mnM Rvbn c⁰qvRb | GB Ae⁻v t⁻tK DEitYi Rb" Avgiv mi^vmwⁱ w⁰c`x mnM wby^qi tKSkj tei Kitz Piv | c^vmtKtj i w¹fR t⁻tK Avgiv t⁻LtZ cvB w⁰c`x we⁻wZi mnM₅ tj vi NvZ n Ges c⁻wU tKv^tbn Ae⁻v^tb AvtQ Zvi Dci w^bf^pk^xj | Avgiv GKwU bZb m^vt^kw^zk w^py $\binom{n}{r}$ w^etePbv Kwi thLvtb 'n' NvZ Ges 'r' c⁻i Ae⁻v^tbi m^vt⁻m^uu⁰KZ | D⁻vniY⁻t^fc hw` $n = 4$ nq, Zte c⁻msL^v nte 5 wU | awi c⁻ cu^pwU h⁻v^ut^g T_1, T_2, T_3, T_4, T_5

Avgiv c⁻M^yj wbtge³ Dcv^tq tj wL |

hLb $n = 4$ c⁻msL^v 5 wU : T_1, T_2, T_3, T_4, T_5 |

Zv^t i mnM₅ wj n^tj v : 1 4 6 4 1

bZb w^py e⁻envi K^ti mnM : $\binom{4}{0} \binom{4}{1} \binom{4}{2} \binom{4}{3} \binom{4}{4}$

GLvtb, $\binom{4}{0} = 1, \binom{4}{1} = \frac{4}{1} = 4$

$\binom{4}{2} = \frac{4 \times 3}{1 \times 2} = 6, \binom{4}{3} = \frac{4 \times 3 \times 2}{1 \times 2 \times 3} = 4$ Ges $\binom{4}{4} = \frac{4 \times 3 \times 2 \times 1}{1 \times 2 \times 3 \times 4} = 1$

[c^vmtKtj i w¹fR t⁻tK m^tRB eStZ cvie]

Dw^j-wLZ bZb w^pyⁱ mⁿv^h" c^vmtKtj i w¹fR ($n = 1, 2, 3, \dots$) Gi Rb" nte :

$$\begin{aligned}
 n=1 & \quad \binom{1}{0} \quad \binom{1}{1} \\
 n=2 & \quad \binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2} \\
 n=3 & \quad \binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3} \\
 n=4 & \quad \binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4} \\
 n=5 & \quad \binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}
 \end{aligned}$$

mŷvi vs Dcti i wĭ fŕ t_ĭK Avgiv Lp mntRB ej tZ cwii $(1+y)^4$ Gi we-wZi ZZxq (T_{2+1}) ct` i mnM $\binom{4}{2}$ Ges $(1+y)^5$ Gi we-wZi ZZxq (T_{2+1}) | PZL[⊙] (T_{3+1}) ct` i mnM h_vµtg $\binom{5}{2}$ Gis $\binom{5}{3}$ |

mvavi Yfvte $(1+y)^n$ Gi we-wZi r Zg ct` i mnM $T_{r+1} = \binom{n}{r}$

GLb, $\binom{n}{r}$ Gi gvb KZ Zv Rvbvi Rb" Avevtiv c`vmtKtj i wĭ fŕ j ħ Kwi | c`vmtKtj i wĭ fŕ Ri `βU
tnj vĭbv cvk[⊙]ĭK Avgiv t` LtZ cvB,

$$\begin{aligned}
 \binom{1}{0} = 1, \quad \binom{2}{0} = 1, \quad \binom{3}{0} = 1, \dots, \quad \binom{n}{0} = 1 \\
 \binom{1}{1} = 1, \quad \binom{2}{2} = 1, \quad \binom{3}{3} = 1, \dots, \quad \binom{n}{n} = 1
 \end{aligned}$$

Avgiv $n=5$ aĭi cvB

$$\begin{aligned}
 \binom{5}{0} = 1, \quad \binom{5}{1} = 5, \quad \binom{5}{2} = \frac{5 \times 4}{1 \times 2} = 10 \\
 \binom{5}{3} = \frac{5 \times 4 \times 3}{1 \times 2 \times 3} = 10, \quad \binom{5}{4} = \frac{5 \times 4 \times 3 \times 2}{1 \times 2 \times 3 \times 4} = 5
 \end{aligned}$$

Ges $\binom{5}{5} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1 \times 2 \times 3 \times 4 \times 5} = 1$

mŷvi vs $\binom{5}{3}$ Gi gvtbi tĭĭĭ ej v hvq

$$\binom{5}{3} = \frac{5 \times (5-1) \times (5-2)}{1 \times 2 \times 3}$$

Ges $\binom{6}{4} = \frac{6 \times (6-1) \times (6-2) \times (6-3)}{1 \times 2 \times 3 \times 4}$

mvavi Y fvte Avgiv vj LtZ cwii ,

$$\binom{n}{0} = 1, \binom{n}{n} = 1$$

$$\binom{n}{r} = \frac{n(n-1)(n-2)\dots(n-r+1)}{1 \times 2 \times 3 \times 4 \dots \times r}$$

উপর্যুক্ত চিহ্ন ব্যবহার করে পাই,

$$(1+y)^4 = \binom{4}{0}y^0 + \binom{4}{1}y^1 + \binom{4}{2}y^2 + \binom{4}{3}y^3 + \binom{4}{4}y^4$$

$$= 1 + 4y + 6y^2 + 4y^3 + y^4$$

$$(1+y)^5 = \binom{5}{0}y^0 + \binom{5}{1}y^1 + \binom{5}{2}y^2 + \binom{5}{3}y^3 + \binom{5}{4}y^4 + \binom{5}{5}y^5$$

$$= 1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$$

এবং $(1+y)^n$ এর বিস্তৃতি

$$(1+y)^n = \binom{n}{0}y^0 + \binom{n}{1}y^1 + \binom{n}{2}y^2 + \binom{n}{3}y^3 + \dots + \binom{n}{n}y^n$$

$$= 1 \cdot y^0 + ny + \frac{n(n-1)}{1 \cdot 2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}y^3 + \dots + 1 \cdot y^n$$

অর্থাৎ দ্বিপদী $(1+y)^n$ এর বিস্তৃতি

$$\therefore (1+y)^n = 1 + ny + \frac{n(n-1)}{1 \cdot 2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}y^3 + \dots + y^n.$$

উদাহরণ ১। $(1+3x)^5$ কে বিস্তৃত কর।

সমাধান : প্যাসকেলের ত্রিভুজের সাহায্যে—

$$\begin{array}{ccccccc} & & & & & & 1 \\ & & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & & 1 & 3 & 3 & 1 \\ & & 1 & 4 & 6 & 4 & 1 \\ 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

$$\therefore (1+3x)^5 = 1 + 5 \cdot 3x + 10 \cdot (3x)^2 + 10(3x)^3 + 5(3x)^4 + 1(3x)^5$$

$$= 1 + 15x + 90x^2 + 270x^3 + 405x^4 + 243x^5$$

দ্বিপদী উপপাদ্যের সাহায্যে -

$$(1+3x)^5 = \binom{5}{0}(3x)^0 + \binom{5}{1}3x + \binom{5}{2}(3x)^2 + \binom{5}{3}(3x)^3 + \binom{5}{4}(3x)^4 + \binom{5}{5}(3x)^5$$

$$\text{বা, } (1+3x)^5 = 1 + \frac{5}{1}(3x) + \frac{5 \cdot 4}{1 \cdot 2} \cdot (3x)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} (3x)^3 + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} (3x)^4 + 1 \cdot (3x)^5$$

$$= 1 + 15x + 90x^2 + 270x^3 + 405x^4 + 243x^5.$$

উদাহরণ ২। $(1-3x)^5$ কে বিস্তৃত কর

সমাধান : প্যাসকেলের ত্রিভুজের সাহায্যে

$$(1-3x)^5 = 1 + 5(-3x) + 10(-3x)^2 + 10(-3x)^3 + 5(-3x)^4 + 1(-3x)^5$$

$$= 1 - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5.$$

					1				
					1	1			
				1	2	1			
			1	3	3	1			
		1	4	6	4	1			
1	5	10	10	5	1				

দ্বিপদী বিস্তৃতির সাহায্যে

$$(1-3x)^5 = \binom{5}{0}(-3x)^0 + \binom{5}{1}(-3x)^1 + \binom{5}{2}(-3x)^2 + \binom{5}{3}(-3x)^3 + \binom{5}{4}(-3x)^4 + \binom{5}{5}(-3x)^5$$

$$= 1 \cdot 1 + \frac{5}{1} \cdot (-3x) + \frac{5 \cdot 4}{1 \cdot 2} (-3x)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} (-3x)^3 + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} (-3x)^4 + 1 \cdot (-3x)^5$$

$$= 1 - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5.$$

মন্তব্য : $(1+3x)^5$ এবং $(1-3x)^5$ এর বিস্তৃতি থেকে দেখা যাচ্ছে যে, উভয় বিস্তৃতি একই। শুধুমাত্র সহগের চিহ্ন পরিবর্তন করে, অর্থাৎ +, -, +, এর মাধ্যমে একটি থেকে অন্যটি পাওয়া সম্ভব।

কাজ :

$(1+2x^2)^7$ এবং $(1-2x^2)^7$ কে বিস্তৃত কর।

উদাহরণ ৩। $\left(1+\frac{2}{x}\right)^8$ কে পঞ্চম পদ পর্যন্ত বিস্তৃত কর।

সমাধান : প্যাসকেলের ত্রিভুজের সাহায্যে নিজে কর।

দ্বিপদী বিস্তৃতি ব্যবহার করে-

$$\left(1+\frac{2}{x}\right)^8 = \binom{8}{0}\left(\frac{2}{x}\right)^0 + \binom{8}{1}\left(\frac{2}{x}\right)^1 + \binom{8}{2}\left(\frac{2}{x}\right)^2 + \binom{8}{3}\left(\frac{2}{x}\right)^3 + \binom{8}{4}\left(\frac{2}{x}\right)^4 + \dots \dots \dots \text{ [৫ম পদ পর্যন্ত]}$$

$$= 1 \cdot 1 + \frac{8}{1} \cdot \frac{2}{x} + \frac{8 \cdot 7}{1 \cdot 2} \cdot \frac{4}{x^2} + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} \cdot \frac{8}{x^3} + \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{16}{x^4} + \dots \dots \dots$$

$$= 1 + \frac{16}{x} + \frac{112}{x^2} + \frac{448}{x^3} + \frac{1120}{x^4} + \dots \dots \dots$$

$$\therefore \left(1+\frac{2}{x}\right)^8 = 1 + \frac{16}{x} + \frac{112}{x^2} + \frac{448}{x^3} + \frac{1120}{x^4} + \dots \dots \dots \text{ [৫ম পদ পর্যন্ত]}$$

উদাহরণ ৪। $\left(1 - \frac{x^2}{4}\right)^8$ এর বিস্তৃতির x^3 ও x^6 এর সহগ নির্ণয় কর।

সমাধান : দ্বিপদী বিস্তৃতির সাহায্যে পাই,

$$\begin{aligned} \left(1 - \frac{x^2}{4}\right)^8 &= \binom{8}{0} \left(-\frac{x^2}{4}\right)^0 + \binom{8}{1} \left(-\frac{x^2}{4}\right)^1 + \binom{8}{2} \left(-\frac{x^2}{4}\right)^2 + \binom{8}{3} \left(-\frac{x^2}{4}\right)^3 + \binom{8}{4} \left(-\frac{x^2}{4}\right)^4 + \dots \\ &= 1 \cdot 1 + \frac{8}{1} \cdot \left(-\frac{x^2}{4}\right) + \frac{8 \cdot 7}{1 \cdot 2} \cdot \frac{x^4}{16} + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} \cdot \left(-\frac{x^6}{64}\right) + \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \left(\frac{x^8}{256}\right) + \dots \\ &= 1 - 2x^2 + \frac{7}{4}x^4 - \frac{7}{8}x^6 + \dots \end{aligned}$$

$\left(1 - \frac{x^2}{4}\right)^8$ এর বিস্তৃতিতে দেখা যাচ্ছে x^3 এর সহগযুক্ত পদ নেই। অর্থাৎ x^3 এর সহগ 0 এবং x^6 এর সহগ $-\frac{7}{8}$

$\therefore x^3$ এর সহগ 0 এবং x^6 এর সহগ $-\frac{7}{8}$.

কাজ : প্যাসকেলের ত্রিভুজের সাহায্যে উদাহরণ ৪ এর সত্যতা যাচাই কর।

উদাহরণ ৫। $(1-x)(1+ax)^6$ কে x^2 পর্যন্ত বিস্তৃত করলে যদি $1+bx^2$ পাওয়া যায়, তাহলে a ও b এর মান নির্ণয় কর।

সমাধান : $(1-x)(1+ax)^6$

$$\begin{aligned} &(1-x) \left[\binom{6}{0} (ax)^0 + \binom{6}{1} (ax)^1 + \binom{6}{2} (ax)^2 + \dots \right] \\ &= (1-x) \left[1 + \frac{6}{1} \cdot ax + \frac{6 \cdot 5}{1 \cdot 2} a^2 x^2 + \dots \right] \\ &= (1-x)(1+6ax+15a^2x^2 + \dots) \\ &= (1+6ax+15a^2x^2 + \dots) + (-x-6ax^2-15a^2x^3 - \dots) \\ &= 1 + (6a-1)x + 15a^2x^2 - 6ax^2 - 15a^2x^3 + \dots \\ &= 1 + (6a-1)x + (15a^2-6a)x^2 - 15a^2x^3 + \dots \end{aligned}$$

প্রশ্নমতে,

$$1 + (6a-1)x + (15a^2-6a)x^2 - 15a^2x^3 + \dots = 1 + bx^2$$

উভয়পক্ষ থেকে x ও x^2 এর সহগ সমীকৃত করে পাই,

$$6a-1=0, 15a^2-6a=b$$

$$\text{বা } a = \frac{1}{6}, \text{ এবং } b = 15 \cdot \frac{1}{36} - 6 \cdot \frac{1}{6} = \frac{5}{12} - 1 = -\frac{7}{12}$$

$$\therefore a = \frac{1}{6}, b = -\frac{7}{12}$$

$$\text{উত্তর : } a = \frac{1}{6}, b = -\frac{7}{12}$$

উদাহরণ ৬। $(1-x)^8(1+x)^7$ এর বিস্তৃতিতে x^7 এর সহগ নির্ণয় কর।

সমাধান :

$$\begin{aligned} (1-x)^8(1+x)^7 &= (1-x)(1-x)^7(1+x)^7 = (1-x)(1-x^2)^7 \\ &= (1-x) \left[\binom{7}{0}(-x^2)^0 + \binom{7}{1}(-x^2)^1 + \binom{7}{2}(-x^2)^2 + \binom{7}{3}(-x^2)^3 + \binom{7}{4}(-x^2)^4 + \dots \right] \\ \therefore (1-x)^8(1+x)^7 &= (1-x)[1-7x^2+21x^4-35x^6+35x^8-\dots] \\ &= (1-7x^2+21x^4-35x^6+35x^8-\dots) + (-x+7x^3-21x^5+35x^7-35x^9+\dots) \\ \therefore (1-x)^8(1+x)^7 &= 1-x-7x^2+7x^3+21x^4-21x^5-35x^6+35x^7-\dots \\ &= (1-x)^8(1+x)^7 = 1-x-7x^2+7x^3+21x^4-21x^5-35x^6+35x^7+35x^8-\dots \\ \therefore (1-x)^8(1+x)^7 \text{ এর বিস্তৃতিতে } x^7 \text{ এর সহগ } &35 \\ \therefore x^7 \text{ এর সহগ } &35 \end{aligned}$$

উদাহরণ ৭। x এর ঘাতের উর্ধ্বক্রম অনুসারে $(2-x)\left(1+\frac{1}{2}x\right)^8$ কে x^3 পর্যন্ত বিস্তৃত কর। উক্ত ফলাফল

ব্যবহার করে $1.9 \times (1.05)^8$ এর মান নির্ণয় কর।

সমাধান : দ্বিপদী বিস্তৃতি ব্যবহার করে পাই,

$$\begin{aligned} (2-x)\left(1+\frac{1}{2}x\right)^8 &= (2-x) \left[\binom{8}{0}\left(\frac{x}{2}\right)^0 + \binom{8}{1}\left(\frac{x}{2}\right)^1 + \binom{8}{2}\left(\frac{x}{2}\right)^2 + \binom{8}{3}\left(\frac{x}{2}\right)^3 + \binom{8}{4}\left(\frac{x}{2}\right)^4 + \dots \right] \\ \text{বা } (2-x)\left(1+\frac{1}{2}x\right)^8 &= (2-x) \left[1 + \frac{8}{1} \cdot \frac{x}{2} + \frac{8 \cdot 7}{1 \cdot 2} \cdot \frac{x^2}{4} + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} \frac{x^3}{8} + \dots \right] \\ &= (2-x)(1+4x+7x^2+7x^3+\dots) \\ &= (2+8x+14x^2+14x^3+\dots) + (-x-4x^2-7x^3-7x^4-\dots) \\ &= 2+7x+10x^2+7x^3+\dots \\ \therefore (2-x)\left(1+\frac{1}{2}x\right)^8 &= 2+7x+10x^2+7x^3+\dots \\ \therefore \text{নির্ণেয় বিস্তৃতি } (2-x)\left(1+\frac{1}{2}x\right)^8 &= 2+7x+10x^2+7x^3+\dots \end{aligned}$$

GLb D^3 we $^{-}wZtZ$ $x = 0.1$ emtq cvB,

$$(2 - \cdot 1) \times \left(1 + \frac{\cdot 1}{2}\right)^8 = 2 + 7 \times (\cdot 1) + 10(\cdot 1)^2 + 7(\cdot 1)^3 + \dots$$

ev, $1 \cdot 9 \times (1 \cdot 05)^8 = 2 + \cdot 7 + 10 \times (\cdot 01) + 7(\cdot 001) + \dots$

ev, $1 \cdot 9 \times (1 \cdot 05)^8 = 2 + \cdot 7 + \cdot 1 + \cdot 007 + \dots$
 $= 2 \cdot 807$ (wZb ` kmgK ^{-}vb ch $^{-}$)

Ans : $1 \cdot 9 \times (1 \cdot 05)^8 = 2 \cdot 807$

KvR : c $^{-}$ vmKtj i w $^{-}$ fRi gva $^{-}$ tq we $^{-}wZwJ$ hvPvB Ki |

Abkxj bx 10-1

1| c $^{-}$ vmKtj i w $^{-}$ fR ev w $^{-}$ c $^{-}$ x we ^{-}wZ e $^{-}$ envi Kti $(1 + y)^5$ Gi we ^{-}wZ wbY $^{-}$ Ki | D^3 we ^{-}wZ mrv $^{-}$ th $^{-}$ (i) $(1 - y)^5$ | (ii) $(1 + 2x)^5$ Gi we ^{-}wZ wbY $^{-}$ Ki |

2| x Gi Nv $^{-}$ Zi EaY $^{-}$ ug Aby $^{-}$ v $^{-}$ i
 (a) $(1 + 4x)^6$, (b) $(1 - 3x)^7$ Gi c $^{-}$ g Pvi c $^{-}$ ch $^{-}$ we ^{-}wZ Ki |

3| $(1 + x^2)^8$ Gi we ^{-}wZ c $^{-}$ g Pvi c $^{-}$ wbY $^{-}$ Ki | D^3 dj v $^{-}$ dj e $^{-}$ envi Kti $(1 \cdot 01)^8$ Gi gvb wbY $^{-}$ Ki |

4| x Gi EaY $^{-}$ ug Aby $^{-}$ v $^{-}$ i w $^{-}$ t $^{-}$ g $^{-}$ w $^{-}$ c $^{-}$ x mg $^{-}$ ni c $^{-}$ g wZb c $^{-}$ wbY $^{-}$ Ki |
 (a) $(1 - 2x)^5$, (b) $(1 + 3x)^9$

Zvi ci, (c) $(1 - 2x)^5(1 + 3x)^9$ tK x^2 ch $^{-}$ we ^{-}Z Ki |

5| w $^{-}$ t $^{-}$ g $^{-}$ we ^{-}wZ mg $^{-}$ ni c $^{-}$ g Pvi c $^{-}$ wbY $^{-}$ Ki | [w $^{-}$ c $^{-}$ x we ^{-}wZ ev c $^{-}$ vmK $^{-}$ vj w $^{-}$ fR Gi th $^{-}$ Kv $^{-}$ bv GKwJ e $^{-}$ envi Kti]

(a) $(1 - 2x^2)^7$ (b) $\left(1 + \frac{2}{x}\right)^4$ (c) $\left(1 - \frac{1}{2x}\right)^7$

6| x^3 ch $^{-}$ -(a) $(1 - x)^6$ Ges (b) $(1 + 2x)^6$ we ^{-}Z Ki | Zvi ci (c) $(1 + x - 2x^3)^6$ tK x^3 ch $^{-}$ we ^{-}Z Ki |

7| x Gi gvb h $^{-}$ t $^{-}$ o tQvU nI qvq x^3 Ges Zvi EaY $^{-}$ v $^{-}$ Zi gvb At $^{-}$ c $^{-}$ v Kiv hvq | c $^{-}$ g $^{-}$ vY Ki th,
 $(1 + x)^5(1 - 4x)^4 = 1 - 11x + 26x^2$.

10-2 w $^{-}$ c $^{-}$ x : $(x + y)^n$ Gi we ^{-}wZ :

Av $^{-}$ g $^{-}$ v G ch $^{-}$ -($1 + y$) n Gi we ^{-}wZ w $^{-}$ t $^{-}$ q Av $^{-}$ tj vPbv Kti wQ | GB ch $^{-}$ q Av $^{-}$ g $^{-}$ v w $^{-}$ c $^{-}$ x we ^{-}wZ m $^{-}$ v $^{-}$ avi Y Av $^{-}$ Kvi $(x + y)^n$ w $^{-}$ t $^{-}$ q Av $^{-}$ tj vPbv Kie thLv $^{-}$ tb n av $^{-}$ Z $^{-}$ K c $^{-}$ Y $^{-}$ msL $^{-}$ v | $(x + y)^n$ Gi we ^{-}wZ m $^{-}$ v $^{-}$ avi YFv $^{-}$ te w $^{-}$ c $^{-}$ x Dccv $^{-}$ b $^{-}$ v $^{-}$ t $^{-}$ g cwi wPZ |

Avgi v Rwb,

$$(1 + y)^n = 1 + \binom{n}{1}y + \binom{n}{2}y^2 + \binom{n}{3}y^3 + \binom{n}{r}y^r + \dots + \binom{n}{n}y^n$$

GLb, $(x + y)^n = \left[x \left(1 + \frac{y}{x} \right) \right]^n = x^n \left(1 + \frac{y}{x} \right)^n$

$$\therefore (x + y)^n = x^n \left[1 + \binom{n}{1} \left(\frac{y}{x} \right) + \binom{n}{2} \left(\frac{y}{x} \right)^2 + \binom{n}{3} \left(\frac{y}{x} \right)^3 + \dots + \binom{n}{n} \left(\frac{y}{x} \right)^n \right]$$

$$\therefore (x + y)^n = x^n \left[1 + \binom{n}{1} \left(\frac{y}{x} \right) + \binom{n}{2} \frac{y^2}{x^2} + \binom{n}{3} \frac{y^3}{x^3} + \dots + \frac{y^n}{x^n} \right] \left[\because \binom{n}{n} = 1 \right]$$

$$= x^n + \binom{n}{1} \left(x^n \cdot \frac{y}{x} \right) + \binom{n}{2} \left(x^n \cdot \frac{y^2}{x^2} \right) + \binom{n}{3} \left(x^n \cdot \frac{y^3}{x^3} \right) + \dots + x^n \cdot \frac{y^n}{x^n}$$

$$\therefore (x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \dots + y^n$$

GiUB n^o Q woc`x Dccv^o i maviY AvKvi | j Yxq, GB we⁻WZ (1 + y)ⁿ Gi Abjfc | GLv^o b x Gi NvZ n t^o tK o ch^o-thM Kiv n^o t^o tQ | Av^o i v j Yxq, c^o Z ct^o x l y Gi Nv^o Zi thM^o dj woc`xi Nv^o Zi mgvb | c^o g ct^o 'x' Gi NvZ n t^o tK i i^o n^o t^o q me^o kl ct^o kb^o | WK we^o cixZ fr^o te y Gi NvZ c^o g ct^o kb^o t^o tK i i^o n^o t^o q tkl ct^o n n^o t^o tQ |

D^o vniY 8 | (x + y)⁵ tK we⁻Z Ki Ges Dnv nB^o Z (3 + 2x)⁵ Gi we⁻WZ wBY^o Ki |

$$\begin{aligned} \text{mgvavb : } (x + y)^5 &= x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + y^5 \\ &= x^5 + 5x^4y + \frac{5 \cdot 4}{1 \cdot 2}x^3y^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3}x^2y^3 + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4}xy^4 + y^5 \\ &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5. \end{aligned}$$

$$\therefore \text{wBY^o we⁻WZ : } (x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5.$$

GLb x = 3 Ges y = 2x emvB

$$\begin{aligned} (2 + 2x)^5 &= 3^5 + 5 \cdot 3^4 \cdot 2x + 10 \cdot 3^3 (2x)^2 + 10 \cdot 3^2 (2x)^3 + 5 \cdot 3 (2x)^4 + (2x)^5 \\ &= 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5 \end{aligned}$$

mZi vs, $(3 + 2x)^5 = 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$

D^o vniY 9 | $\left(x + \frac{1}{x^2} \right)^6$ tK x Gi Nv^o Zi Aat^o ug Ab^o m^o v^o ti PZ^o L^o c^o ch^o-we⁻Z Ki Ges x gj^o B

c^o w kb^o v^o Ki |

mgvavb : woc`x Dccv^o Ab^o m^o v^o ti cvB,

$$\begin{aligned} \left(x + \frac{1}{x^2}\right)^6 &= (x)^6 + \binom{6}{1}x^5 \left(\frac{1}{x^2}\right) + \binom{6}{2}x^4 \left(\frac{1}{x^2}\right)^2 + \binom{6}{3}x^3 \left(\frac{1}{x^2}\right)^3 + \dots \\ &= x^6 + 6x^3 + \frac{6 \cdot 5}{1 \cdot 2}x^4 \cdot \frac{1}{x^4} + \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}x^3 \frac{1}{x^6} + \dots \\ &= x^6 + 6x^3 + 15 + 20\frac{1}{x^3} + \dots \end{aligned}$$

D'ei : $x^6 + 6x^3 + 15 + \frac{20}{x^3} + \dots$

Ges x g³ c` 15

D`vniY 10 | x Gi Nv³Zi EaY³g Abm³ti $\left(2 - \frac{x}{2}\right)^7$ Gi we³Zi c³g Pvi WJ c` w³Y³ Ki | D³

we³Zi m³v³th` (1.995)⁷ Gi g³v³b Pvi `k³w³K `v³b ch³S-w³Y³ Ki |

$$\begin{aligned} \text{mgvavb} : \left(2 - \frac{x}{2}\right)^7 &= 2^7 + \binom{7}{1}2^6 \left(-\frac{x}{2}\right) + \binom{7}{2}2^5 \left(-\frac{x}{2}\right)^2 + \binom{7}{3}2^4 \left(-\frac{x}{2}\right)^3 + \dots \\ &= 128 + 7 \cdot 64 \left(-\frac{x}{2}\right) + \frac{7 \cdot 6}{1 \cdot 2} \cdot 32 \cdot \frac{x^2}{4} + \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} \cdot 16 \left(-\frac{x}{2}\right)^3 + \dots \end{aligned}$$

$$\therefore \left(2 - \frac{x}{2}\right)^7 = 128 - 224x + 168x^2 - 70x^3 + \dots$$

$$\therefore \text{w³t³Y³ we³ w³i} \left(2 - \frac{x}{2}\right)^7 = 128 - 224x + 168x^2 - 70x^3 + \dots$$

GLb, $2 - \frac{x}{2} = 1.995$

ev, $\frac{x}{2} = 2.000 - 1.995$

ev, $x = 0.01$ e³m³t³q c³v³B

$$\left(2 - \frac{0.01}{2}\right)^7 = 128 - 224 \times (0.01) + 168 \times (0.01)^2 - 70 \times (0.01)^3 + \dots$$

ev, $(1.995)^7 = 125.7767$ (Pvi `k³w³K `v³b ch³S)

10.3 $n!$ Ges n_c Gi g³v³b w³Y³ :

w³t³Pi D`vniY³ t³j v³j ¶ Kwi :

$2 = 2 \cdot 1$

$6 = 3 \cdot 2 \cdot 1$

$24 = 4 \cdot 3 \cdot 2 \cdot 1$

$120 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Wwbow` tKi , Ydj mgn`K Avgiv GLb mst`¶t`c GKwU mvst`KwZK wPtyi gva`tg cKvk Ki tZ cwii |

$$2 = 2 \cdot 1 = 2 !$$

$$6 = 3 \cdot 2 \cdot 1 = 3 !$$

$$24 = 4 \cdot 3 \cdot 2 \cdot 1 = 4 !$$

$$120 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5 !$$

GLb j ¶ Kwi

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 4 \cdot (4-1) \cdot (4-2) \cdot (4-3)$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 5 \cdot (5-4) \cdot (5-2) \cdot (5-3) \cdot (5-4)$$

∴ mvavi Yfvte wj LtZ cwii , $n! = n(n-1)(n-2)(n-3)\dots\dots\dots 3 \cdot 2 \cdot 1$

Ges $n!$ tK tdt±wii qvj (*Factorial*) n ej v nq |

Z`æc, 3! tK tdt±wii qvj wZb,

4! tK tdt±wii qvj Pvi BZ`w` cov nq |

Avevi j ¶ Kwi :

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(1 \cdot 2 \cdot 3) \cdot (2 \cdot 1)}$$

$$= \frac{5!}{3! \times 2!} = \frac{5!}{3!(5-3)!}$$

$$\binom{7}{4} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4!}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4! \times (3 \cdot 2 \cdot 1)} = \frac{7!}{4! \times 3!}$$

$$= \frac{7!}{4!(7-4)!}$$

∴ mvavi Yfvte Avgiv ej tZ cwii $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Wwbow` cvt`ki tdt±wii qvj mgn`K th cZxK Øviv cKvk Kiv nq Zv ntj v,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = {}^n c_r$$

$$\therefore \binom{7}{4} = \frac{7!}{4!(7-4)!} = {}^7 c_4$$

Ges $\binom{5}{3} = \frac{5!}{3!(5-3)!} = 5 \cdot c_3$

mZi vs, $\binom{n}{r} = n \cdot c_r$

A_#r, $\binom{n}{r} | n \cdot c_r$ Gi gvb mgvb |

$\therefore \binom{n}{1} = n \cdot c_1, \binom{n}{2} = n \cdot c_2$
 $\binom{n}{3} = n \cdot c_3 \dots \dots \binom{n}{n} = n \cdot c_n$

Avgiv Rmb $\binom{n}{n} = 1 = n \cdot c_n$

GLb $n \cdot c_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!(0)!} = \frac{1}{0!}$

$\therefore 1 = \frac{1}{0!}$

A_#r, $0! = 1$.

gtb i vL tZ nte

$\therefore n! = n(n-1)(n-2)(n-3) \dots \dots \dots 3 \cdot 2 \cdot 1$

$\binom{n}{r} = n \cdot c_r, \quad n \cdot c_n = 1,$

$\binom{n}{r} = n \cdot c_r = \frac{n!}{r!(n-r)!}, \quad \binom{n}{0} = n \cdot c_0 = 1$

$\binom{n}{n} = n \cdot c_n = 1, \quad 0! = 1.$

GLb $(1+y)^n = 1 + n \cdot c_1 y + n \cdot c_2 y^2 + n \cdot c_3 y^3 + \dots + n \cdot c_r y^r + \dots + n \cdot c_n y^n$

$(1+y)^n = 1 + n \cdot c_1 y + n \cdot c_2 y^2 + n \cdot c_3 y^3 + \dots + n \cdot c_r y^r + \dots + n \cdot c_n y^n$

el, $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!} y^2 + \frac{n(n-1)(n-2)}{3!} y^3 + \frac{n(n-1)(n-2)(n-3)}{4!} y^4 \dots + y^n$

A_#r $(1+y)^n = 1 + ny + \frac{n(n-1)}{1 \cdot 2} y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} y^3 + \dots + y^n$

Ges Abjfcvte,

$(x+y)^n = x^n + n \cdot c_1 x^{n-1} y + n \cdot c_2 x^{n-2} y^2 + n \cdot c_3 x^{n-3} y^3 + \dots + n \cdot c_r x^{n-r} y^r + \dots + n \cdot c_n y^n$

$$\text{ev } (x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \dots + y^n$$

j ¶Yxq : abvZK cYfnsL v n Gi Rb

$$1 | \text{w}c`x \text{w}e^{-Z} (1 + y)^n \text{ Gi mvaviY c` ev } r \text{Zg c` } T_{r+1} = \binom{n}{r} y^r \text{ ev } n_{c_r} y^r$$

$$\text{GLvfb, } \binom{n}{r} \text{ ev } n_{c_r} \text{ w}c`x \text{ mnM}$$

$$\begin{aligned} (x + y)^n &= x^n + n_{c_1} x^{n-1}y + n_{c_2} x^{n-2}y^2 + n_{c_3} x^{n-3}y^3 + n_{c_4} x^{n-4}y^4 + \dots + y^n \\ &= x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \frac{n(n-1)(n-2)(n-3)}{4!}x^{n-4}y^4 + \dots + y^n \end{aligned}$$

mvaviY c` ev rZg c`

$$T_{r+1} = \binom{n}{r} x^{n-r} y^r \text{ ev } n_{c_r} x^{n-r} y^r$$

$$\text{thLvfb } \binom{n}{r} \text{ ev } n_{c_r} \text{ w}c`x \text{ mnM}$$

$$\text{D`vniY 11} | \left(x - \frac{1}{x^2}\right)^5 \text{ tK w}e^{-Z} \text{ Ki}$$

mgavb : w}c`x Dccv` e`envi Kti

$$\begin{aligned} \left(x - \frac{1}{x^2}\right)^5 &= x^5 + 5_{c_1} x^{5-1} \left(-\frac{1}{x^2}\right) + 5_{c_2} x^{5-2} \left(-\frac{1}{x^2}\right)^2 + 5_{c_3} x^{5-3} \left(-\frac{1}{x^2}\right)^3 + 5_{c_4} x^{5-4} \left(-\frac{1}{x^2}\right)^4 + \left(-\frac{1}{x}\right) \\ &= x^5 - 5x^4 \cdot \frac{1}{x^2} + \frac{5 \cdot 4}{1 \cdot 2} x^3 \cdot \frac{1}{x^4} - \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} x^2 \cdot \left(\frac{1}{x^6}\right) + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} x \cdot \left(\frac{1}{x^8}\right) - \frac{1}{x^{10}} \\ &= x^5 - 5x^3 + \frac{10}{x} - \frac{10}{x^4} + \frac{5}{x^7} - \frac{1}{x^{10}} \end{aligned}$$

$$\text{D`vniY 12} | \left(2x^2 - \frac{1}{2x}\right)^8 \text{ Gi w}e^{-Z} \text{ c}g \text{ PviU c` w}bYq \text{ Ki}$$

mgavb :

$$\begin{aligned} \left(2x^2 - \frac{1}{2x}\right)^8 &= (2x^2)^8 + 8_{c_1} (2x^2)^7 \left(-\frac{1}{2x}\right) + 8_{c_2} (2x^2)^6 \left(-\frac{1}{2x}\right)^2 + 8_{c_3} (2x^2)^5 \left(-\frac{1}{2x}\right)^3 + \dots \\ &= 256x^{16} - 512x^{13} + 448x^{10} - 224x^7 + \dots \end{aligned}$$

Abkxj bx 10.2

1) i $\delta_{c_0} = \delta_{c_8}$

ii $\binom{n}{r} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r}$

iii $(1+x)^n$ Gi we $^{-}$ wZtZ wZxq c` wJ
 $= \frac{n(n-1)}{2!} x^2$

wbtpi tkvbwJ mWVK ?

K. i l ii

L. ii l iii

M. i l iii

N. i, ii l iii

2) $(a+x)^n$ -Gi we $^{-}$ wZtZ $(n+1)$ msL`K c` itqfQ| GLvfb n GKwJ

K. AFYvZK i wvk

L. abvZK i wvk

M. FYvZK i wvk

N. fMvsk

3) $(x+y)^5$ -Gi we $^{-}$ wZtZ wZc` x mnM, wj ntj v :

K. 5, 10, 10, 5

L. 1, 5, 10, 10, 5, 1

M. 10, 5, 5, 10

N. 1, 2, 3, 3, 2, 1

4) $(1-x)(1+\frac{x}{2})^8$ -Gi we $^{-}$ wZtZ x Gi mnM

K. -1

L. $\frac{1}{2}$

M. 3

N. $-\frac{1}{2}$

5) $(x^2 + \frac{1}{x^2})^4$ -Gi we $^{-}$ wZtZ x gy³ c` KZ?

K. 4

L. 6

M. 8

N. 0

6) $(2-x)(1+ax)^5$ tk x^2 chS-we $^{-}$ Z Ki tj hw` $2+9x+cx^2$ cvl qv hvq, Zte a l c Gi gvb

K. a = 1, c = 15

L. a = 5, c = 15

M. a = 15, c = 1

N. a = 1, c = 0

wbtpi Zt`i Avtj vtK 7 l 8 bs cOkie DEi `vl :

$$n_{c_r} = \frac{n!}{r!(n-r)!} ntj$$

7) $n_{c_0} =$ KZ?

K. 0

L. 1

M. n

N. wBYq Kiv hvq bv

8) $n = r = 100$ ntj n_{c_r} Gi gvb

K. 0

L. 1

M. 100

N. 200

9) $(x+y)^4$ we $^{-}$ wZi mnM, wj i mvRvtj Avgiv cvB-

K.

4

L.

1

1 4 1

1 2 1

1 5 5 1

1 3 3 1

1 6 10 6 1

1 4 6 4 1

M.	2	N.	6
	2 3 2		6 12 6
	1 5 5 2		6 18 18 6
	2 7 10 7 2		6 24 36 24 6

10 | $(2+x^2)^5$ cōZiU t'q'f'f' we-Z Ki :

(a) $(2+x^2)^5$ (b) $\left(2-\frac{1}{2x}\right)^6$

11 | $(2+3x)^6$ we-wZmgf'ni cōg PviU c` wBY' Ki |

(a) $(2+3x)^6$ (b) $\left(4-\frac{1}{2x}\right)^5$

12 | $\left(p-\frac{1}{2}x\right)^6 = r-96x+5x^2+\dots\dots\dots$ n'tj , p Ges r Gi gvb wBY' Ki |

13 | $\left(1+\frac{x}{2}\right)^8$ Gi we-wZi x^3 Gi mnM wBY' Ki |

14 | x Gi Nv'tZi Ea'p'g Abyn'ti $\left(2+\frac{x}{4}\right)^8$ tK x^3 chS-we-Z Ki | Dnvi mnv'th" $(1.9975)^6$ Gi Avmb'g'v' Pvi `k'g'K `v' chS-wBY' Ki |

15 | $(1.99)^5$ Dccv` e`envi K'ti $(1.99)^5$ Gi gvb Pvi `k'g'K `v' chS-wBY' Ki |

16 | $\left(1+\frac{1}{4}\right)^n$ Gi we-wZi ZZxq c't` i mnM PZl`c't` i mn'tMi w`Y | n Gi gvb wBY' Ki |
we-wZi c`msL`v I ga`c` wBY' Ki |

17 | (a) $\left(k-\frac{x}{3}\right)^7$ Gi we-wZ'tZ k^3 Gi mnM 560 n'tj x Gi gvb wBY' Ki |

(b) $\left(x^2+\frac{k}{x}\right)^6$ Gi we-wZ'tZ x^3 Gi mnM 160 n'tj k Gi gvb wBY' Ki |

mRbk'j c'k'e

18 | t` I qv Av'tQ,

$P = (a+bx)^6$ (i)

$Q = (b+ax)^5$ (ii)

$R = (a+x)^n$ (iii)

K. (iii) Gi we-wZiU t'j L Ges m'f'U cōq'vM K'ti (i) Gi we-wZ wBY' Ki |

L. h'w (i) Gi we-wZi wZxq I ZZxq c` h`y'p'tg (ii) Gi we-wZi wZxq I ZZxq c't` i Ab'c'v'tZi mgvb nq Z'te t`Lvl th, a:b = $\sqrt{5} : 2$ | Dcwi D³ Dw³ i `c't'q' GK'U D`vni Y`vl |

M. t`Lvl th, (ii) Gi we-wZi t'Rvo `v'bxq cig a'eK`'ij i thvM'dj we't'Rvo `v'bxq cig a'eK`'ij i thvM'd'tj i mgvb | Z'ug G'gb GK'U wZc`x i w'k D't'j L Ki, hvi t'q'f'f' I Dcwi - D³ we'l q'U mZ` nq |

GKv`k Aa`vq `vbr¼ R`vngwZ

we`y mij`tiLv l eµtiLvi exRMwYwZK cKvktK R`vngwZi th Astk Aa`qb Kiv nq, ZvB `vbr¼ R`vngwZ
bvtg cwi wPZ | R`vngwZi GB Ask wetkly R`vngwZ (*Analytic Geometry*) bvtgl cwi wPZ | mgZtj we`y
cvZtbi gva`tg mij ev eµtiLv A_ev Gt`i Øviv `Zwi R`vngwZK t¶t h_v, wî fR, PZfR, eE BZ`w` wPÎ
cKvk Kiv nq | mgZtj we`y cvZtbi c×wZi mPbv Ktib weL`vZ divmx MwYZwe` *Rene Descartes*
(tWKvZ`bvtg cwi wPZ) | tWKvZP cewZ R`vngwZi GB `vbr¼ (*Coordinates*) cUv Zwi B bvgvbmvti
KvZfmxq `vbr¼ (*Cartesian Coordinates*) bvtg cwi wPZ | `vbr¼ R`vngwZ l wetkly R`vngwZ gj Z
KvZfmxq `vbr¼ wbfP | ZvB tWKvZfK wetkly R`vngwZi cEZR ej v hvq |

GB Aa`vtqi cUg Astk wK¶v_¶ i mgZtj KvZfmxq `vbr¼i avi Yv cUvtbi gva`tg `BwU we`y ga`eZP
`tZi wY¶qi tKSkj Avtj vPbv Kiv nte | wZxq Astk mij`tiLvi gva`tg mP thtKvfbv wî fR l PZfRi
t¶t dj wY¶qi c×wZ Avtj vPbv Kiv nte Ges ZZxq Astk mij`tiLvi Xij wY¶ Ges `BwU we`y msthtM
mij`tiLvi mij`xKiY wY¶qi tKSkj e`vL`v Kiv nte | eµtiLv Øviv mP tKvb R`vngwZK wPÎ ev mgxKiYi
Avtj vPbv GLvtb Kiv nte bv | D`PZi tk¶tZ G mspvš`weL` Avtj vPbv Kiv nte |

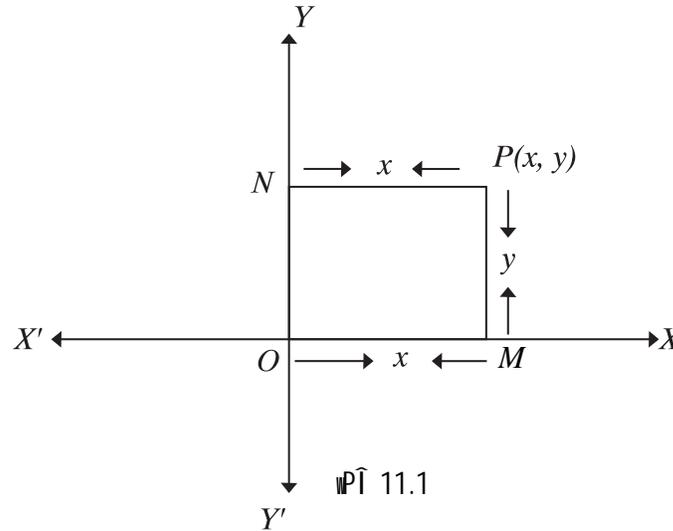
Aa`vq tk¶t wK¶v_¶ –

- mgZtj KvZfmxq `vbr¼i avi Yv e`vL`v KiZ cvi te |
- `BwU we`y ga`eZP` tZi wY¶ KiZ cvi te |
- mij`tiLvi Xvtj i avi Yv e`vL`v KiZ cvi te |
- mij`tiLvi mgxKiY wY¶ KiZ cvi te |
- evui `N`wY¶qi gva`tg wî fR l PZfRi t¶t dj wY¶ KiZ cvi te |
- we`y cvZtbi gva`tg wî fR l PZfR mspvš`R`vngwZK wPÎ A¼b KiZ cvi te |
- mij`tiLvi mgxKiY tj LwPÎ Dc`vcb KiZ cvi te |

11.1 AvqZvKvi KvZfmxq `vbr¼ (*Rectangular Cartesian Coordinates*)

mgZtj i avi Yv ceZP tk¶tZ t` l qv ntqtQ | GKwU tUwtj i Dcwi fVM, Nti i tgtS, eB-Gi Dcwi fVM
Ggb wK th KvMtRi Dci tj Lv nq Gt`i cØZ`tKB mgZj | GKwU dUwtj i Dcwi fVM ev GKwU tevZtj i
Dcwi fVM ntj v eµZj | GB Astk mgZtj Aew`Z tKvfbv we`y mWk Ae`vb wY¶qi tKSkj Avtj vPbv Kiv
nte | mgZtj Aew`Z tKvfbv wbr`Ø we`y mWk Ae`vb wY¶qi Rb` H mgZtj Aw¼Z `BwU ci`úi tQ`x
mij`tiLv ntZ wbr`Ø we`y `tZi Rvbr cØqvRb | Gi KviY wntmte ej v hvq ci`úi tQ`x `BwU mij`tiLv ntZ
tKvfbv wbr`Ø `tZi tKej gvÎ GKwU we`B`vKtZ cvi |

tKv^tbv mgZ^tj ci^ˆui mg^tKv^tY tQ^ˆ K^ti Gi^fc `BwU mij^tiLv XOX' Ges YOY' AwK^tj XOX' tK x A^ql (x-axis), YOY' tK y A^ql (y-axis) Ges tQ^ˆ we^ˆy 00^ˆ tK gj we^ˆy (Origin) ej v nq|



W^PT 11.1

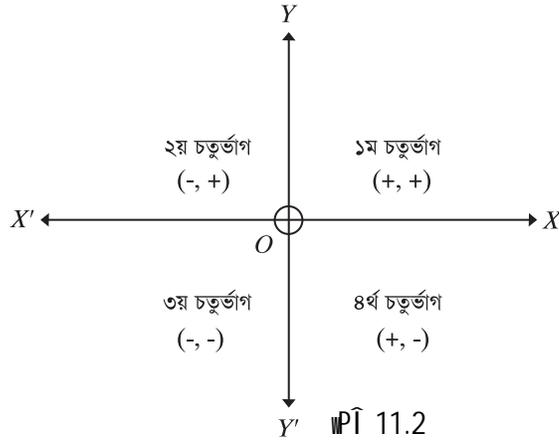
GLb atⁱ wB A^ql^tqi mgZ^tj th^tKv^tbv we^ˆy P | D³ P we^ˆy t^ˆK XOX' A^ˆ, x A^ql Ges YOY' A^ˆ y A^qlⁱ Dci j^ˆ h^ˆv^ˆutg PM Ges PN | Zv^tj y-A^ql n^tZ P we^ˆy `iZⁱ = NP = OM = x tK P we^ˆy j fR (abscissa) ev x^ˆ v^ˆlv^ˆ (x-coordinate) etj | Aveⁱ x A^ql n^tZ P we^ˆy j `iZⁱ = MP = ON = y tK P we^ˆy j tKwU (Ordinate) ev y^ˆ v^ˆlv^ˆ (y-coordinate) ej v nq| fR l tKwU^tK GK mv^t v^ˆlv^ˆ ej v nq| mZⁱ vs W^PT P we^ˆy j v^ˆlv^ˆ ej tZ y A^ql l x A^ql n^tZ P we^ˆy j j^ˆ iZⁱ tevSvq Ges Zv^t i^tK x l y 0vⁱ v^ˆlv^ˆ R K^ti P we^ˆy j v^ˆlv^ˆ P(x, y) cZ^xK 0vⁱ cK^vk Kⁱ v nq|

we^ˆy j v^ˆlv^ˆ mPK (x, y) GKwU μgtRvo eSvq hvi cU^gwU fR l wZ^xqW tKwU v^ˆlv^ˆ R K^ti | ZvB (x, y) l (y, x) 0vⁱ v^ˆlv^ˆ wfbwe^ˆy eSvq| mZⁱ vs ci^ˆui mg^tKv^tY tQ^ˆ K^ti Gi^fc GK^tRvov A^qlⁱ mv^tc^tq^l tKv^tbv we^ˆy j v^ˆlv^ˆ tK AvqZvKvi Kv^tZ^ˆq v^ˆlv^ˆ ej v nq| we^ˆy j y A^qlⁱ W^tb v^ˆK^tj fR av^ˆZ^ˆK l ev^tg v^ˆK^tj fR FYvZ^ˆK n^te| Aveⁱ we^ˆy j x A^qlⁱ Dci v^ˆK^tj tKwU av^ˆZ^ˆK Ges b^xtP v^ˆK^tj tKwU FYvZ^ˆK n^te| x A^qlⁱ Dci tKwU kb^ˆ Ges y A^qlⁱ Dci fR kb^ˆ n^te|

mZⁱ vs tKv^tbv we^ˆy j fR l tKwU h^ˆv^ˆutg OX l OY eivei ev Zv^t i mgv^ˆs^ˆv^ˆj w^ˆ tK v^ˆK^te| GKb^fv^ˆte FYvZ^ˆK fR ev tKwU OX' l OY' eivei hv Zv^t i mgv^ˆs^ˆv^ˆj w^ˆ tK v^ˆK^te|

Kv^tZ^ˆq v^ˆlv^ˆ A^ql^tq 0vⁱ mgZ^tj XOY, YOX', X'OY', Y'OX GB PⁱwU fv^tM we^f3 nq| G^t i cU^ˆZ^ˆKwU^tK PZ^fM (Quadrant) ej v nq|

XOY PZ^fM^tK cU^g aiv nq Ges Nwoi Ku^lvi AveZ^ˆbi we^cixZ w^ˆ tK ch^ˆq^ˆutg wZ^xq, ZZ^xq l PZ^ˆ PZ^fM aiv nq| tKv^tbv we^ˆy j v^ˆlv^ˆ W^PY Av^ˆnv^ˆi we^ˆy j Ae^ˆv^ˆb we^fbaPZ^fM v^ˆtK|



11.2 দুই বিন্দুর মধ্যে দূরত্ব (Distance between two Points)

গণনা করি, $P(x_1, y_1)$ বিন্দু $Q(x_2, y_2)$ বিন্দুর মধ্যকার দূরত্ব। P ও Q বিন্দু x -অক্ষের উপর M ও N বিন্দুতে PM ও QN লম্বাংশ আঁকি। P বিন্দু থেকে QN লম্বাংশের উপর PR লম্বাংশ আঁকি।

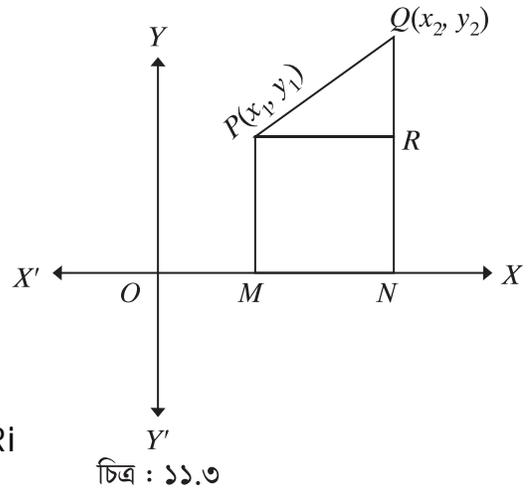
সুতরাং P বিন্দুর x -অক্ষের উপর $OM = x_1$

বিন্দু P থেকে x -অক্ষের উপর $MP = y_1$

Q বিন্দুর x -অক্ষের উপর $ON = x_2$ ও $QN = y_2$

∴ $PR = MN = ON - OM = x_2 - x_1$

$$QR = NQ - NR = NQ - MP = y_2 - y_1$$



এখন PQR ত্রিভুজের ক্ষেত্রফল PQ হипোটেনুজের দৈর্ঘ্য

অনুসরণ করে Z বিন্দু P থেকে Q বিন্দু পর্যন্ত দূরত্ব

$$PQ^2 = PR^2 + QR^2$$

$$\text{এবং } PQ = \pm \sqrt{PR^2 + QR^2}$$

$$\therefore PQ = \pm \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore P \text{ বিন্দু থেকে } Q \text{ বিন্দু পর্যন্ত দূরত্ব}$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

এখন Z বিন্দু P থেকে Q বিন্দু পর্যন্ত দূরত্ব P বিন্দু থেকে Q বিন্দু পর্যন্ত দূরত্ব

অনুসরণ করে Q বিন্দু থেকে P বিন্দু পর্যন্ত দূরত্ব

$$QP = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

∴ $PQ = QP$.

P we`yntZ Q we`yev Q we`yntZ P we`j `iZi mgvb |

A_ŕ, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = QP$.

Abjmxvš: gj we`y(0,0) ntZ mgZtj Aew`Z th tKvb we`y $P(x, y)$ Gi `iZi

$$PQ = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2}$$

D`vniY 1 | (1, 1) Ges (2,2) we`y `BwU GKwU mgZtj wPwYZ Ki |

Gt`i ga`eZŕ` iZi wBYŕ Ki |

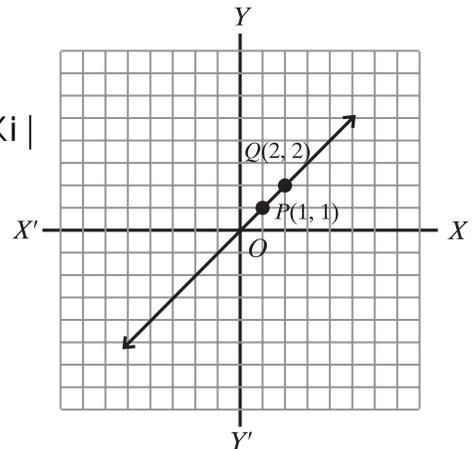
awi, $P(1,1)$ Ges $Q(2,2)$ cŕE we`ŕq |

wPŕŕ, xy mgZtj we`ŕqtK wPwYZ Kiv ntj v |

we`ŕtqi ga`eZŕ` iZi $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(2-1)^2 + (2-1)^2} \text{ GKK |}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2} \text{ GKK |}$$



wPŕ 11.4

D`vniY 2 | gj we`y $O(0,0)$ Ges Aci `BwU we`y $P(3,0)$ I $Q(0,3)$ mgZtj wPwYZ Ki | cŕZŕKi ga`eZŕ` iZi wBYŕ Ki | wZbwU we`ythvM Ki tj th R`wgvwZK wPŕ AswKZ nq Zvi bvg Kx Ges tKb ?

mgvavb : $O(0,0), P(3,0)$ I $Q(0,3)$ we`ywZbwU Ae`vb xy mgZtj t`Lvŕbv ntj v :

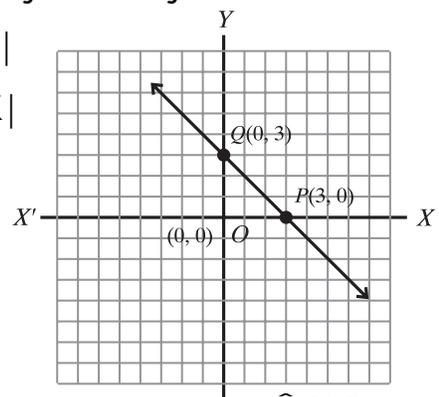
`iZi $OP = \sqrt{(3-0)^2 + (0-0)^2} = \sqrt{3^2 + 0^2} = \sqrt{3^2} = 3 \text{ GKK |}$

`iZi $OQ = \sqrt{(0-0)^2 + (3-0)^2} = \sqrt{0^2 + 3^2} = \sqrt{3^2} = 3 \text{ GKK |}$

`iZi $PQ = \sqrt{(3-0)^2 + (0-3)^2} \text{ GKK |}$

$$= \sqrt{y^2 + y^2} = \sqrt{9+9} \text{ GKK |}$$

$$= \sqrt{18} = \sqrt{2 \cdot 9} = 3\sqrt{2}. \text{ GKK |}$$

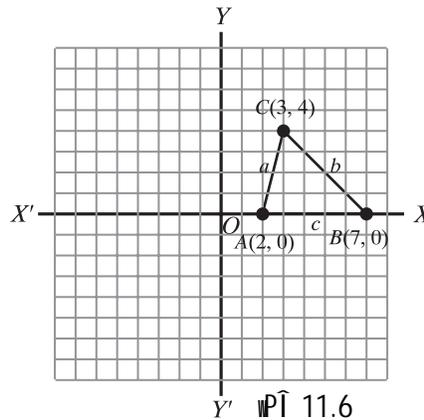


wPŕ 11.5

R`wgvwZK wPŕ wU i bvg mgwŕevu wŕ fŕR KviY Gi `B evu OP Ges OQ Gi `iZi mgvb |

D`vniY 3 | GKwU wŕ fŕRi wZbwU kxl we`yh_vµtg $A(2,0), B(7,0)$ I $C(3,4)$ | mgZtj Gt`i Ae`vb t`LvI Ges wŕ fŕRwU A¼b Ki | wŕ fŕRwU cw i mxgv cuP `kvgK `vb chS`wBYŕ Ki |

mgvavb : xy mgZtj $A(2, 0)$, $B(7, 0)$ | $C(3, 4)$ Gi Ae⁻vb t⁻ Lv⁺bv ntj v :



ABC w⁺l f⁺ri

w⁺l 11.6

AB ev⁺i $\sqrt{(7-2)^2 + (0-0)^2} = \sqrt{5^2} = 5$ GKK

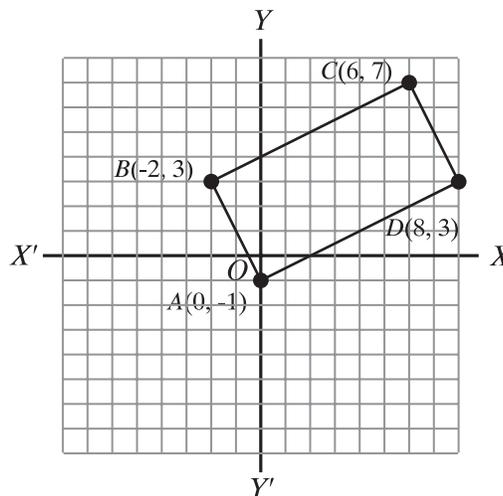
BC ev⁺i $\sqrt{(3-7)^2 + (4-0)^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = 4\sqrt{2}$ GKK

AC ev⁺i $\sqrt{(3-2)^2 + (4-0)^2} = \sqrt{1^2 + 4^2} = \sqrt{17}$ GKK

\therefore w⁺l f⁺ri cwi m⁺gv = $AB + BC + AC$ ev⁺l tqi $\sqrt{a^2 + b^2 + c^2}$ mgw⁺o
 = $(a + b + c)$
 = $(5 + 4\sqrt{2} + \sqrt{17})$ GKK
 = 14.77996 GKK (c⁺lq)

D⁻niY 4 | t⁻lvi th, $(0, -1)$, $(-2, 3)$, $(6, 7)$ Ges $(8, 3)$ we⁻y⁺ t⁻ v GKw⁺ AvqZt⁺q⁺t⁺i Pviw⁺ kxl⁺ e⁻y⁺

gtb Kw⁺i, $A(0, -1)$, $B(-2, 3)$, $C(6, 7)$ Ges $D(8, 3)$ c⁺l E⁻ we⁻y⁺ mgv⁺ | xy mgZtj G⁺t⁻ i Ae⁻vb t⁻ Lv⁺bv ntj v :



w⁺l 11.7

AB ev⁺i $\sqrt{(-2-0)^2 + \{3-(-1)\}^2} = \sqrt{(-2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ GKK |

CD ev⁺i $\sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ GKK |

$\therefore AB \text{ ev\u00fci } \hat{\hat{N}}^\circ = CD \text{ ev\u00fci } \hat{\hat{N}}^\circ$

Avevi,

$AD \text{ ev\u00fci } \hat{\hat{N}}^\circ = \sqrt{(8-0)^2 + (3-(-1))^2} = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5} \text{ GKK}$

$BC \text{ ev\u00fci } \hat{\hat{N}}^\circ = \sqrt{\{6-(-2)\}^2 + (7-3)^2} = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5} \text{ GKK}$

$\therefore AD \text{ ev\u00fci } \hat{\hat{N}}^\circ = BC \text{ ev\u00fci } \hat{\hat{N}}^\circ$

$\therefore \text{wecixZ ev\u00fci } \hat{\hat{N}}^\circ \text{mgvb}$

mZivs ej v hvq, $ABCD$ GKwJ mvgvŠwi K ev AvqZt'ŋŋĪ |

$BD \text{ Kt'Y} \hat{\hat{N}}^\circ = \sqrt{(8-(-2))^2 + (3-3)^2} = \sqrt{(10)^2 + (0)^2} = \sqrt{100} = 10 \text{ GKK}$

GLb, $BD^2 = 100, AB^2 = (2\sqrt{5})^2 = 20, AD^2 = (4\sqrt{5})^2 = 80$

$\therefore BD^2 = AB^2 + AD^2 = 20 + 80 = 100$

$\therefore BD^2 = AB^2 + AD^2$

$\therefore \text{cx_v} \hat{\hat{M}} \text{v} \hat{\hat{t}} \text{mi Dccv} \hat{\hat{A}} \text{b} \hat{\hat{h}} \text{v} \hat{\hat{q}} \text{ } ABC \text{ GKwJ mg} \hat{\hat{t}} \text{KvYx w} \hat{\hat{I}} \text{ f} \hat{\hat{R}} \text{ Ges } \angle BAD \text{ mg} \hat{\hat{t}} \text{KvY}$

mZivs G ōvi v cōyWYZ ntj v th, $ABCD$ GKwJ AvqZt'ŋŋĪ |

D`vniY 5 | t`LvL th, $(-3, -3), (0, 0) \text{ I } (3, 3)$ we`yWZbwU ōvi v tKvb wĪ fR `Zwi Kiv hvq bv | mgravb :

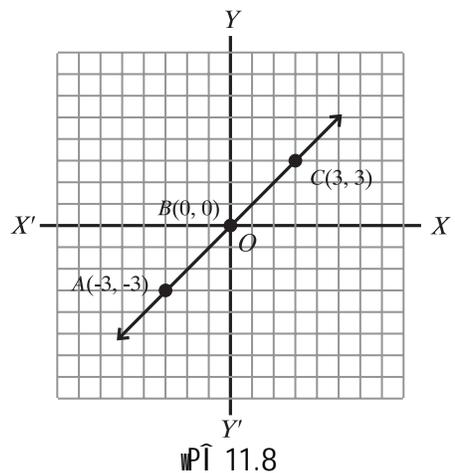
awi, $A(-3, -3), B(0, 0) \text{ I } C(3, 3)$ cŌ Ē we`yngn | xy mgZtj Zvt` i Ae`vb t`Lvfbv ntj v :

Avgiv Rwb, thtKvfbv wĪ fRi `B ev\u00fci mgw\u00f2 ZZxq ev\u00fci Atc'ŋv eo | at\u00fci w\u00f2B ABC GKwJ wĪ fR | $AB, BC \text{ I } AC$ Gi wZbwU ev\u00fci |

GLb, $AB \text{ ev\u00fci } \hat{\hat{N}}^\circ = \sqrt{\{0-(-3)\}^2 + \{0-(-3)\}^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2} \text{ GKK}$

$BC \text{ ev\u00fci } \hat{\hat{N}}^\circ = \sqrt{(3-0)^2 + (3-0)^2} = \sqrt{18} = 3\sqrt{2} \text{ GKK}$

$AC \text{ ev\u00fci } \hat{\hat{N}}^\circ = \sqrt{(3+3)^2 + (3+3)^2} = \sqrt{72} = 6\sqrt{2} \text{ GKK}$



t`Lv hv`Q, $AB + BC = 3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2} = AC$

A_ŋ `B ev\u00fci mgw\u00f2 ZZxq ev\u00fci mgvb |

$\therefore \text{we} \hat{\hat{y}} \text{WZbwU GKB mij } \hat{\hat{t}} \text{L} \text{v} \text{q Ae} \hat{\hat{v}} \text{b K} \hat{\hat{t}} \text{ Ges G} \hat{\hat{t}} \text{ i } \hat{\hat{O}} \text{vi v tKvfbv wĪ fR MvB m} \hat{\hat{a}} \hat{\hat{e}} \text{ b} \hat{\hat{q}} |$

Abkxj bx 11.1

- 1| cÖZt¶¶t cÖË we`mg¶ni ga'eZP` tZ;wbYq Ki |
 (i) (2, 3) | (4, 6) (ii) (-3, 7) | (-7, 3) (iii) (a, b) | (b, a)
 (iv) (0, 0) | (sinθ, cosθ) (v) $\left(-\frac{3}{2}, -1\right)$ | $\left(\frac{1}{2}, 2\right)$
- 2| GKwJ wî f¶Ri kxI ¶q h_vµtg A(2, -4), B(-4, 4) | C(3, 3) | wî f¶Rw A¼b Ki Ges t`Lvl th, GwJ GKwJ mgvðevù wî f¶R |
- 3| A(2, 5), B(-1, 1) | C(2, 1) GKwJ wî f¶Ri kxI ¶q | wî f¶Rw AwK | t`Lvl th GwJ GKwJ mg¶KvYx wî f¶R |
- 4| A(1, 2), B(-3, 5) | C(5, -1) we`j¶q Øviv wî f¶R MVb Ki v hvq wKbv hvPvB Ki |
- 5| gj we`yt_tK (-5, 5) | (5, k) we`Øq mg`ieZPntj k Gi gvb wbYq Ki |
- 6| t`Lvl th, A(2, 2), B(-2, -2) Ges C(-2√3, 2√3) GKwJ mgevù wî f¶Ri kxI ¶q | Gi cwi mxgv wZb`kugK`vb chS`wbYq Ki |
- 7| t`Lvl th, A(-5, 0), B(5, 0), C(5, 5) | D(-5, 5) GKwJ eM¶¶t¶i Pviw kxI ¶q |
- 8| A(-2, -1), B(5, 4), C(6, 7) Ges D(-1, 2) Øviv MwZ PZf¶Rw mgvšwi K bv AvqZt¶¶t Zv wbYq Ki |
- 9| A(10, 5), B(7, 6), C(-3, 5) we`y_tjvi gta` tKvbw P(3, -2) Gi met¶tq wbKUeZP | tKvbw met¶tq `ieZ¶ |
- 10| P(x, y) we`yt_tK y -At¶i `tZ;Ges Q(3, 2) we`j `tZ;mgvb | cÖvY Ki th,
 $y^2 - 4y - 6x + 13 = 0$.

11.3 wî f¶Rt¶¶t i t¶¶t dj (Area of Triangles)

Avgiv Rwb wZbw wfbo we`yGKB mij ti Lvq Ae`vb bv Kijtj H wZbw we`jK mij ti Lv Øviv thwM Kijtj GKwJ wî f¶Rt¶¶t cvl qv hvq | D³ wî f¶Rt¶¶t evùtft` Ges tKvYtft` wfbo wfboentZ cvti | GB Astk Avgiv GKwJ gvT m¶t i mnvth` thtKvbw wî f¶Ri evùt `N°wbY¶qi gva`tg wî f¶Rw i t¶¶t dj wbYq Kijtj m¶tg ne | GKB m¶t i mnvth` thtKvbw PZf¶RtK`Bw wî f¶R t¶¶t wef³ Kti PZf¶R t¶¶t i t¶¶t dj wbYq Kivl mæe nte | Gt¶¶t Avgiv wî f¶R t¶¶t wJi cwi mxgv (evù_tjvi `N¶ mgvð) Ges evùt `N¶

gva'tg t'q'Idj wby'q Kie | th'Kv'tbv w' f'R Av'k'wZ ev t'Kv'Yv'Kw' i R'ngi t'q'Idj wby'q' GB c'x'wZ A_ f' ev'ui ^ N' gva'tg t'q'Idj wby'q AZ'š- i "Zp'Y' A_ f' R'ngi t'q'Idj wby'q' G'w' L'p'B K'v'h'K' i | K'v'Y w'nt'm'te ej'v hv'q w' t'Kv'Yv'K'v' hv' t'P'S'K'v'b'v'K'v' R'ngi k'x'l' e' y' t'j' v' i 'v'b's'K' R'v'b'v' b'v'B ev m'æ' e' b'q w'K'š' h'w' 'v'b's'K' R'v'b'v' v't'K' Z'v'nt'j' Avg'iv' Av' i l' m'nt'R' t'q'Idj wby'q' m'q'lg' ne | GB A's't'k' Avg'iv' ' b'w' c'x'wZ' i gva'tg w' f'R' ev e'uf'f'R' i t'q'Idj wby'q' Kie |

c'x'wZ 1 :

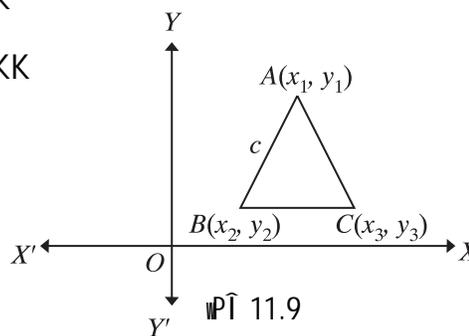
t'q'Idj wby'q' m'f' : cv't'k'p' w'p't' ABC G'k'w' w' f'R' t' L'v't'bv' n't'q't'Q | $A(x_1, y_1), B(x_2, y_2) | C(x_3, y_3)$ w'Z'b'w' w'f'b'w'e' y' Ges $AB, BC | CA$ w' f'f'R' i w'Z'b'w' ev' | ' t'Z' wby'q' m'f' i m'v'n'v't'h' m'nt'R'B

$AB, BC | CA$ ev'ui ^ N' wby'q' m'æ' | th'gb :

AB ev'ui ^ N' 'c' a'ti $c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ GKK

BC ev'ui ^ N' 'a' a'ti $a = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$ GKK

AC ev'ui ^ N' 'b' a'ti $b = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$ GKK



GL'b w' f'R'w' i c'w' m'x'g'v'2s' a'ti f

$2s = a + b + c$ [c'w' m'x'g'v' = ev' w'Z'b'w' ^ N' m'g'w'o']

A_ f' $s = \frac{1}{2}(a + b + c)$ GKK

GL'v't'b s n't'j'v' w' f'f'R' i c'w' m'x'g'v' i A't'a'K' |

Avg'iv' 's' Ges a, b, c Gi m'v'n'v't'h' m'nt'R'B th'Kv't'bv w' f'f'R' i t'q'Idj wby'q' K' i t'Z' c'w' i |

w' f'R' t'q'Idj i t'q'Idj wby'q' m'f' :

w' f'R' ABC Gi AB ev'ui ^ N' 'c', BC ev'ui ^ N' 'a' Ges CA ev'ui ^ N' 'b' Ges c'w' m'x'g'v' '2s'

n't'j' ΔABC Gi t'q'Idj $\sqrt{s(s-a)(s-b)(s-c)}$ e'M'GKK | [gva'w'g'K' 'f' i M'v'YZ R'v'w'g'w'Z' Gi c'w' w'g'w'Z' A's't'k' c'g'v' t' l' q'v' Av't'Q | w'K'v'v'v' c'g'v'Y'w' t' t'L' w't'e |]

w' b't'g'e' D' v'ni Ym'g'f'ni gva'tg m'f'w' i e'env' i m'nt'R'B e'f'v' h'r't'e |

j' q'Y'x'q : w'w'f'b'w'w' f'f'R' i t'q'Idj wby'q' w'w'f'b'w'w' i t'q't'Q, w'K'š' G'k'w' g'v' i m'f' i m'v'n'v't'h' Avg'iv' GL'v't'b th'Kv't'bv w' f'f'R' i t'q'Idj wby'q' m'q'lg' ne |

D' v'ni Y 1 | $A(2, 5), B(-1, 1)$ Ges $C(2, 1)$ G'k'w' w' f'f'R' i w'Z'b'w' k'x'l' e' y' | w' f'f'R'w' i G'k'w' t'g'v'U'v'g'w' w'p't' A'w'K' Ges c'w' m'x'g'v' l' ev'ui ^ N' gva'tg t'q'Idj wby'q' K' i | w' f'f'R'w' i t'K'v'b' a' i t' b' i w' f'R' w'p't' t' t'L' Av'v'R' K' i Ges Z'v' i 'c't'q' h'y' v' l' |

m'g'v'v'b : cv't'k' i w'p't' w' f'f'R'w' i t' L'v't'bv' n't'j' v' :

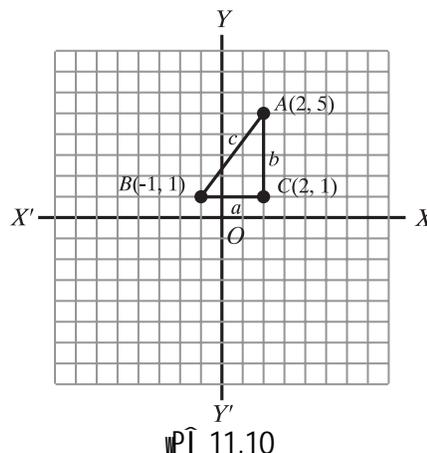
$$AB \text{ evüi } \hat{\hat{N}}^\ominus, c = \sqrt{(-1-2)^2 + (1-5)^2} = \sqrt{9+16} = 5 \text{ GKK}$$

$$BC \text{ evüi } \hat{\hat{N}}^\ominus, a = \sqrt{(2+1)^2 + (1-1)^2} = \sqrt{9+0} = 3 \text{ GKK}$$

$$AC \text{ evüi } \hat{\hat{N}}^\ominus, b = \sqrt{(2-2)^2 + (1-5)^2} = \sqrt{0+16} = 4 \text{ GKK}$$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(5+3+4) = \frac{12}{2} = 6 \text{ GKK}$$

$$\begin{aligned} \therefore \text{t} \hat{\hat{I}} \hat{\hat{d}} \text{j} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{6(6-5)(6-3)(6-4)} \text{ eM}^\ominus \text{GKK} \\ &= \sqrt{6 \cdot 1 \cdot 3 \cdot 2} \text{ eM}^\ominus \text{GKK} \\ &= 6 \text{ eM}^\ominus \text{GKK} \end{aligned}$$



wPÎ t` tL Avgiv eStZ cwii GwU GKwU mgtkvYx wÎ fR|
cx_vfMvi vfmî Dccv t` "i mnvvt h" GwU mn tRB cgvY Kiv hvq

$$AB^2 = c^2 = 5^2 = 25$$

$$BC^2 = a^2 = 3^2 = 9$$

$$CA^2 = b^2 = 4^2 = 16$$

$$\therefore AB^2 = 25 = BC^2 + CA^2 = 9 + 16 = 25$$

$$\therefore ABC \text{ GKwU mgtkvYx wÎ fR| } AB \text{ AwZ fR } | \angle ACB \text{ mgtkvY|}$$

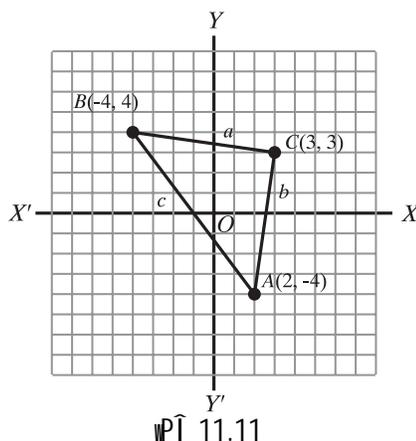
D`vniY 2| A(2, -4), B(-4, 4) Ges C(3, 3) GKwU wÎ fRi wZbwU kxI q wÎ fRwU AwK Ges evüi
N^wY qî gva t g Gi t d j wY q Ki | wPÎ t` tL wÎ fRwU GKwU bvg `vI Ges Gi t` t q h y³
t` Lvl |

mgvavb : ABC wÎ fRwU wPÎ AwKv ntj v :

$$AB = c = \sqrt{(-4-2)^2 + \{4-(-4)\}^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ GKK}$$

$$BC = a = \sqrt{(3-(-4))^2 + (3-4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \text{ GKK}$$

$$CA = b = \sqrt{(2-3)^2 + (-4-3)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2} \text{ GKK}$$



GLb, $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(10+5\sqrt{2}+5\sqrt{2}) = \frac{1}{2}(10+10\sqrt{2}) = 5+5\sqrt{2}$ GKK

$$\begin{aligned} \therefore \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(5+5\sqrt{2})(5+5\sqrt{2}-10)(5+5\sqrt{2}-5\sqrt{2})(5+5\sqrt{2}-5\sqrt{2})} \text{ eM}^\circ\text{GKK} \\ &= \sqrt{(5+5\sqrt{2})(5\sqrt{2}-5) \cdot 5 \cdot 5} \text{ eM}^\circ\text{GKK} \\ &= 5\sqrt{(5\sqrt{2}+5)(5\sqrt{2}-5)} \text{ eM}^\circ\text{GKK} \\ &= 5\sqrt{(5\sqrt{2})^2 - 5^2} = 5\sqrt{50-25} = 5\sqrt{25} \text{ eM}^\circ\text{GKK} \\ &= 5 \cdot 5 = 25 \text{ eM}^\circ\text{GKK} \end{aligned}$$

c0 È wî fRiU GKwU mgw0evû wî fR | tKbbv $BC = CA = 5\sqrt{2}$ GKK | A_ŕ, wî fRiU i `ßU evû mgvb | Avevi, $AB^2 = 10^2 = 100$

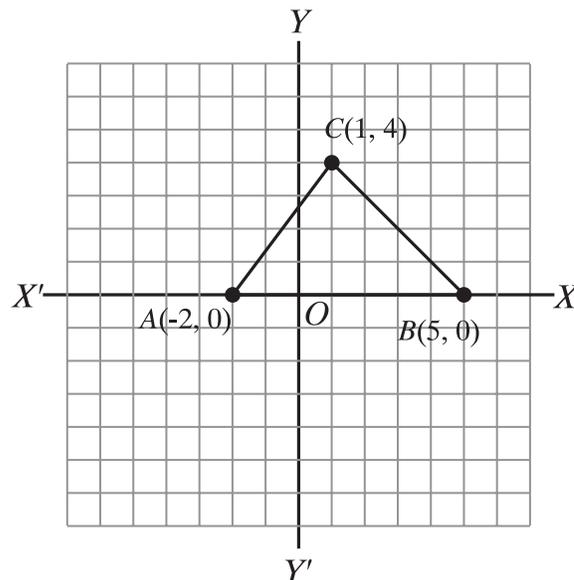
$$BC^2 + CA^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 = 50 + 50 = 100$$

$$\therefore AB^2 = BC^2 + CA^2$$

∴ GwU GKwU mg†KvYx wî fR |

A_ŕ $\triangle ABC$ GKwU mg†KvYx | mgw0evû wî fR |

D`vniY 3 | GKwU wî fRi kxIŕq h_vµ†g $A(-2, 0)$, $B(5, 0)$ Ges $C(1, 4)$ c0Z`KwU evûi `N°wbYŕ Ki Ges wî fRiU i tŕI dj wbYŕ Ki | wî fRiU Kx ai†Yi Abgvb Ki Ges `c†ŕ hy³ `vl |



mgvavb : wî fRiwi (wPÎ : 11.12) wPÎ t` Lvfbv ntjv :

$$AB = c = \sqrt{(5 - (-2))^2 + (0 - 0)^2} = \sqrt{49} = 7 \text{ GKK}$$

$$BC = a = \sqrt{(1 - 5)^2 + (4 - 0)^2} = \sqrt{16 + 16} = 4\sqrt{2} \text{ GKK}$$

$$CA = b = \sqrt{(-2 - 1)^2 + (0 - 4)^2} = \sqrt{9 + 16} = 5 \text{ GKK}$$

$$S = \frac{1}{2}(a + b + c) = \frac{1}{2}(7 + 4\sqrt{2} + 5) = \frac{1}{2}(12 + 4\sqrt{2}) = 6 + 2\sqrt{2} \text{ GKK}$$

$$\begin{aligned} \therefore \text{t}^{\wedge}\text{d}j &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(6 + 2\sqrt{2})(6 + 2\sqrt{2} - 7)(6 + 2\sqrt{2} - 4\sqrt{2})(6 + 2\sqrt{2} - 5)} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{(6 + 2\sqrt{2})(2\sqrt{2} - 1)(6 - 2\sqrt{2})(2\sqrt{2} + 1)} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{(6 + 2\sqrt{2})(6 - 2\sqrt{2})(2\sqrt{2} + 1)(2\sqrt{2} - 1)} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{(6^2 - (2\sqrt{2})^2)((2\sqrt{2})^2 - 1^2)} = \sqrt{28 \cdot 7} = 14 \text{ eM}^{\circ}\text{GKK} \end{aligned}$$

c0 E wî fRiwi GKiw wel gevû wî fRi | Kvi Y Gi tKvfbv evûB Aci tKvb evûi mgvb bq|

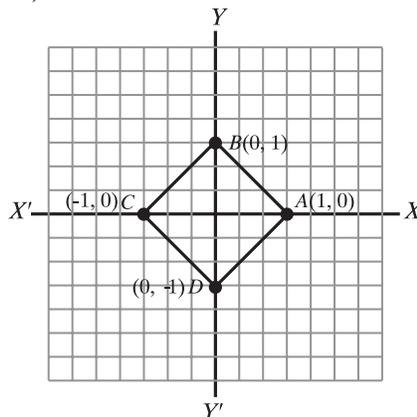
j Yxq : th wZbw wî fRi t^{\wedge}i t^{\wedge}dj wbyq Kiv ntjv Zvi 1gw mgtkvYx, 2qiw mgw0evû I ZZxqiw wel gevû wî fRi | GKiw gvÎ mfi mrvth` c0Z`Kiw wî fRi t^{\wedge}dj wbyq mae ntqtQ | Ab` thtKvfbv wî fRi t^{\wedge}i WK GKbfvte t^{\wedge}dj wbyq Kiv mae nte | Abjxj bxtZ G iKg Avil KtqKiw wî fRi t^{\wedge}dj msuvš-mgmiv_vKte|

G chiq Avgiv PZfRi t^{\wedge}i t^{\wedge}dj GKB mfi e`envi Kti wbyqi tKškj Avtj vPbv Kie|

D`vniY 1 | GKiw PZfRi 4iw kxl`h_vutg A(1, 0), B(0, 1), C(-1, 0) Ges D(0, -1) | PZfRi wPÎ Awk Ges thtKvfbv `B evû I KtYp` N`wbyqi gva'tg Gi t^{\wedge}dj wbyq Ki |

mgvavb : cvtkp wPÎ we`ycvZtbi gva'tg ABCD PZfRi t`Lvfbv ntjv | AB, BC, CD Ges DA PZfRi Pviw evû Ges AC I BD PZfRi `Biu KYq

$$\text{evû } AB = c = \sqrt{(1 - 0)^2 + (0 - 1)^2} = \sqrt{2} \text{ GKK}$$



$$\text{evù } BC = a = \sqrt{(0+1)^2 + (1-0)^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ GKK}$$

$$\text{KY}^{\circ}AC = b = \sqrt{(1+1)^2 + (0-0)^2} = \sqrt{2^2} = 2 \text{ GKK}$$

$$\therefore AC^2 = 4$$

$$\text{evù } CD = c = \sqrt{(-1-0)^2 + (0-1)^2} = \sqrt{2} \text{ GKK}$$

$$\text{evù } DA = \sqrt{(0-1)^2 + (-1-0)^2} = \sqrt{2} \text{ GKK}$$

$$\dagger \text{ Lv hv}^{\circ}Q, AB = BC = CD = DA = \sqrt{2} \text{ GKK}$$

\therefore PZfRw GKw eM^{ev} i p^m |

$$\text{GLb, } AC^2 = AB^2 + BC^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4$$

\therefore PZfRw GKw eM^q |

$$\therefore \text{PZfR } ABCD \text{ Gi } \dagger\ddagger\text{dj} = 2 \times \widehat{\text{w}} \text{ fR } ABC \text{ Gi } \dagger\ddagger\text{dj} |$$

$$\text{GLb } \widehat{\text{w}} \text{ fR } ABC \text{ Gi } \text{cwi mxgv, } 2s = AB + AB + BC = 2 + \sqrt{2} + \sqrt{2} = 2 + 2\sqrt{2} \text{ KK}$$

$$s = \frac{1}{2}(2 + 2\sqrt{2}) = 1 + \sqrt{2} \text{ GKK |}$$

$$\begin{aligned} \therefore \widehat{\text{w}} \text{ fR } ABC \text{ Gi } \dagger\ddagger\text{dj} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(1+\sqrt{2})(1+\sqrt{2}-\sqrt{2})(1+\sqrt{2}-2)(1+\sqrt{2}-\sqrt{2})} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{(\sqrt{2}+1) \cdot 1 \cdot (\sqrt{2}-1) \cdot 1} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{(\sqrt{2})^2 - 1^2} \text{ eM}^{\circ}\text{GKK} \\ &= \sqrt{2-1} \text{ eM}^{\circ}\text{GKK} \\ &= 1 \text{ eM}^{\circ}\text{GKK} \end{aligned}$$

$$\begin{aligned} \therefore \text{PZfR}\dagger\ddagger\text{ } ABCD \text{ Gi } \dagger\ddagger\text{dj} &= 2 \times 1 \text{ eM}^{\circ}\text{GKK} \\ &= 2 \text{ eM}^{\circ}\text{GKK} | \end{aligned}$$

gšē : eM^p evù i ^ˆN^qK eM^qti i [†]†dj cvl qv hvq | AvqZ Gi ^ˆN^oI c^o'_Y Kti i [†]†dj cvl qv hvq | wKš[†]th[†]Kv[†]bv PZfRi [†]†dj wby^q Kiv hvq bv |

D`vniY 2 | A(-1, 1), B(2, -1), C(3, 3) Ges D(1, 6) Øviv MvZ PZfRw A^{1/2}b Kti Gi cizw evù I GKw KtY^p ^ˆN^owby^q Ki Ges PZfRw i [†]†dj wby^q Ki |

mgvavb : we`ycvZtbi gva'tg xy-mgZtj ABCD PZfRwU t`Lv'tbv ntjv :

ABCD PZfRwU

evu, $AB = a = \sqrt{3^2 + 2^2} = \sqrt{13}$ GKK

evu, $BC = b = \sqrt{1^2 + 4^2} = \sqrt{17}$ GKK

evu, $CD = d = \sqrt{2^2 + 3^2} = \sqrt{13}$ GKK

evu, $DA = e = \sqrt{2^2 + 5^2} = \sqrt{29}$ GKK

KY^o, $AC = c = \sqrt{(3+1)^2 + (3-1)^2}$ GKK

$= \sqrt{4^2 + 2^2} = \sqrt{20}$ GKK

wi fR ΔABC G $2s = a + b + c = \sqrt{13} + \sqrt{17} + \sqrt{20}$ GKK

$= 12 \cdot 2008 = 3 \cdot 6056 + 4 \cdot 1231 + 4 \cdot 472$ GKK

$\Rightarrow s = 6 \cdot 1004$ GKK

wi fR ABC Gi t'qit' dj = $\sqrt{s(s-a)(s-b)(s-c)}$ eM^oGKK

$= \sqrt{6 \cdot 1004 \times 2 \cdot 4948 \times 1 \cdot 9773 \times 1 \cdot 6283}$ eM^oGKK

$= \sqrt{49 \cdot 000}$ eM^oGKK

$= 7$ eM^oGKK

ΔACD G $2s = c + d + e = \sqrt{20} + \sqrt{13} + \sqrt{29}$ GKK

$= 4 \cdot 4721 + 3 \cdot 6056 + 5 \cdot 3852$ GKK

$= 13 \cdot 4429$ GKK

$\therefore s = 6 \cdot 7312$ GKK |

$\therefore \Delta ACD$ Gi t'qit' dj = $\sqrt{s(s-e)(s-d)(s-c)}$

$= \sqrt{6 \cdot 7312 \times 2 \cdot 2591 \times 3 \cdot 1256 \times 1 \cdot 3460}$ eM^oGKK

$= \sqrt{63 \cdot 9744}$ eM^oGKK

$= 7 \cdot 9983$ eM^oGKK

$\therefore ABCD$ PZfR t'qit' i t'qit' dj = $(7 \cdot 000 + 7 \cdot 998)$ eM^oGKK

$= 14 \cdot 998$ eM^oGKK

$= 15$ eM^oGKK (c'q) |

gSe` : ABCD PZfRwU eM^oev AvqZ ev mgvS^hi K ev i s'm t'Kv'tbvUB bq | G ai'tbi welg AvKv'ti i Rngi t'qit' dj wby'q G c'wZ AZ`S-KvhRi |

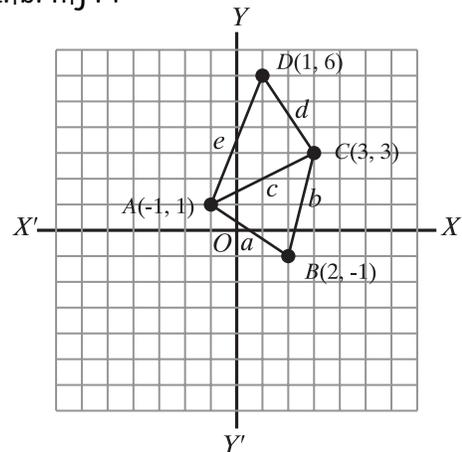
D`vniY 3 | PviwU we`j `vbw¼ h_vu'tg A(2, -3), B(3, 0), C(0, 1) Ges D(-1, -2)

(a) t`Lvl th, ABCD GKwU i s'm |

(b) AC I BD Gi `N^owby'q Ki Ges ABCD GKwU eM^oKbv hvPvB Ki

(c) wi fR t'qit' i gva'tg PZfRwU t'qit' dj wby'q Ki |

mgvavb : ABCD PZfRwU we`ycvZtbi gva'tg wP^t : 11.1.5 G t`Lv'tbv ntjv :



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(a) awi a, b, c, d h_vµtǵ AB, BC, CD Ges DA evüi $\hat{\hat{N}}^{\circ}$ Ges $KY^{\circ}AC = e$ | $KY^{\circ}BD = f$.

$$\text{Zvntǵ} , a = \sqrt{(3-2)^2 + (0+3)^2} = \sqrt{1^2 + 3^2} = \sqrt{10} \text{ GKK}$$

$$b = \sqrt{(0-3)^2 + (1-0)^2} = \sqrt{3^2 + 1^2} = \sqrt{10} \text{ GKK}$$

$$c = \sqrt{(-1-0)^2 + (-2-1)^2} = \sqrt{1^2 + 3^2} = \sqrt{10} \text{ GKK}$$

$$d = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{3^2 + 1^2} = \sqrt{10} \text{ GKK}$$

thñZl $a = b = c = d = \sqrt{10}$ GKK

$\therefore ABCD$ GKµW i µñ |

$$KY^{\circ}AC = e = \sqrt{(0-2)^2 + (1+3)^2} = \sqrt{4+16} = \sqrt{20} \text{ GKK}$$

$$\text{Ges } KY^{\circ}BD = f = \sqrt{(-1-3)^2 + (-2-0)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} \text{ GKK}$$

\therefore t`Lv hvñ"Q $AC = BD$ A_µñ, $KY^{\circ}q$ mgvb

$$AC^2 = (\sqrt{20})^2 = 20$$

$$AB^2 + BC^2 = (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$$

$$AC^2 = AB^2 + BC^2.$$

\therefore cx_vñMvi vñmi Dccv` Abhvqx $\angle ABC$ mgñKvY |

\therefore PZfRµW GKµW eM^q

$\therefore ABCD$ GKµW eM^q

PZfR $ABCD$ Gi tñññ dj = $2 \times$ wñ fR ABC Gi tñññ dj
GLvñb $\triangle ABC$ Gi tñññ

$$\begin{aligned} s &= \frac{a+b+c}{2} = \frac{\sqrt{10} + \sqrt{10} + \sqrt{20}}{2} \\ &= \frac{2\sqrt{10} + 2\sqrt{5}}{2} = \sqrt{10} + \sqrt{5} \text{ GKK} | \end{aligned}$$

$\therefore \triangle ABC$ Gi tñññ dj = $\sqrt{s(s-a)(s-b)(s-f)}$

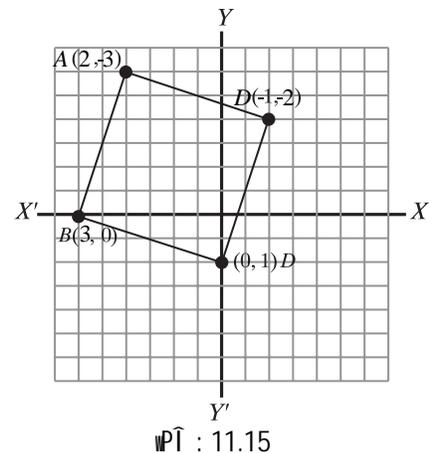
$$= \sqrt{(\sqrt{10} + \sqrt{5})(\sqrt{10} + \sqrt{5} - \sqrt{10})(\sqrt{10} + \sqrt{5} - \sqrt{10})(\sqrt{10} + \sqrt{5} - \sqrt{20})} \text{ eM}^{\circ}\text{GKK}$$

$$= \sqrt{(\sqrt{10} + \sqrt{5}) \cdot \sqrt{5} \cdot \sqrt{5} (\sqrt{10} - \sqrt{5})} \text{ eM}^{\circ}\text{GKK}$$

$$= \sqrt{5 \{(\sqrt{10})^2 - (\sqrt{5})^2\}} = \sqrt{5 \cdot (10 - 5)} \text{ eM}^{\circ}\text{GKK}$$

$$= \sqrt{5 \cdot 5} = 5 \text{ eM}^{\circ}\text{GKK}$$

$\therefore ABCD$ eñM^q tñññ dj = 2×5 eM^qGKK = 10 eM^qGKK |



gše : mnR c×wZ : ABCD eMŪi tñŪdj $(\sqrt{10})^2 = 10$ eM©GKK |

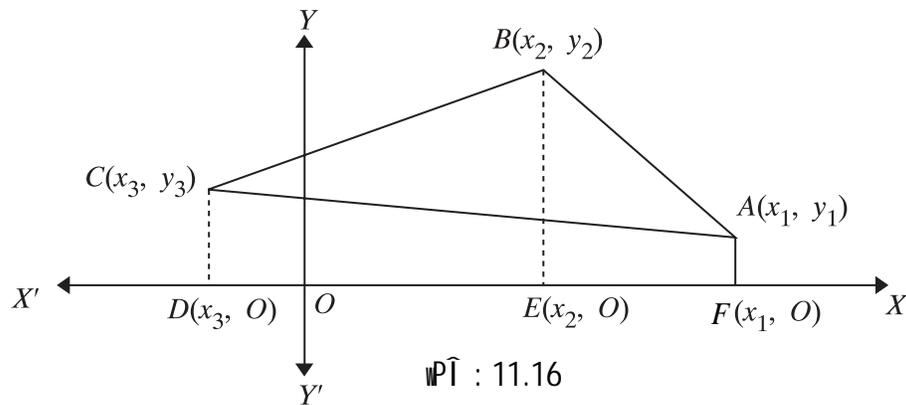
wŪ fR tñŪi tñŪdj wBYŪ

c×wZ 2 : kxŪj 'vbstKi mnvth' tñŪdj wBYŪ :

GB c×wZtZ GKwŪ wŪ fRi wZbŪ kxŪj 'vbstKi mnvth' Lp mntRB wŪ fRtñŪi tñŪdj wBYŪ Kiv hvq | GKBFvte thKvtbv eufRi kxŪj 'vbstK Rvbn vKtj | eufRi tñŪdj wBYŪ mæe | ev'e tñŪŪ GB c×wZ e'envi Kiv mæe nq bv | Gi KviY ntjv Avgiv hw GKwŪ Rngi tñŪdj wBYŪ KitZ PvB Ges RngwŪ hw wŪ tKvYvKvi ev tPstKvYvKvi nq Zvntj GB c×wZtZ tñŪdj wBYŪ Kiv hvte bv | thtnZt tKšwYK wŪ mgñi 'vbn¼ Avgv' i Rvbn bvB ev Rvbn mæe bq | wKš' RngwŪi evüi 'N' Avgiv mntRB tçtç wbtZ cwi Ges 1 bs c×wZtZ tñŪdj wBYŪ KitZ cwi | ZvB Dfç c×wZ mæutKkŪv' i aviYv vKv cŪqvRb | 2 bs c×wZi mnvth' wŪ fR | eufRi tñŪdj wBYŪqi tKškj D`vniYi mnvth' evLv Kiv ntjv :

wŪ fRtñŪi tñŪdj wBYŪqi mvaviY mŪ :

awi , $A(x_1, y_1)$, $B(x_2, y_2)$ Ges $C(x_3, y_3)$ wŪ fR ABC Gi wZbŪ kxŪj | wPŪ 11.15 Gi Abjç $A, B | C$ wŪ yNwoi KwŪvi wecixZ wŪ tK mvRvtbv |



wPŪ t_ŪK Avgiv cvB,

$$\begin{aligned} \text{eufR } ABCDF \text{ Gi tñŪdj} &= \text{wŪ fRtñŪ } ABC \text{ Gi tñŪdj} + \text{UñcwRqvgtñŪ } ACDF \text{ Gi tñŪdj} \\ &= \text{UñcwRqvgtñŪ } ABEF \text{ Gi tñŪdj} + \text{UñcwRqvgtñŪ } BCDE \text{ Gi tñŪdj} \end{aligned}$$

mŪZivs Avgiv cvB,

$$\text{wŪ fRtñŪ } ABC \text{ Gi tñŪdj} = \text{UñcwRqvgtñŪ } ABEF \text{ Gi tñŪdj} + \text{UñcwRqvgtñŪ } BCDE \text{ Gi tñŪdj} - \text{UñcwRqvgtñŪ } ACDF \text{ Gi tñŪdj}$$

∴ wŪ fRtñŪ ABC Gi tñŪdj

$$\begin{aligned} &= \frac{1}{2} \times (BE + AF) \times EF + \frac{1}{2} \times (CD + BE) \times DE - \frac{1}{2} \times (CD + AF) \times DF \\ &= \frac{1}{2} (y_2 + y_1)(x_1 - x_2) + \frac{1}{2} (y_3 + y_2)(x_2 - x_3) - \frac{1}{2} (y_3 + y_1)(x_1 - x_3) \end{aligned}$$

$$= \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$$

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ eM}^\text{GKK}$$

thLvfb,

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = (x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$$

thLvfb „Ydtj i w`K ↘ abvZK wPy wntmte wbtq cvl qv tMtQ $x_1y_2 + x_2y_3 + x_3y_1$ Ges „Ydtj i w`K ↗ FYvZK wPy wntmte wbtq cvl qv tMtQ $-x_2y_1 - x_3y_2 - x_1y_3$

mYzivs, ΔABC Gi t¶¶dj = $\frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

gše“ : gtb i vLv AZ“š-, i “ZpYth G c×wZtZ t¶¶dj wby¶qi t¶¶t we`yng¶ni vbvsk

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ Aek`B Nnoi Kulvi wecixZ w`tK wbtZ nte}$$

D`vniY 1| $A(2, 3)$, $B(5, 6)$ Ges $C(-1, 4)$ kxl¶wko ABC w¶f¶Ri t¶¶dj wby¶ Ki |

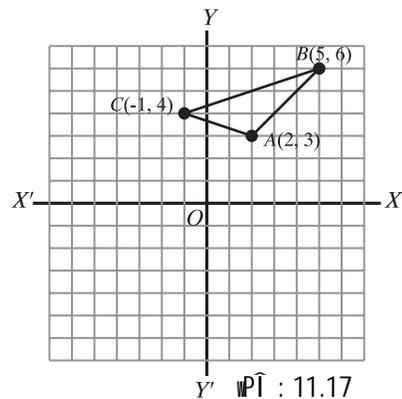
mgvavb : $A(2, 3)$, $B(5, 6)$ Ges $C(-1, 4)$ kxl¶wZbw¶tK Nnoi Kulvi wecixZ w`tK tbl qv ntj v|

$$\Delta ABC \text{ Gi t¶¶dj} = \frac{1}{2} \begin{vmatrix} 2 & 5 & -1 & 2 \\ 3 & 6 & 4 & 3 \end{vmatrix} \text{ eM}^\text{GKK}$$

$$= \frac{1}{2}(12 + 20 - 3 - 15 + 6 - 8) \text{ eM}^\text{GKK}$$

$$= \frac{1}{2}(12) \text{ eM}^\text{GKK}$$

$$= 6 \text{ eM}^\text{GKK}$$



Y' w¶t : 11.17

D`vniY 2| GKw¶ w¶f¶Ri wZbw¶ kxl¶A(1, 3), B(5, 1) Ges C(3, r) | ΔABC Gi t¶¶dj 4 eM^GKK ntj 0r¶Gi m¶¶e` gvbmg¶ wby¶ Ki |

mgvavb : $A(1, 3)$, $B(5, 1)$ Ges $C(3, r)$ kxl¶wZbw¶ Nnoi Kulvi wecixZ w`tK wetePbv Kti ΔABC Gi t¶¶dj

$$= \frac{1}{2} \begin{vmatrix} 1 & 5 & 3 & 1 \\ 3 & 1 & r & 3 \end{vmatrix} \text{ eM}^\text{GKK}$$

$$= \frac{1}{2}(1+5r+9-15-3-r) \text{ eM}^{\text{GKK}}$$

$$= \frac{1}{2}(4r-8) = (2r-4) \text{ eM}^{\text{GKK}}$$

ckg†Z, $(2r-4) = \pm 4$

A_ŕ, $2r = 0$ ev, 8

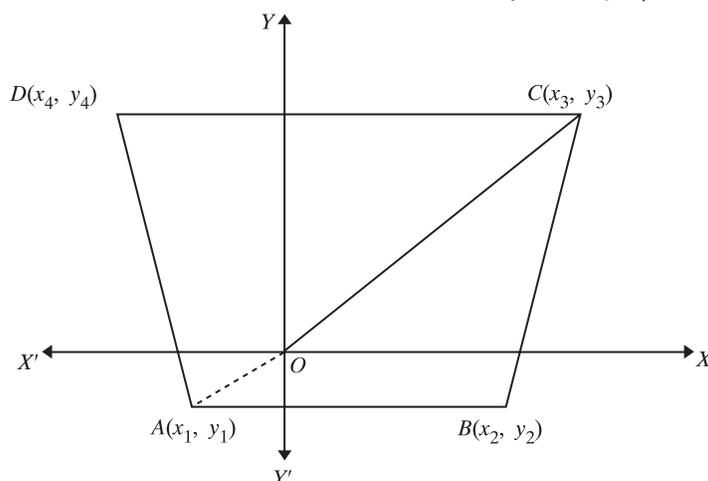
$\therefore r = 0$ ev, 4

DĒi : $r = 0, 4$

PZfR†ŕ†Ī i †ŕ†Ī dj

wPĪ 11.17 G ABCD GKŪ PZfR | PZfRŪi PviŪ kxl^h vµtg $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$

Ges $D(x_4, y_4)$ Ges A, B, C, D †K Nwoi Kulvi mecixZ w` K Abjv†i ntq†Q |



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GLb PZfR†ŕ†Ī ABCD Gi †ŕ†Ī dj

= w†fR†ŕ†Ī ABC Gi †ŕ†Ī dj + w†fR†ŕ†Ī ACD Gi †ŕ†Ī dj

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} x_1 & x_3 & x_4 & x_1 \\ y_1 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1x_3)$$

$$+ \frac{1}{2}(x_1y_3 + x_3y_4 + x_4y_1 - x_3y_1 - x_4y_3 - x_1y_4)$$

$$= \frac{1}{2}x_1y_2 + x_2y_3 + x_4y_4 + x_4y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4)$$

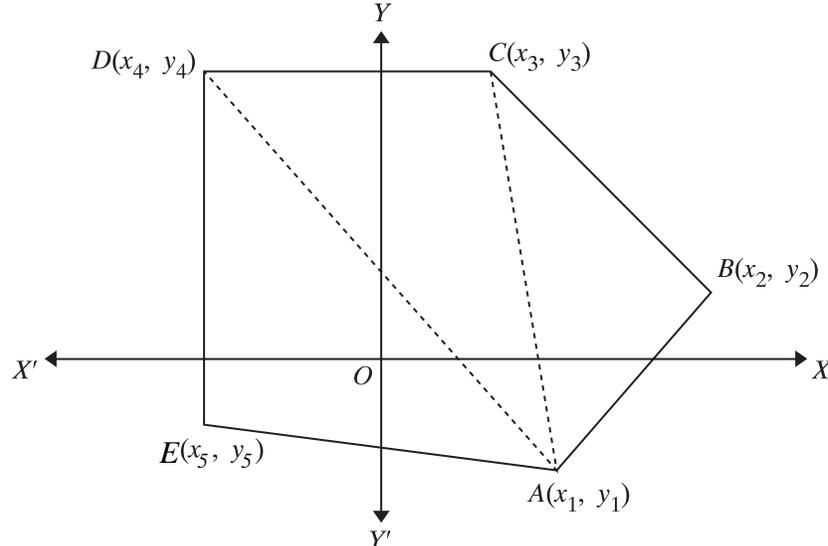
$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$\text{m}^{\text{Z}}\text{i vs PZfRt}^{\text{q}}\text{ ABCD Gi t}^{\text{q}}\text{d} = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

Abjfcvte GKwJ cÅfR ABCDE (wPÎ : 11.18)Gi kxl e`y,tjv hw` A(x₁, y₁), B(x₂, y₂), C(x₃, y₃), D(x₄, y₄) | E(x₅, y₅) nq Ges wPÎ i gZ kxl e`y,tjv hw` Nnoi KulUvi wecixZ w` tK nq, Zte cÅfR ABCDE Gi t^qdj wZbwU wÎ fR t^q ABC, ACD | ADE Gi t^qd t^q i mgwó i mgvb | wÎ fR t^q | PZfR t^q i wK Abjfcvte cÅfR t^q ABCDE Gi t^qdj

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{vmatrix}$$

GKbfvte thtKvfbv eufRi kxl e`yngtñi vbsK Rvbv vKtj mntRB Dctiv³ c×wZtZ t^qdj wBYe Kiv hvq |



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KvR : PZfR t^q i t^qdj wBYeqi c×wZi mwnvth` cÅfR | lofR t^q i t^qdj wBYeqi m` cÅZcv` b Ki |

D`vniY 3 | A(1, 4), B(-4, 3), C(1, -2) Ges D(4, 0) kxl e`wko PZfR t^q ABCD Gi t^qdj wBYe Ki |

mgvavb : we`yngtñK Nnoi KulUvi wecixZ w` tK wbtq PZfR t^q ABCD Gi t^qdj

$$\begin{aligned} &= \frac{1}{2} \begin{vmatrix} 1 & -4 & 1 & 4 & 1 \\ 4 & 3 & -2 & 0 & 4 \end{vmatrix} \text{ eM}^{\text{GKK}} \\ &= \frac{1}{2} (3+8+0+16+16-3+8-0) \text{ eM}^{\text{GKK}} \\ &= \frac{1}{2} (48) = 24 \text{ eM}^{\text{GKK}} \end{aligned}$$

Abkxj bx 11.2

- 1| $A(-2, 0)$, $B(5, 0)$, $C(1, 4)$ h_vµtḡ ΔABC Gi kxl 9e`y|
 (i) AB , BC Ges CA evüi %N°Ges ΔABC Gi cwi mxgv wbyḡ Ki |
 (ii) wî fRwUi tḡĪdj wbyḡ Ki ;
- 2| wbtḡe³ cĪZtḡĪĪ ABC wî fḡRi tḡĪdj wbyḡ Ki :
 (i) $A(2, 3)$, $B(5, 6)$ Ges $C(-1, 4)$;
 (ii) $A(5, 2)$, $B(1, 6)$ Ges $C(-2, -3)$;
- 3| t`Lvl th, $A(1, 1)$, $B(4, 4)$, $C(4, 8)$ Ges $D(1, 5)$ we`y,tjv GKwU mvgvšwi tḡKi kxl 9e`y|
 $AC \perp BD$ evüi ^N°wbyḡ Ki | mvgvšwi KwUi tḡĪdj wî fḡRi gva'tḡ wZb `kvgK `vb chS
 wbyḡ Ki |
- 4| $A(-a, 0)$, $B(0, -a)$, $C(a, 0)$ Ges $D(0, a)$ kxl 9e`kó $ABCD$ PZfRwUi tḡĪdj KZ ?
- 5| t`Lvl th, $(0, -1)$, $(-2, 3)$, $(6, 7)$ Ges $(8, 3)$ we`y,tjv GKwU AvqZtḡĪĪi PwiwU kxl 9
 KYḡtḡi ^N°Ges AvqZwUi tḡĪdj wbyḡ Ki |
- 6| wZbwU we`j `vbr¼ h_vµtḡ $A(-2, 1)$, $B(10, 6)$ Ges $C(a, -6)$ | $AB = BC$ ntḡ a Gi mæte`
 gvbmgḡ wbyḡ Ki | 'a' Gi gvḡbi mḡvḡth` th wî fR MwZ nq Gi tḡĪdj wbyḡ Ki |
- 7| A, B, C wZbwU we`j `vbr¼ h_vµtḡ $A(a, a+1)$, $B(-6, -3)$ Ges $C(5, -1)$ |
 AB Gi ^N° AC Gi ^N° wḡy ntḡ 'a' Gi mæte` gvb Ges wî fRwUi 9e`kó wbyḡ Ki |
- 8| wbtḡe³ PZfRmgḡni tḡĪdj wbyḡ Ki [c×wZ 2 e`envi Ki] :
 (i) $(0, 0)$, $(-2, 4)$, $(6, 4)$, $(4, 1)$;
 (ii) $(1, 4)$, $(-4, 3)$, $(1, -2)$, $(4, 0)$;
 (iii) $(1, 0)$, $(-3, -3)$, $(4, 3)$, $(5, 1)$;

- 9) t`Lvl th, $A(2, -3), B(3, -1), C(2, 0), D(-1, 1)$ Ges $E(-2, -1)$ kxl eukó eũfRi t`I d j 11 eM GKK |
- 10) GKW PZ fRi PviW kxl $A(3, 4), B(-4, 2), C(6, -1)$ Ges $D(p, 3)$ Ges kxl fgn Nwoi KuLvi weciX w` tK AvenZ | $ABCD$ PZ fRi t`I d j w` fR ABC Gi t`I d j i w` Y ntj P Gi gvb wBY q Ki |

11.4 mij ti Lvi Xvj (Gradient or slope of a line)

vbv¼ R`wgnZi (Coordinate Geometry) GB Astki c`tg Avgiv mij ti Lvi Xvj (Gradient or slope) ej tZ Kx eSvq Ges mij ti Lvi Xvj wBY q i tKŠkj Avtj vPbv Kie | Xvtj i aviYv e`envi Kti GKW mij ti Lvi exRMwYwZK ijc Kx nq Zv Avtj vPbv Kiv nte | tKv tlv mij ti Lv `Bw we` yw` t q AwZµg Kti t tmB mij ti Lvi Xvtj i cKwZ I D^3 mij ti Lvi mgxKiY wBY q KivB gj Z GB Astki gj Avtj vPbv i w` q | `Bw mij ti Lv tKv tlv we` tZ wgvj Z ntj ev tQ` Kti t tmB tQ` we` y `vbv¼ wBY q i gva`tg wZbwU mgxKiY Øviv w t` KZ ti Lvi gva`tg MwZ w` fR w t q | Avtj vPbv Kiv nte |

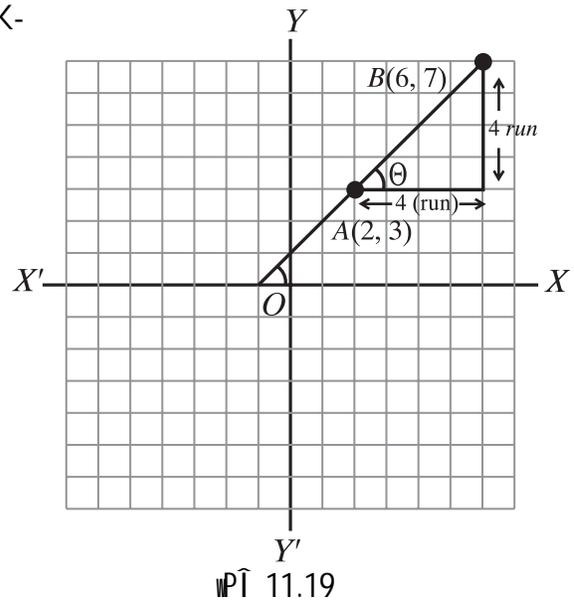
GLv tlv Avgiv t`Lve exRMwY tZ `B Pj tKi GKvZ mgxKiY mij ti Lv w t ` R Kti Ges Zv t` i mgvavb ntj v tmB tQ` we` y |

Xvj (Gradient or slope)

wP 11.19 G AB mij ti LwU wetePbv Kwi | ti LwU $A(2, 3)$ I $B(6, 7)$ `yU we` yw` t q AwZµg Kti tQ | wP t vbyv t i ti LwU x A t` i avZK w` tKi mv t θ tKvY DrcbæKti tQ | GB tKvY θ ntj v Abf w gK x -A t` i mv t AB mij ti LwU Kx cwi gvY AvbZ ntq tQ Zvi cwi gvC | vbv¼ R`wgnZ tZ Avgiv AB ti Lvi Xvj (Gradient) m tK w t g e³ fv t e cwi gvC Kti w K-

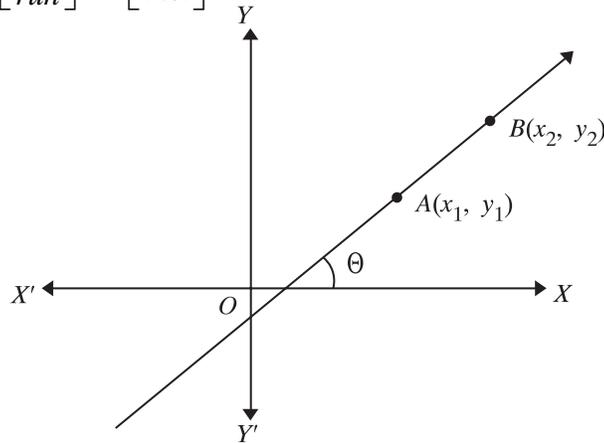
$$m = \frac{y \text{ vbstKi cwi eZ}}{x \text{ vbstKi cwi eZ}} = \frac{7-3}{6-2} = \frac{4}{4} = 1$$

$\therefore AB$ ti Lvi Xvj (m) = 1.



mvaviYZ, GKUW mij ti Lv AB hLb $A(x_1, y_1)$ I $B(x_2, y_2)$ we`y w` tq AwZµg Kti ZLb Gi Xvj (m) tK Avgiv

$$m = \frac{y_2 - y_1}{x_2 - x_1} \left[\frac{\text{rise}}{\text{run}} \right] = \left[\frac{\text{I Vv}}{\text{nuUv}} \right] \text{ Øviv cKvk Kti } _ \text{vK}$$



ev weKct¶, tKvtbv mij ti Lv Øviv x At¶i avZµK w` tKi mvt_ DrcbætKvY Ø I Xvj m Gi gta` m`úK n`j v, $m = \tan \theta$

¶ 11.19 G AB ti Lvi t¶i¶ mij ti Lvi Xvj $m = 1$ A_¶, $\tan \theta = 1$

ev, $\theta = 45^\circ$ (GKUW m`¶KvY)

D`vniY 1 | wbtgæ cZ¶¶i¶ wbt`KZ we` Øq Øviv AwZµvš-mij ti Lvi Xvj wBY¶ Ki |

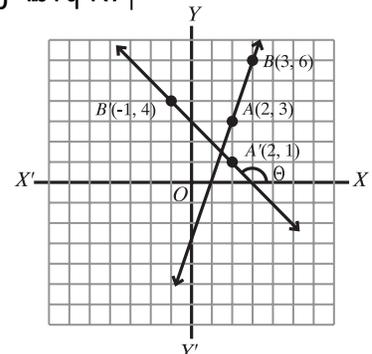
(a) $A(2, 3)$ Ges $B(3, 6)$

(b) $A'(2, 1)$ Ges $B'(-1, 4)$

mgravb :

(a) AB ti Lvi Xvj $= \frac{\text{rise}}{\text{run}} = \frac{\text{I Vv}}{\text{nuUv}} = \frac{6-3}{3-2} = \frac{3}{1} = 3$

(b) $A'B'$ ti Lvi Xvj $= \frac{\text{I Vv}}{\text{nuUv}} = \frac{4-1}{-1-2} = \frac{3}{-3} = -1$



চিত্র : ১১.২০

j ¶Yxq : ¶ 11.20 t_¶K t`Lv hv¶`0, AB ti Lvi Xvj (Gradient) avZµK Ges DrcbætKvY GKUW m`¶KvY | Avevi, GKB ¶ 11.20 t_¶K GuU cwí `vi th A'B' ti Lvi Xvj FYvZµK Ges DrcbætKvY GKUW `j¶KvY |

mZivs Dctiv³ Avtj vPbv t_¶K Avgiv GB wmv¶š-Avm¶Z cwí th, Xvj avZµK n`j ti Lv Øviv x-At¶i avZµK w` tKi mvt_ DrcbætKvb m`¶KvY Ges Xvj FYvZµK n`j ti Lv Øviv x-At¶i avZµK w` tKi mvt_ DrcbætKvY GKUW `j¶KvY |

DrcbætKvY kb` A_ev mg¶KvY n`j Xvj wK nte Zv wbtgæ³ D`vni¶Yi mrvvth` e`vL`v Kiv n`j v :

D`vniY 2| A, B Ges C wZbWU we`j `vbw¼ h`vµtg (2, 2), (5, 2) Ges (2, 7) | KvZñxq Ztj AB | AC tiLv A¼b Ki | mæe ntj AB | AC tiLvi Xvj wbYq Ki |

mgvavb : KvZñxq Ztj AB | AC tiLv A¼b Kiv ntjv :
 wPÎ t`tk t`Lv hvq th, AB tiLv x-Atñi mgvš+vj Ges
 AC tiLv y-Atñi mgvš+vj | AB tiLvi Xvj ,

$$m = \frac{I\ V}{n\ Uv} = \frac{2-2}{5-2} = \frac{0}{3} = 0$$

AC tiLvi Xvj $m = \frac{y_2 - y_1}{x_2 - x_1}$ mF Øviv wbYq Kiv hvte bv, Kvi Y

$$x_1 = x_2 = 2 \text{ Ges } x_2 - x_1 = 0$$

hw` $x_1 = x_2$ nq Zte tiLvi Xvj wbYq Kiv hvq bv wKŠ' tiLwU y-Atñi mgvš+vj nq|

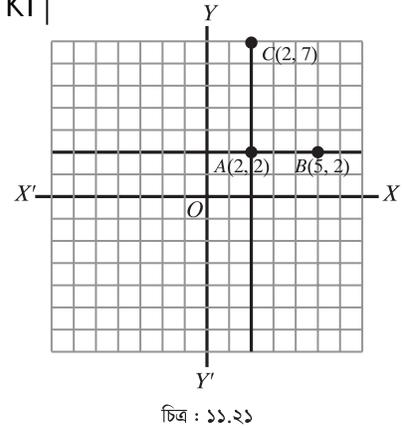
mvaviYZ tKvfbv mij tiLv $A(x_1, y_1)$ | $B(x_2, y_2)$ we` yw` tq AwZµg Ki tj

$$Xvj, m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{ev, } m = \frac{y_1 - y_2}{x_1 - x_2} \text{ hw` } x_1 \neq x_2 \text{ nq|}$$

jñ Kwi : hw` $x_1 = x_2$ nq, Zvntj tiLwU y-Atñi mgvš+vj A_ñ x-Atñi Dci j æ^nq| GB iKg j æ^ tiLv eivei ev Lvov tiLv eivei nuUv mæe bq| ZvB Xvj wbYq Kivi mæe bq|

gŠe` : wPÎ 11.21 G AB tiLvi thtKvfbv we` jZ tkwU A_ñ, $y = 2$ Ges AC tiLvi thtKvfbv we` jZ fR A_ñ, $x = 2$ ZvB AB mij tiLvi mgxKiY $y = 2$ Ges AC mij tiLvi mgxKiY $x = 2$ |



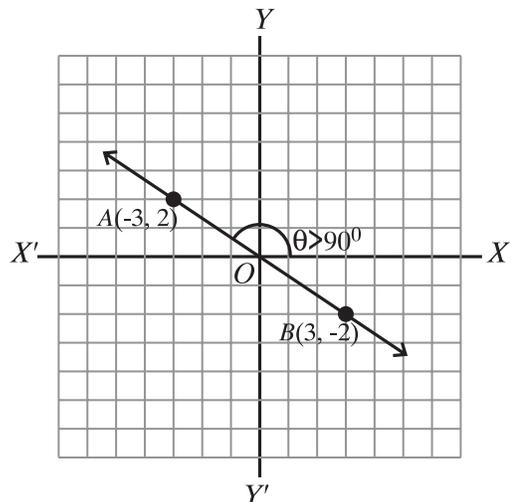
चित्र : ११.२१

D`vniY 3| $A(-3, 2)$ Ges $B(3, -2)$ we` yw` tq AwZµgKvix tiLvi Xvj wbYq Ki |

mgvavb : AB tiLvi Xvj m ntj ,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{I\ V}{n\ Uv} = \frac{2 - (-2)}{-3 - 3} = \frac{4}{-6} = -\frac{2}{3}$$

Xvj FYvZñK nI qvq tiLwU x-Atñi abvZñK
 w`tki mv`_ `j tkvY DrcbæKti tQ|



चित्र : ११.२२

D`vniY 4| $A(1, -1)$, $B(2, 2)$ Ges $C(4, t)$ we` ywZbWU
 mgtiL ntj t Gi gv b KZ ?

mgvarb : $A, B \mid C$ mgfiL nlqvq $AB \mid BC$ tiLvi Xvj GKB nte| mZivs,Avgiv cvB-

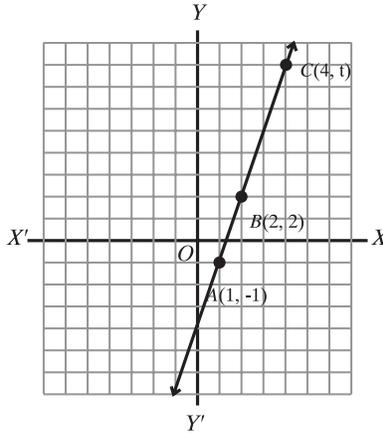
$$\frac{2+1}{2-1} = \frac{t-2}{4-2}$$

ev, $\frac{3}{1} = \frac{t-2}{2}$

ev, $t-2 = 6$

ev, $t = 8.$

mZivs t Gi gvb 8 |



চিত্র : ১১.২০

D`vniY 5| $A(t, 3t), B(t^2, 2t), C(t-2, t)$ Ges $D(1,1)$ PviwU wfbome>`y| AB Ges CD tiLv mgvšivj ntj t Gi m`te` gvb wbyq Ki |

mgvarb : AB tiLvi Xvj $m_1 = \frac{2t-3t}{t^2-t} = \frac{-t}{t(t-1)} = \frac{-1}{1-t}.$

$$CD \text{ tiLvi Xvj } m_2 = \frac{1-t}{1-t+2} = \frac{1-t}{3-t}.$$

$AB \mid CD$ tiLv mgvšivj etj , $AB \mid CD$ tiLvi Xvj mgvb A`fr, $m_1 = m_2$

ev, $\frac{-1}{1-t} = \frac{1-t}{3-t}.$

ev, $(1-t)^2 = -(3-t)$

ev, $1-2t+t^2 = -3+t$

ev, $t^2 - 3t + 4 = 0$

ev, $t = -1$ Ges 2

mZivs t Gi m`te` gvbmgñ $-1, 2$

Abkxj bx 11.2

1| wbtgub` cizwU t`f`f` A \ B we>` Mvgx mij fiLvq Xvj wbyq Ki |

(K) $A(5, -2)$ Ges $B(2, 1)$

(L) $A(3, 5)$ Ges $B(-1, -1)$

(M) $A(t, t)$ Ges $B(t^2, t)$

(N) $A(t, t+1)$ Ges $B(3t, 5t+1)$

- 2| $wZbwU wfbwew`y A(t, 1), B(2, 4)$ Ges $C(1, t)$ mgtiL ntj t Gi gvb wbY@ Ki |
- 3| $f`Lvl th, A(0, -3), B(4, -2)$ Ges $C(16, 1)$ we`y wZbwU mgtiL |
- 4| $A(1, -1), B(t, 2)$ Ges $C(t^2, t+3)$ mgtiL ntj t Gi m`e` gvb wbY@ Ki |
- 5| $A(3, 3p)$ Ges $B(4, p^2 + 1)$ we`y Mvgx tiLvi Xvj -1 ntj P Gi gvb wbY@ Ki |
- 6| $cgvY Ki th, A(a, 0), B(0, b)$ Ges $C(1, 1)$ mgtiL nte, hw` $\frac{1}{a} + \frac{1}{b} = 1$ nq |
- 7| $A(a, b), B(b, a)$ Ges $C(\frac{1}{a}, \frac{1}{b})$ mgtiL ntj $cgvY Ki th, a + b = 0$.

11.5 mij tiLvi mgxKiY :

awi , GKwU vbow` @ mij tiLvi 'L' `BwU vbow` @ we`y $A(3, 4)$ Ges $B(5, 7)$ w` tq AwZµg Kti | wPÎ 11.24 G tiLwU t` Lvfbv ntj v |

Zvntj AB mij tiLvi Xvj $m_1 = \frac{7-4}{5-3} = \frac{3}{2}$ (1)

gtb KwI , $P(x, y)$ mij tiLvi L Gi Dci GKwU we`y | Zvntj AP tiLvi Xvj

$$m_2 = \frac{y-4}{x-3} \dots\dots\dots(2)$$

wKŠ'AP | AB GKB mij tiLvi nI qvq Df`tqi Xvj mgvb | A_{f} ,

$$m_1 = m_2$$

ev, $\frac{3}{2} = \frac{y-4}{x-3}$ [(1) | (2) t`tk cvB]

ev, $3x - 9 = 2y - 8$

ev, $2y - 8 = 3x - 9$

ev, $2y = 3x - 1$

ev, $y = \frac{3}{2}x - \frac{1}{2} \dots\dots\dots(3)$

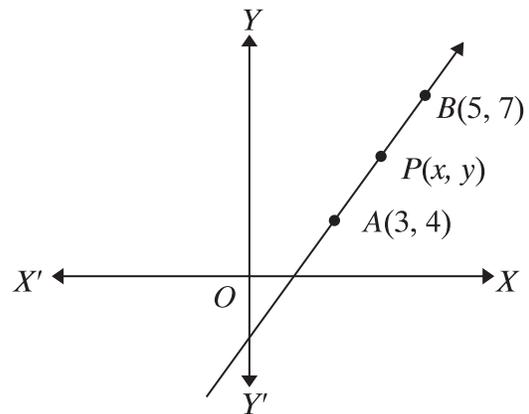
Avevi , PB tiLvi Xvj m_3 atI

$$m_3 = \frac{7-y}{5-x} \dots\dots\dots(4)$$

AB Ges PB tiLvi Xvj mgvb etj [(1) | (4) t`tk cvB]

$$m_1 = m_3$$

ev, $\frac{3}{2} = \frac{7-y}{5-x}$ [(1) | (4) t`tk cvB]



চিত্র : ১১.২৪

$$\begin{aligned} \text{ev, } 15 - 3x &= 14 - 2y \\ \text{ev, } 2y + 15 &= 3x + 14 \\ \text{ev, } 2y &= 3x - 1 \\ \text{ev, } y &= \frac{3}{2}x - \frac{1}{2} \dots\dots\dots(5) \end{aligned}$$

mgxKiY (3) | (5) GKB mgxKiY | mZivs mgxKiY (3) ev (5) n"Q mij tiLv L Gi KvZfmxq mgxKiY | j ¶ | Ki tj f`Lv hvte mgxKiY (3) ev (5) x Ges y Gi GKNvZ mgxKiY Ges GwJ GKwJ mij tiLv wbt` R Kti | ZvB wbtm` tn ejv hvq x Ges y Gi GKNvZ mgxKiY me mgq GKwJ mij tiLv wbt` R Kti | mgxKiY (3) ev (5) tK wbgjfc cKvk Kiv hvq-

$$\begin{aligned} y &= \frac{3}{2}x - 1 \dots\dots\dots (3) \text{ ev } (5) \\ \frac{y-4}{x-3} &= \frac{3}{2} \quad A_{\text{ev}} \quad \frac{y-7}{x-5} = \frac{3}{2} \\ A_{\text{fr}}, \quad \frac{y-4}{x-3} &= \frac{7-4}{5-3} \quad A_{\text{ev}} \quad \frac{y-7}{x-5} = \frac{7-4}{5-3} \\ A_{\text{fr}}, \quad \frac{y-4}{x-3} &= m \quad A_{\text{ev}} \quad \frac{y-7}{x-5} = m \end{aligned}$$

mZivs mvaviYfvte ejv hvq, hw` `BwJ wbw` 0 we`y A(x₁, y₁) Ges B(x₂, y₂) tKvb mij tiLvi Dci Aew`Z nq, Zvntj Xvj

$$m = \frac{y_2 - y_1}{x_2 - x_1} \left[\begin{matrix} \text{rise} \\ \text{run} \end{matrix} \right] \text{ ev } \left[\begin{matrix} \text{I W} \\ \text{nuUv} \end{matrix} \right]$$

Ges D³ mij tiLvi KvZfmxq mgxKiY nte-

$$\begin{aligned} \frac{y - y_1}{x - x_1} &= m \dots\dots\dots(6) \\ \text{ev, } \frac{y - y_2}{x - x_2} &= m \dots\dots\dots(7) \end{aligned}$$

mgxKiY (6) nZ cvB-

$$y - y_1 = m (x - x_1) \dots\dots\dots(8)$$

mgxKiY (7) nZ cvB,

$$y - y_2 = m (x - x_2) \dots\dots\dots(9)$$

∴ (8) Ges (9) nZ Avgiv ejtZ cwii GKwJ mij tiLvi Xvj m ntj Ges tiLwU wbw` 0 we` (x₁, y₁) hv (x₂, y₂) w`tq AwZµg Ki tj tiLwUi KvZfmxq mgxKiY (8) A_{ev} (9) Øviv wbyQ Kiv hvte |

Aci mgxKiY (6) Ges (7) nZ Avgiv cvB-

$$m = \frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} \dots\dots\dots(10)$$

mgxKiY (10) nřZ úófvte ejv hvq, GKwU mij ři Lvi BwU wbow`@ we`y $A(x_1, y_1)$ Ges $B(x_2, y_2)$ w` řq AwZmug Ki řj Gi KvřZřwq mgxKiY nře

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \quad \text{ev} \quad \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1} \dots\dots\dots(11)$$

thřnZl $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$

Dctiv³ Avřj vPbv wbtge³ D`vniYmgřni mrvřth` e`vL`v Kiv nřjv hvřZ wkvřv_řiv mij ři Lvi Xvj I mgxKiY mnřRB eřřZ cvři |

D`vniY 1 | $A(3, 4)$ I $B(6, 7)$ we` řřqi mřřhvMKvix ři Lvi mgxKiY wbyř Ki |

mgvavb : AB ři Lvi Xvj $m = \frac{I W}{nuUv} = \frac{7 - 4}{6 - 3} = \frac{3}{3} = 1$

mgxKiY (8) e`envi Kři AB ři Lvi mgxKiY

$$\begin{array}{l} y - 4 = 1(x - 3) \\ \text{ev, } y - 4 = x - 3 \\ \text{ev, } y = x + 1 \end{array} \quad \left| \quad y - y_1 = m(x - x_1) \dots\dots\dots (8) \right.$$

mgxKiY (9) e`envi Kři AB ři Lvi mgxKiY

$$\begin{array}{l} y - 7 = 1(x - 6) \\ \text{ev, } y = x + 1 \end{array} \quad \left| \quad y - y_2 = m(x - x_2) \dots\dots\dots(9) \right.$$

mgxKiY (11) e`envi Kři AB ři Lvi mgxKiY

$$\begin{array}{l} \frac{y - 4}{x - 3} = \frac{4 - 7}{3 - 6} \\ \text{ev, } \frac{y - 4}{x - 3} = \frac{-3}{-3} = 1 \\ \text{ev, } y - 4 = x - 3 \\ \text{ev, } y = x + 1 \end{array}$$

j řřYxq : mř (8) ev (9) ev (11) thřKvřbwU e`envi Kři BwU wbow`@ we` Mvqx ři Lvi mgxKiY wbyř Ki hvq | wkvřv_řiv mřeavgZ thřKvřbwU e`envi Ki řřZ cvři ře |

D`vniY 2 | GKwU wbow`@ mij ři Lvi Xvj 3 Ges ři LwU $(-2, -3)$ we` Mvqx | ři LwUi mgxKiY wbyř Ki |

mgvavb : ř I qv AvřQ, Xvj $m = 3$

wbow`@ we`y $(x_1, y_1) = (-2, -3)$

∴ ři LwUi mgxKiY,

$$\begin{array}{l} (y - y_1) = m(x - x_1) \\ \text{ev, } y - (-3) = 3\{x - (-2)\} \\ \text{ev, } y + 3 = 3(x + 2) \\ \text{ev, } y = 3x + 3 \end{array}$$

D`vniY 3 | mij ti Lv $y = 3x + 3$ wvwi` 8 we`y $P(t, 4)$ we`y w`tq AmZµg Kti | P we`j `vbr¼ wbyq Ki | ti LwU x Ges y A¶tk h_vµtg A | B we`fZ tQ` Kti | A | B we`j `vbr¼ wbyq Ki |

mgvavb : $P(t, 4)$ we`yU $y = 3x + 3$ ti Lvi Dci Aew`Z nl qvq P we`j `vbr¼ ti Lvi mgxKiYtk wmx (satisfy) Kite |

$$A_{\text{ur}} 4 = 3t + 3$$

$$\text{ev, } 3t = 4 - 3$$

$$\text{ev, } t = \frac{1}{3}$$

$$\therefore P \text{ we`j `vbr¼ } P(t, 4) = P\left(\frac{1}{3}, 4\right).$$

$y = 3x + 3$ ti LwU x A¶tk A we`fZ tQ` Kti | KvtrB A we`j tkwU ev y `vbr¼ 0 [thtnZl x A¶ti mKj we`fZ y Gi gvb kb`]

$$\therefore 0 = 3x + 3$$

$$\text{ev, } x = -1.$$

$$\therefore A \text{ we`j `vbr¼ } (-1, 0).$$

Avevi, $y = 3x + 3$ ti LwU y A¶tk B we`fZ tQ` Kivq B we`j fr ev x `vbr¼ 0 | [thtnZl y A¶ti mKj we`fZ x Gi gvb kb`]

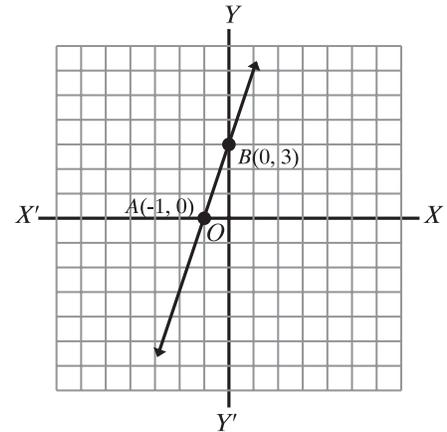
$$\therefore y = 3 \cdot 0 + 3$$

$$\text{ev, } y = 3$$

$$\therefore B \text{ we`j `vbr¼ } (0, 3)$$

GLb KvZfxq Ztj AB ti LwU A ¼b Kw |

AB ti LwU x A¶tk $(-1, 0)$ we`fZ Ges y A¶tk $(0, 3)$ we`fZ tQ` Kti tQ | A_{ur} , x Gi gvb hLb -1 ZLb $y = 3x + 3$ ti LwU x A¶tk tQ` Kti tQ | Avevi y Gi gvb hLb 3 ZLb ti LwU y A¶tk tQ` Kti tQ | mZivs ti LwU x tQ` $K - 1$ Ges y tQ` $K 3$ |



চিত্র : ১১.২৫

Dj wK bq (Non Vertical) Ggb mij ti Lvi mvaviY mgxKiYtk wbtg³i fñc cKvk Kiv nq |

$$y = mx + c$$

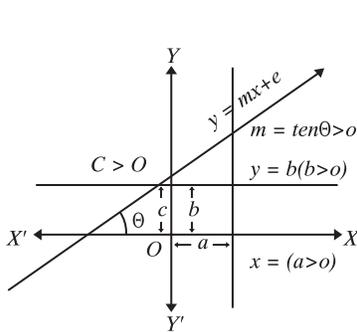
GLvrb m ti LwU X vj Ges c , y A¶ti tQ` Kvsk | $m > 0$ Ges $c > 0$ Gi Rb` ti LwU 11.26 wPñT ð` Lvrbv ntj v |

Avevi y A¶ti mgvst+vj A_{ur} , x A¶ti Dci j æti Lvi mvaviY mgxKiY $x = a$ | wPñT 11.26

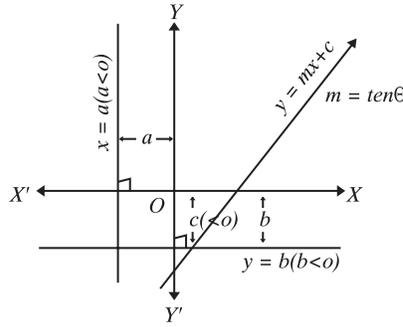
GKbfvte x A¶ti mgvst+vj A_{ur} , y A¶ti Dci j æti Lvi mvaviY mgxKiY $y = b$ wPñT 11.26

j ¶Yxq 'c' Gi gvb avZK nl qvq $y = mx + c$ ti LwU y A¶ti avZK w`tk c GKK `ñi tQ` Kti tQ | m Gi gvb avZK ($m = \tan\theta > 0$) nl qvq $y = mx + c$ ti Lv Øviv DrcbœtkvYwU m²tkvY | 'a' | 'b' Gi gvb avZK nl qvq $x = a$ ti LwU y A¶ti Wwb w`tk Ges $y = b$ ti LwU x A¶ti Dci ð` Lvrbv ntj tQ |

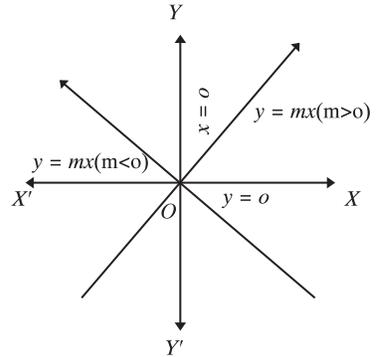
'a', 'b' I 'c' Gi FYZK gvbi tej vq ti Lv, tj vi Ae⁻vb 11.27 wP⁺ t⁻ Lv⁺bv ntj v |



চিত্র : ১১.২৬



চিত্র : ১১.২৭



চিত্র : ১১.২৮

wP⁺ 11.26 I 11.27 Ges Dctiv³ Avtj vPbv t⁻ K Avgiv⁻ úó KtiB ej tZ cwi c = 0 ntj y = mx ti LwU gj we⁻ y(0, 0) w⁻ tq hvte, a = 0 ntj ti LwU y A⁺ Ges b = 0 ntj ti LwU x A⁺ | wP⁺ 11.28 mZivs x A⁺ i mgxKiY y = 0 Ges y A⁺ i mgxKiY x = 0

D⁻ nviY 4 | y - 2x + 3 = 0 ti Lvi Xvj I y A⁺ i tQ⁻ Kvsk wby⁺ Ki | KvZ⁺ Ztj ti LwU G⁺ K t⁻ Lvi |

mgvavb : y - 2x + 3 = 0

ev. y = 2x - 3 [y = mx + c AvKvi]

∴ Xvj m = 2

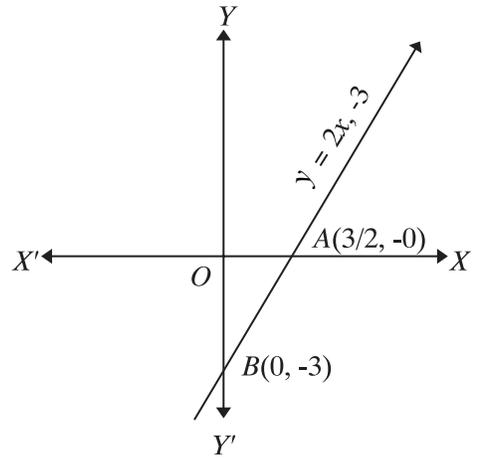
y A⁺ i tQ⁻ Kvsk c = -3

GLb ti LwU x I y A⁺ i K A I B we⁻ tZ tQ⁻ Kiti cwiB,

A we⁻ j⁻ vbr¼ (3/2, 0) [x A⁺ i y = 0 ewmtq x = 3/2]

B we⁻ j⁻ vbr¼ (0, -3) [y A⁺ i x = 0 ewmtq (y = -3)]

KvZ⁺ Ztj ti LwU G⁺ K t⁻ Lv⁺bv ntj v |



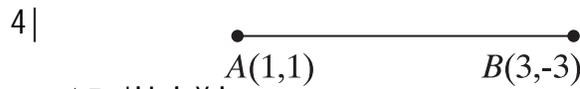
চিত্র : ১১.২৯

D⁻ nviY 5 | A(-1, 3) Ges B(5, 15) we⁻ tqi msthvM ti Lv x-A⁺ I y A⁺ i K P I Q we⁻ tZ tQ⁻ Kiti | PQ ti Lvi mgxKiY wby⁺ Ki Ges PQ Gi⁻ N⁺ wby⁺ Ki |

mgvavb : AB ti Lvi mgxKiY $\frac{y-3}{x+1} = \frac{3-15}{-1-5} = \frac{-12}{-6} = 2$

ev, y - 3 = 2x + 2

ev, y = 2x + 5.....(1)



AB ti Lvi Xvj

- K. 2
- M. 0
- L. -2
- N. 6

5 | $x - 2y - 10 = 0$ Ges $2x + y - 3 = 0$ ti Lv^otqi Xvj ^otqi Ydj

- K. -2
- M. -2
- L. 2
- N. -1

6 | $y = \frac{x}{2} + 2$ Ges $2x - 10y + 20 = 0$ mgxKi Y^oq

- K. $\frac{1}{2}$ eM^oGKK
- M. ti Lv^oq mgvšivj
- L. GKB ti Lv^oq R Kti
- N. ti Lv^oq ci $\frac{1}{2}$ eM^oGKK

7 | $y = x - 3$ Ges $y = -x + 3$ Gi tQ^o we^o y

- K. (0, 0)
- M. (3, 0)
- L. (0, 3)
- N. (-3, 3)

vb^otPi Z^oi Av^otj v^otK 8 I 9 bs c^okie DEi ^ovl :

$x = 1, y = 1$

8 | ti Lv^oq x A^ot^o K th we^o y tQ^o Kti Zvi $\frac{1}{2}$

- K. (0, 1)
- M. (0, 0)
- L. (1, 0)
- N. (1, 1)

9 | ti Lv^oq A^ot^o tqi mv^o th t^oq^o $\frac{1}{2}$ Zwi Kti Zvi t^oq^o dj

- K. $\frac{1}{2}$ eM^oGKK
- M. 2 eM^oGKK
- L. 1 eM^oGKK
- N. 4 eM^oGKK

10 | GK^oW mij ti Lvi mgxKi Y^o vb^oq Ki hv (2, -1) we^o y^o t^oq hvq Ges hvi Xvj 2.

11 | vb^ot^o we^o $\frac{1}{2}$ w^o t^oq Av^oZ^o μ vš-mij ti Lvi mgxKi Y^o vb^oq Ki |

- (a) A(1, 5), B(2, 4)
- (b) A(3, 0), B(0, -3)
- (c) A(a, o), B(2a, 3a)

12 | vb^ot^o c^oi Z^o t^oq^o mij ti Lvi mgxKi Y^o vb^oq Ki |

- (a) Xvj 3 Ges y tQ^o K -5
- (b) Xvj -3 Ges y tQ^o K -5
- (c) Xvj 3 Ges y tQ^o K 5
- (d) Xvj -3 Ges y tQ^o K 5

Dc^otiv^o Pvi^o ti Lv GKB mgZ^otj G^otK t^o Lvl |

[GB ti Lv mg^oni gva^o t^oq Xvj tevSv hvte Ges y - A^ot^o t^o K Kvstki v^ot^o y^o Rb^o ti Lv tKvb PZ^o v^o M Ae^o vb Ki t^oq]

- 13| $\text{wb}tq\text{e}^3$ ti Lvmgna x A \uparrow tiK I y A \uparrow tiK tKvb we^y tQ` Kti $\text{wbY}q$ Ki | Zvi ci ti Lvmgna GtK t` Lvl |
 (a) $y = 3x - 3$
 (b) $2y = 5x + 6$
 (c) $3x - 2y - 4 = 0$
- 14| $(k, 0)$ we^y Mvgn I k Xvj $\text{we}nk\acute{o}$ mij ti Lvi mgxKiY k Gi gva^tq $\text{wbY}q$ Ki | $hw`$ ti LwU $(5, 6)$ we^y Mvgn nq Zte k Gi gvb $\text{wbY}q$ Ki |
- 15| $(k^2, 2k)$ we^y Mvgn Ges $\frac{1}{k}$ Xvj $\text{we}nk\acute{o}$ ti Lvi mgxKiY $\text{wbY}q$ Ki | $hw`$ ti LwU $(-2, 1)$ we^y w^tq AwZ μ g Kti Zte k Gi m^{ae} gvb $\text{wbY}q$ Ki |
- 16| GKwU ti Lv $A(-2, 3)$ we^y w^tq hvq hvi Xvj $\frac{1}{2}$ | Avevi ti LwU $hw`$ $(3, k)$ we^y w^tq hvq Zte k Gi gvb KZ ?
- 17| 3 Xvj $\text{we}nk\acute{o}$ GKwU ti Lv $A(-1, 6)$ we^y w^tq hvq Ges x A \uparrow tiK B we^y tK tQ` Kti | A we^y Mvgn Ab` GKwU ti Lv x A \uparrow tiK C(2, 0) we^y tZ tQ` Kti |
 (a) AB I AC ti Lvi mgxKiY $\text{wbY}q$ Ki |
 (a) ΔABC Gi t \uparrow ti dj $\text{wbY}q$ Ki |
- 18| t` Lvl th, $y - 2x + 4 = 0$ Ges $3y = 6x + 10$ ti Lv θ q ci $\text{u}i$ tQ` Kti bv | ti Lv θ tqi $\text{w}P\hat{\text{T}}$ GtK e v L v Ki tKb mgxKiY B $\text{w}U$ mgvavb bvB |
- 19| $y = x + 5, y = -x + 5$ Ges $y = 2$ mgxKiY wZbwU GKwU $\text{w}i$ f \ddot{r} i wZbwU e v wb^t R Kti | $\text{w}i$ f \ddot{r} $\text{w}U$ $\text{w}P\hat{\text{T}}$ AvK Ges t \uparrow ti dj $\text{wbY}q$ Ki |
- 20| $y = 3x + 4$ Ges $3x + y = 10$ ti Lv θ tqi tQ` we^y v v $\frac{1}{4}$ $\text{wbY}q$ Ki | ti Lv θ tqi $\text{w}P\hat{\text{T}}$ AvK Ges x A \uparrow mg^tq MvZ $\text{w}i$ f \ddot{r} i t \uparrow ti dj $\text{wbY}q$ Ki |
- 21| $\text{c}g\text{vY}$ Ki th, $2y - x = 2, y + x = 7$ Ges $y = 2x - 5$ ti Lv wZbwU mgwe^y (Concurrent) A f GKb we^y w^tq AwZ μ g Kti |
- 22| $y = x + 3, y = x - 3, y = -x + 3$ Ges $y = -x - 3$ GKwU PZ f f r i PviwU e v wb^t R Kti | PZ f f r AvK Ges t \uparrow ti dj wZbwU w f b c \times wZ^t $\text{wbY}q$ Ki |
- 23| t` I qv AvtQ,
 $3x + 2y = 6$
 K. $\text{c}\acute{o}$ \acute{E} ti LwU A \uparrow θ q \ddot{t} K th th we^y tZ tQ` Kti Zv $\text{wbY}q$ Ki |
 L. A \uparrow θ tqi Lw \acute{U} Z Astki cwi gvY $\text{wbY}q$ Ki Ges ti LwU A \uparrow θ tqi mv^t th $\text{w}i$ f \ddot{r} Drcba Kti Zvi t \uparrow ti dj $\text{wbY}q$ Ki |
 M. A \uparrow θ q Ges ti LwU \ddot{t} K avi we^t Pbv Kti Gi Dci GKwU 5 GKK D PZv $\text{we}nk\acute{o}$ Nbe v Zvi Kiv ntjv hvi Kx g we^y v Dcti | Nbe v w $\text{mgM}\acute{O}$ Z tj i t \uparrow ti dj Ges AvqZb $\text{wbY}q$ Ki |
- 24| t` I qv AvtQ, A (1, 4a) Ges B (5, $a^2 - 1$) we^y Mvgn ti Lvi Xvj = -1
 K. t` Lvl th, a Gi w gvb i tq^t Q |
 L. a Gi gvb θ tqi Rb` th PviwU we^y y cvl qv hvq, ai Zviv P, Q, R I S, PQRS-Gi t \uparrow ti dj $\text{wbY}q$ Ki |
 M. PZ f f r w mgv^s w K bv AvqZ? G e v vcv^t tZ v g v g h m e v L v Ki |

Øv`k Aa`vq mgZj xq tf±i

Avgiv t`j vi iwvk Ges Zvi Dci weifbæcKvi MvYwZK cØµqvi cØqvM wktLwQ | wKŠ`i ay t`j vi iwvk mæútk©avi bv
_vKtj B ^`bw`b Rxeþbi AþbK Kvhþug e`vLv Kiv hvq bv | Gt¶¶t¶ Avgt`i tf±i iwki avi Yv cØqvRb nq |
GB Aa`vtq Avgiv tf±i iwvk mæútk©Avtj vPbv Ki þev |

Aa`vq tkþl wK¶v_xPv

- t`j vi iwvk I tf±i iwvk eYØv Ki þZ cvi þe |
- t`j vi iwvk I tf±i iwvk cØZtKi mrvnþh` e`vLv Ki þZ cvi þe |
- mgvb tf±i, wecixZ tf±i I Ae`vb tf±i e`vLv Ki þZ cvi þe |
- tf±ti i thvM I thvMweia e`vLv Ki þZ cvi þe |
- tf±ti i weþqM e`vLv Ki þZ cvi þe |
- tf±ti i t`j vi µWYZK I GKK tf±i e`vLv Ki þZ cvi þe |
- tf±ti i t`j vi µWYZK I eÈbweia e`vLv Ki þZ cvi þe |
- tf±ti i mrvnþh` weifbæR`wgvZK mgm`vi mgvavb Ki þZ cvi þe |

12.1 | t`j vi iwvk I tf±i iwvk

^`bw`b Rxeþb cØq met¶¶t¶B e`i cwi gvþci cØqvRb nq | 5 tm.wg., 3 wgvbU, 12 UvKv, 5 wj Uvi, 6° C
BZ`w` Øviv h_vµtg e`i ^`N°, mgþqi cwi gvY, UvKvi cwi gvY, AvqZþbi cwi gvY I ZvcgvÎvi cwi gvY
tevSvþbv nq | Gme cwi gvþci Rb` tKej gvÎ GKKmn cwi gvY DþjL Ki þj B Pþj | Avevi hw` ej v nq
GKwU tj vK GKwe`yt_þK hvÎv Kþi cØtg 4 wg. I cþi 5 wg. tMj, Zvntj hvÎvwe`yt_þK Zvi `þZjwbYØ
Ki þZ tMþj cØtg Rvbv `i Kvi tj vKwU MvZi w`K Kx? MvZi mwVK w`K bv Rvbv chS-hvÎvwe`yt_þK
tj vKwU KZ`þ wMþqþQ Zv mwVKfvþe wbYØ mæþ bq |

th iwvk tKej gvÎ GKKmn cwi gvY Øviv mæúYþc tevSvþbv hvq, ZvþK t`j vi ev Aw`K ev wbow`R iwvk ej v
nq | ^`N°, fi, AvqZb, `wZ, ZvcgvÎv BZ`w` cØZþKB t`j vi iwvk |

th iwvkþK mæúYþc cKvk Kivi Rb` Zvi cwi gvY I w`K Dfþqi cØqvRb nq, ZvþK tf±i ev mw`K iwvk
ej v nq | miY, teM, ZjY, I Rb, ej BZ`w` cØZþKB tf±i iwvk |

12.2 | tf±i iwki R`wgvZK cØZifc: w`K wbt`RK ti Lvsk

tKvþbv ti Lvstki GK cØŠþK Aw`we`y (initial point) Ges Aci cØŠþK AŠwe`y (terminal point)
wntþte wPwýZ Ki þj H ti LvskþK GKwU w`K wbt`RK ti Lv (directed line segment) ej v nq | tKvþbv
w`K wbt`RK ti Lvstki Aw`we`yA Ges AŠwe`yB ntj H w`K wbt`RK ti LvstþK \overline{AB} Øviv mwPZ Kiv

nq | c0Z`K w` K wbt` RK ti Lvsk GKwU tf±i iwk, hvi cwi gvY H ti Lvstki ~ N°(| \overline{AB} | ev mst¶t c AB Øviv mPZ) Ges hvi w` K A we`yntZ AB ti Lv eivei B we`ywb` RKvix w` K |

wecixZµtg thKvfbv tf±i iwk tK GKwU w` K wbt` RK ti Lvsk Øviv cKvk Kiv hvq, thLvfb ti LvskwU ~ N° iwk wU cwi gvY Ges ti LvskwU Aw we`yntZ AŠwe`ywb` RKvix w` K c0 È tf±i iwki w` K |

ZvB, tf±i iwk I w` K wbt` RK ti Lvsk mgv_ R aviYv | w` K wbt` RK tK R`vugwZK tf±i etj I D t j L Kiv nq | Avgv` i Avtj vPbv GKB mgZtj Aew`Z tf±ti i gta` mxgve× _vKte |

aviK ti Lv t tKvb tf±i (w` K wbt` RK ti Lvsk) th Amxg mij ti Lvi Ask we tkl, ZvtK H tf±ti i aviK ti Lv ev i ayaviK (support) ej v nq |

mPivPi GKwU tf±i tK GKwU A¶i w` t q mPZ Kiv nq;

thgb $\overline{AB} = \underline{u}$ wKŠ' \overline{AB} wj Ltj thgb tevSv hvq th, tf±iwU Aw we`y A I AŠwe`y B, \underline{u} wj Ltj tZgb tKvfbv Z_` cvl qv hvq bv |

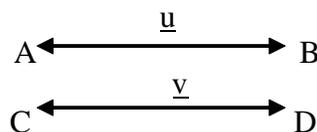
KvR: 1 | tZvgvi ewo ntZ `g tmvRv `w¶tY 3 wK. wg. `ti Aew`Z | ewo ntZ tntU `tj thtZ GK NvUv mgq j vMtj tZvgvi MwZteM KZ?

2 | `g QuU ci mvBtKtj 20 wgvbU ewo Gtj Gt¶t tZvgvi MwZteM KZ?

12.3 | tf±ti i mgZv, wecixZ tf±i

mgvb tf±i t GKwU tf±i \underline{u} -tK Aci GKwU tf±i \underline{v} -Gi mgvb ej v nq hw`

(i) $|\underline{u}| = |\underline{v}|$ (\underline{u} Gi ~ N° mgvb \underline{v} Gi ~ N°)



(ii) \underline{u} -Gi aviK, \underline{v} -Gi avi tKi m½ AwfbøA_ev mgvŠ+vj nq,

(iii) \underline{u} -Gi w` K \underline{v} -Gi w` tKi m½ GKgtLx nq | mgZvi GB msAv th wbtPi wbgg, t j v t g t b P t j , Zv mntRB tevSv hvq :

- (1) $\underline{u} = \underline{v}$
- (2) $\underline{u} = \underline{v}$ ntj $\underline{v} = \underline{u}$
- (3) $\underline{u} = \underline{v}$ Ges $\underline{u} = \underline{w}$ ntj $\underline{u} = \underline{w}$

\underline{u} -Gi aviK Ges \underline{v} -Gi aviK ti LvØq Awfbøev mgvŠ+vj ntj , Avgiv mst¶t c ej e th \underline{u} Ges \underline{v} mgvŠ+vj tf±i |

`be` t th tKvb we`yt_tK c0 È th tKvb tf±ti i mgvb Kti GKwU tf±i Uvbv hvq |

tKbbv, we`y P Ges tf±i \underline{u} t` l qv _vKtj , Avgiv P we`y w` t q \underline{u} Gi avi tKi mgvŠ+vj Kti GKwU mij ti Lv Uwb, Zvici P we`y t_tK \underline{u} Gi w` K eivei (\underline{u}) Gi mgvb Kti PQ ti Lvsk tKtU wB |

Zvntj A½b Abhvqx $\overline{PQ} = \underline{u}$ nq |

wecixZ tf±i : \underline{v} tK \underline{u} -Gi wecixZ tf±i ej v nq, hw`

(i) $|\underline{v}| = |\underline{u}|$

(ii) \underline{v} -Gi avi K, \underline{u} -Gi avi tKi m½ Awfbæev mgvš+vj nq|

(iii) \underline{v} -Gi w K \underline{u} -Gi w tKi wecixZ nq|

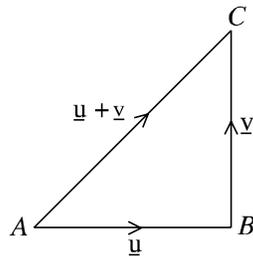
\underline{v} hw` \underline{u} Gi wecixZ tf±i nq, Zte \underline{u} nte \underline{v} Gi wecixZ tf±i | mgZvi msÁv t_tK tevSv hvq th, \underline{v} Ges \underline{w} cØZ_tK \underline{u} Gi wecixZ tf±i tevSv_tZ N nq|

$$\underline{u} = \overrightarrow{AB} \text{ n}tj \quad -\underline{u} = \overrightarrow{BA}$$

12.4| tf±t+i i thvM I we tqvM

1| (K) tf±i thv_tMi wî fR wewa

tf±i thv_tMi msÁv t tKvb \underline{u} tf±t+i cØšwe`yt_tK Aci GKwJ tf±i \underline{v} AvKv n_tj $\underline{u} + \underline{v}$ Øviv Gifc tf±i tevSvq hvi Aw`we`y \underline{u} Gi Aw`we`y Ges hvi cØšwe`y \underline{v} Gi cØšwe`y|



g_tb Kwî, $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{BC} = \underline{v}$ Gifc `BwJ tf±i th, \underline{u} Gi cØšwe`y \underline{v} Gi Aw`we`y| Zvntj \underline{u} Gi Aw`we`y Ges \underline{v} Gi cØšwe`y msthvRK \overrightarrow{AC} tf±i \underline{u} I \underline{v} tf±i Øtqi mgwó ej v nq Ges $\underline{u} + \underline{v}$ Øviv mwPZ nq|

\underline{u} I \underline{v} mgvš+vj bv n_tj \underline{u} , \underline{v} Ges $\underline{u} + \underline{v}$ tf±i Îq Øviv wî fR Drcbævq etj Dctiv³ thvRb c×wZ_tK wî fR wewa ej v nq|

(L) tf±i thv_tMi mgvšwî K wewa

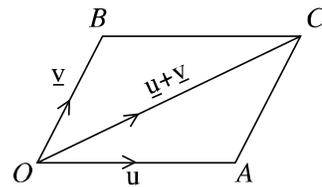
tf±i thv_tMi wî fR wewai Abym×vš-wntmte tf±i thv_tMi mgvšwî K wewa wbaifct tKvb mgvšwî tKi `BwJ mwbmNZ evü Øviv `BwJ tf±i \underline{u} I \underline{v} Gi gvb I w K mwPZ n_tj, H mgvšwî tKi th KY© \underline{u} I \underline{v} tf±i Øtqi mPK ti Lvi tQ`we`y Mgx Zv Øviv $\underline{u} + \underline{v}$ tf±t+i i gvb I w K mwPZ nq|

cØyv t g_tb Kwî, th_tKv_tbv we`yt_tK Aw¼Z \underline{u}

Ges \underline{v} tf±i Øq \overrightarrow{OA} Ges \overrightarrow{OB} Øviv mwPZ n_tq_tQ|

OACB mgvšwî K I Zvi \overrightarrow{OC} KY©A¼b Kwî |

Zvntj H mgvšwî tKi OC KY©Øviv \underline{u} Ges \underline{v} Gi thvMdj mwPZ nte|



$$A_\text{f} \overrightarrow{OC} = \underline{u} + \underline{v} \text{ (tf±i `vbvš_t+i i gva`tg)}$$

OACB mgvšwî tKi OB I AC mgvb I mgvš+vj |

$$\therefore \overrightarrow{AC} = \overrightarrow{OB} = \underline{v} \text{ (tf±i `vbvš_t+i i gva`tg)}$$

$\therefore \underline{u} + \underline{v} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ [wĭ fĴR weia Abjvŋi]

`be" t (1) `β ev ZtZwaK tf±i i thvMdj tK Zv` i j wäl ej v nq| ej ev tetMi j wä wbyĴqi tĴtĴ tf±i thvMi c×wZ AbjviY Ki tZ nq|

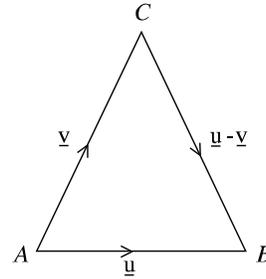
(2) `βwJ tf±i mgvšivj ntj Zv` i thvMi tĴtĴ mgvšivi K weia cĴhvR" bq, wKš" wĭ fĴR weia mKj tĴtĴ cĴhvR" |

2| tf±i i wetqM t

u Ges v tf±i tĴqi wetqMdj u Ń v ej tZ

u Ges (-v) (v Gi weciX tf±i) tf±i tĴqi

thvMdj u + (Ń v) tevSvq|



tf±i wetqM i wĭ fĴR weia

$\underline{u} = \overrightarrow{AB}, \underline{v} = \overrightarrow{AC}$ ntj $\underline{u} - \underline{v} = \overrightarrow{CB}$; A_ŋ $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

K_vq t u Ges v Gi Aw" we" yGKB ntj u Ń v tmB tf±i, hvi Aw" we" ynt"Q v Gi Ašwe" yGes hvi Ašwe" ynt"Q u Gi Ašwe" y|

mstĴt c t GKB Aw" we" yenkó `βwJ tf±i i wetqMdj nt"Q Ašwe" Ĵq ōviv weciXZμtg MWZ tf±i |

cĴvY t CA ti LvsktK GgbfĴte ewaZ Kwi thb AE = CA nq| AEFB mgvšivi K MvB Kwi | tf±i

thvMi mgvšivi K weia AbĴvqx, $\overrightarrow{AE} + \overrightarrow{AB} = \overrightarrow{AF}$

Avevi AFBC GKwJ mgvšivi K, tKbbv BF = AE = CA

Ges BF || AE etj BF || CA.

$\therefore \overrightarrow{AF} = \overrightarrow{CB}$ tf±i vbvšĴ,

wKš' $\overrightarrow{AE} = -\underline{v}$ Ges $\overrightarrow{AB} = \underline{u}$

mĴi vs $\underline{u} + (-\underline{v}) = \overrightarrow{CB}$ cĴwYZ nj |

3| kb" tf±i : th tf±i i ciggvb kb" Ges hvi w" K wbyĴ Ki v hvq bv Zv`K kb" tf±i etj |

u th tKvb tf±i ntj u + (Ń u) Kx nte?

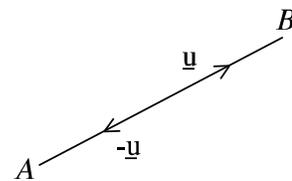
awi, $\underline{u} = \overrightarrow{AB}$ ZLb $\underline{u} = \overrightarrow{BA}$ dtj

$\underline{u} + (\underline{u}) = \overrightarrow{AB} + \overrightarrow{BA}$

$= \overrightarrow{AA}$ (wĭ fĴR weia AbĴvqx)

wKš' \overrightarrow{AA} Kx ai tbi tf±i? GwJ GKwJ we" ytf±i, A_ŋ Gi

Aw" we" y| Ašwe" yGKB we" y mĴi vs " N" kb" |



A₁ AA 0viv A we` tKB eStZ nte| Gifc tfti (hvi `N° kb`) tK kb` tfti ejv nq Ges 0
cZxK 0viv mPZ Kiv nq| GB GKgvI tfti hvi tKvb wv` 0 w` K ev avi K ti Lv tbB|

kb` tfti i AeZviYvi dtj Avgiv ej tZ cwi th, $\underline{u} + (-\underline{u}) = \underline{0}$

Ges $\underline{u} + \underline{0} = \underline{0} + \underline{u} = \underline{u}$

e`Z kb` tfti i m½ tktlv³ Aft` wvwnZ i tqtQ|

12.5| tfti thvMi weWamgn

1| tfti thvMi weWbgq weWa (Commutative Law)

th tKvb \underline{u} , \underline{v} tfti i Rb` $\underline{u} + \underline{v} = \underline{v} + \underline{u}$

c0vY t gtb Kwi, $\overrightarrow{OA} = \underline{u}$ Ges $\overrightarrow{OB} = \underline{v}$, OACB mgvšwi K | Zvi KY°OC A¼b Kwi | OA |
BC mgvb | mgvš+vj Ges OB | AC mgvb | mgvš+vj |

∴ $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \underline{u} + \underline{v}$

Avei, $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{OA} = \underline{v} + \underline{u}$

∴ $\underline{u} + \underline{v} = \underline{v} + \underline{u}$

∴ tfti thvRb weWbgq weWa wv× Kti |

tfti thvRtbi msthvM weWa (Associative Law)

th tKvb \underline{u} , \underline{v} , \underline{w} Gi Rb` $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$

c0vY t gtb Kwi, $\overrightarrow{OA} = \underline{u}$, $\overrightarrow{AB} = \underline{v}$, $\overrightarrow{BC} = \underline{w}$

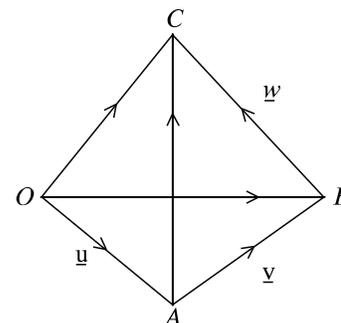
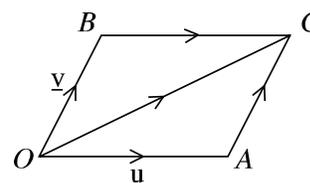
A₁ u Gi c0šwe` yt_tK v Ges v Gi c0šwe` yt_tK w A¼b Kiv ntqtQ| O,C Ges A, C
thvM Kwi |

Zvntj $(\underline{u} + \underline{v}) + \underline{w} = (\overrightarrow{OA} + \overrightarrow{AB}) + \overrightarrow{BC}$
= $\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$

Avei, $\underline{u} + (\underline{v} + \underline{w}) = \overrightarrow{OA} + (\overrightarrow{AB} + \overrightarrow{BC})$
= $\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

∴ $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$

mYZivs tfti thvRb msthvM weWa wv× Kti |



Abjmvš-t tKvfbv wI fRi wZbvU evüi GKB µg 0viv mPZ tfti tfti thvMdj kb` |

Dcti i wPftI, $\overrightarrow{OB} + \overrightarrow{BA} = \overrightarrow{OA} = (-\overrightarrow{AO})$

∴ $\overrightarrow{OB} + \overrightarrow{BA} + \overrightarrow{AO} = \overrightarrow{OA} + \overrightarrow{AO} = -\overrightarrow{AO} + \overrightarrow{AO} = \underline{0}$

3| t f±i thvMi eR³ wewa (Cancellation Law)

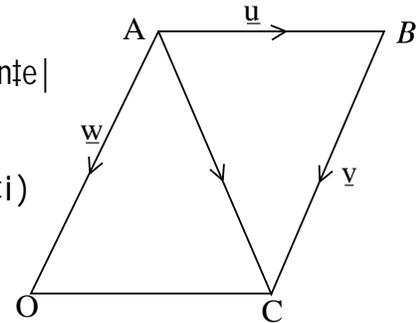
thtKv_u u, v, w t f±i i Rb³ $\underline{u} + \underline{v} = \underline{u} + \underline{w}$ ntj , $\underline{v} = \underline{w}$ nte|

c³Y t GLv_u $\underline{u} + \underline{v} = \underline{u} + \underline{w}$

∴ $\underline{u} + \underline{v} + (-\underline{u}) = \underline{u} + \underline{w} + (-\underline{u})$ (Dfqc³¶ N_u thvM K_u)

ev, $\underline{u} - \underline{u} + \underline{v} = \underline{u} - \underline{u} + \underline{w}$

∴ $\underline{v} = \underline{w}$



12.6| t f±i i msL³ v_u WYZK ev t⁻ j vi_u WYZK (Scalar multiple of a vector)

u thtKv_u t f±i Ges m th tKv_u ev⁻ e msL³ v_u ntj $m\underline{u}$ Øv_u tKv_u t f±i tevSvq, v_u tP Zv evL³ v_u Kiv nj |

(1) $m = 0$ ntj , $m\underline{u} = \underline{0}$,

(2) $m \neq 0$ ntj , $m\underline{u}$ Gi av_u K u Gi av_u tKi m_u Avfb_u

$m\underline{u}$ Gi [^] N³ u Gi [^] tN³ (m) v_u Ges

(K) $m > 0$ ntj $m\underline{u}$ Gi v_u K u Gi v_u tKi m_u tM GKg_ux

(L) $m < 0$ ntj $m\underline{u}$ Gi v_u K u Gi v_u tKi v_u ecixZ |

ðe³ t (1) $m = 0$ Av_u $\underline{u} = \underline{0}$ ntj

$m\underline{u} = \underline{0}$

(2) $1\underline{u} = \underline{u}$, $(-1)\underline{u} = -\underline{u}$

Dcwi D³ ms³ Av_u tZ t⁻ Lv hvq, $m(\underline{n\underline{u}}) = n(\underline{m\underline{u}}) = mn(\underline{u})$

mn Df_u t_u > 0, Df_u t_u < 0, GK_u > 0 Aciv_u < 0, GK_u ev Df_u q 0, G mKj t³ ¶ t³ c_u K c_u K f_u te v_u t_u P_u v_u t_u m_u t_u RB m_u t_u v_u i ev⁻ eZv m_u t_u K_u v_u t_u v_u t_u Z n l qv hvq | v_u t_u P Gi GK_u D³ v_u niY t⁻ qv nj t

g_u t_u Kwi $\overrightarrow{AB} = \overrightarrow{BC} = \underline{u}$

AC tK G ch_u S-Gi f_u t_u c eva_u Z Kwi thb

CD = DE = EF = FG = AB nq |

ZLb $\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FG}$

$$= \underline{u} + \underline{u} + \underline{u} + \underline{u} + \underline{u} + \underline{u} = 6\underline{u}$$

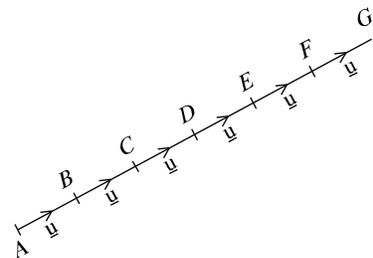
Av_u v_u t_u K $\overrightarrow{AG} = \overrightarrow{AC} + \overrightarrow{CE} + \overrightarrow{EG}$

$$= 2\underline{u} + 2\underline{u} + 2\underline{u}$$

$$= 3(2\underline{u})$$

Ges $\overrightarrow{AG} = \overrightarrow{AD} + \overrightarrow{DG} = 3\underline{u} + 3\underline{u} = 2(3\underline{u})$

∴ $2(3\underline{u}) = 3(2\underline{u}) = 6\underline{u}$



be t BW tfti aviK tiLv Awfbæev mgvšivj ntj, G`i GKWtK Aciwji mvsL`wYZK AvKvfi cKvk Kiv hvq|

ev`te $AB \parallel CD$ ntj,

$$\overrightarrow{AB} = m\overrightarrow{CD}, \text{ thLvfb, } |m| = \frac{|\overrightarrow{AB}|}{|\overrightarrow{CD}|} = \frac{AB}{CD}$$

$m > 0$ ntj, $\overrightarrow{AB} \parallel \overrightarrow{CD}$ mggtLx nq,

$m < 0$ ntj, $\overrightarrow{AB} \parallel \overrightarrow{CD}$ weciXZgtLx nq|

12.7| tfti i mvsL`wYZK msµvš-eĒb mĤ

(Distributive laws concerning scalar multiples of vectors)

m, n BW t`j vi Ges $\underline{u}, \underline{v}$ BW tfti ntj,

$$(1) (m + n)\underline{u} = m\underline{u} + n\underline{u}$$

$$(2) m(\underline{u} + \underline{v}) = m\underline{u} + m\underline{v}$$

cġvY t (1) m ev n kb` ntj mĤW Aek`B LvĤU|

gtb Kwi, m, n Dftq abvZK Ges $\overrightarrow{AB} = m\underline{u}$

$$\therefore |\overrightarrow{AB}| = m|\underline{u}|$$

AB tK C chS-eiaZ Kwi thb $|\overrightarrow{BC}| = n|\underline{u}|$ nq|

$$\therefore \overrightarrow{BC} = n\underline{u} \text{ Ges}$$

$$|\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}| = m|\underline{u}| + n|\underline{u}| = (m + n)|\underline{u}|$$

$$\therefore \overrightarrow{AC} = (m + n)\underline{u}$$

$$\text{WŠ' } \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

$$\therefore m\underline{u} + n\underline{u} = (m + n)\underline{u}$$

m, n Dftq FYvZK ntj $(m + n)\underline{u}$ Gi `N`nte

$|m + n||\underline{u}|$ Ges w`K nte \underline{u} Gi w`tki weciXZ w`K, ZLb $m\underline{u} + n\underline{u}$ tftiwji `N`nte

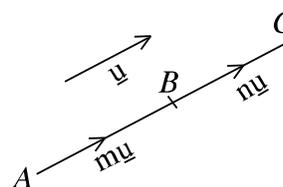
$|m||\underline{u}| + |n||\underline{u}| = (|m| + |n|)|\underline{u}|$ [$\because m\underline{u}, n\underline{u}$ tftiġq GKB w`tk KvR Kfi] Ges w`K nte \underline{u} Gi weciXZ

w`K| WŠy $m < 0$ Ges $n < 0$ ni qvq $|m| + |n| = |m + n|$, tmĤnZi GtĤĤĤ $(m + n)\underline{u} = m\underline{u} + n\underline{u}$

cvi qv tMj |

meĤkĤl m Ges n Gi gĤa` cġgW > 0 , Aciw < 0 ntj $(m + n)\underline{u}$ Gi `N`nte $|m + n||\underline{u}|$ Ges

w`K nte



(K) \underline{u} Gi w`tki mvt_ GKgtLx hLb $|m| > |n|$

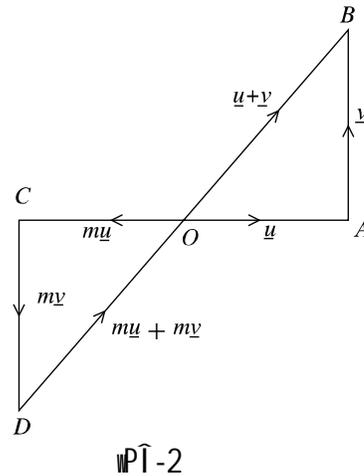
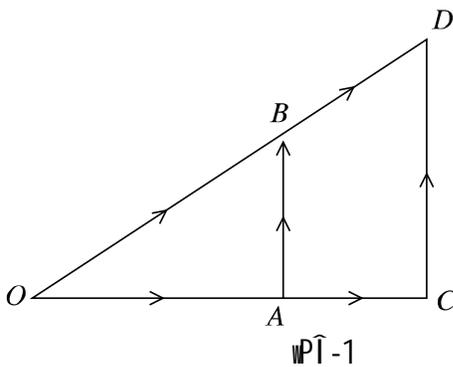
(L) \underline{u} Gi weciXZ w`K hLb $|m| < |n|$

ZLb $m\underline{u} + n\underline{u}$ tf±iWJl `N°1 w`tk $(m + n)\underline{u}$ Gi mvt_ GKgtLx nte|

`be` t wZbwU we`y A, B, C mgtiL nte hw` Ges tKej hw` $\overrightarrow{AC}, \overrightarrow{AB}$ Gi mvsL`_wYZK nq|

gše` t (1) `BwU tf±ti i aviK tiLv AwfbœA_ev mgvš±vj ntj Ges Zvt` i w`K GKB ntj, Zvt` i m`k (similar) tf±i ej v nq|

(2) th tf±ti i `N°1 GKK, ZvtK (w`K wbt` RK) GKK tf±i ej v nq|



g`tb Kwî, $\overrightarrow{OA} = \underline{u}, \overrightarrow{AB} = \underline{v}$

Zvntj $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \underline{u} + \underline{v}$

OA tK C chš-ewaZ Kwî thb $OC = m \cdot OA$ nq| C we`y w`tq AwZ AB Gi mgvš±vj CD tiLv OB Gi ewaZvsktK D we`tZ tQ` Kti | thtnZl OAB Ges OCD w`fRØq m`k,

$$\text{tmtnZl } \frac{|\overrightarrow{OC}|}{|\overrightarrow{OA}|} = \frac{|\overrightarrow{CD}|}{|\overrightarrow{AB}|} = \frac{|\overrightarrow{OD}|}{|\overrightarrow{OB}|} = m$$

$$\therefore \overrightarrow{CD} = m\overrightarrow{AB} = m\underline{v}$$

wPÎÑ1 G m abvZK, wPÎ-2 G m FYvZK

$$\therefore OC = m \cdot OA, CD = m \cdot AB, OD = m \cdot OB$$

$$G\uparrow\uparrow Y \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD} \text{ ev, } m(\overrightarrow{OA}) + m(\overrightarrow{AB}) = m(\overrightarrow{OB})$$

$$\therefore m\underline{u} + m\underline{v} = m(\underline{u} + \underline{v})$$

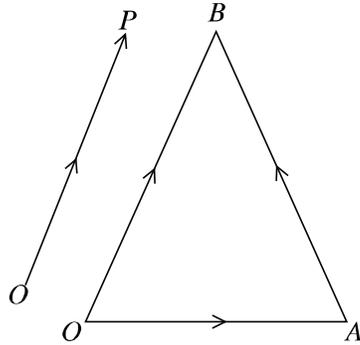
`be` t m Gi mKj g`tbi Rb` Dctiv³ m`f mZ`|

KvR: $m \mid n$ Gi wefwbœcKvi mvsL`K gvb wbtq \underline{u} tf±ti i Rb` $(m + n)\underline{u} = m\underline{u} + n\underline{u}$ m`fWJ hvPvB Ki |

12.8 | Ae⁻vb tf±i (Position Vector)

mgZj tKv⁺bv vbw⁺ O we⁺ j mvtct⁺ H mgZtj i thtKv⁺bv P we⁺ j Ae⁻vb \vec{OP} Øviv vbw⁺ Kiv hvq | \vec{OP} tK O we⁺ j mvtct⁺ P we⁺ j Ae⁻vb tf±i ejv nq Ges O we⁺ tK tf±i i gj we⁺ y (origin) ejv nq |

g⁺tb Kw⁺i, tKv⁺bv mgZtj OGKw⁺ vbw⁺ O we⁺ y Ges GKB mgZtj A Aci GKw⁺ we⁺ y O, A thvM Kijt Drcbæ \vec{OA} tf±i O we⁺ j cwi⁺tc⁺ tZ A we⁺ j Ae⁻vb tf±i ejv nq | Abj⁺cfvte, GKB O we⁺ j tç⁺ tZ GKB mgZtj Aci B we⁺ j Ae⁻vb tf±i \vec{OB} . A, B thvM Kw⁺i |



g⁺tb Kw⁺i, $\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$

Zvntj $\vec{OA} + \vec{AB} = \vec{OB}$ A⁺ $\underline{a} + \vec{AB} = \underline{b}$

$$\therefore \vec{AB} = \underline{b} - \underline{a}$$

mgZivs, ⁺ Bw⁺ we⁺ j Ae⁻vb tf±i Rv⁺bv vKtj Zv⁺ i mst⁺hvRK ti Lv Øviv m⁺PZ tf±i H tf±i i c⁺Š we⁺ j Ae⁻vb tf±i t⁺ tK Aw⁺ we⁺ j Ae⁻vb tf±i wet⁺qm Kti cvl qv hvte |

⁺ b⁺e⁺ : gj we⁺ y wfbæwfbæ Ae⁻vb⁺ vKtj GKB we⁺ j Ae⁻vb tf±i wfbæwfbænt⁺Z cvti | tKvb vbw⁺ O c⁺Š cv⁺ wel⁺ tqi mgvavtb G wel⁺ tqi wetePbvaxb mKj we⁺ j Ae⁻vb tf±i GKB gj we⁺ j mvtct⁺ aiv nq |

KvR : tZigvi LvZiq GKw⁺ we⁺ tK gj we⁺ y O ati w⁺fbæ Ae⁻vb Avi | cvPw⁺ we⁺ y vbtq O we⁺ j mvtct⁺ G⁺ t⁺ vi Ae⁻vb tf±i wPw⁺YZ Ki |

12.9 | KwZcq D⁻ vni Y

D⁻ vni Y | t⁻ Lvl th, (K) $-(-\underline{a}) = \underline{a}$

(L) $-m(\underline{a}) = m(-\underline{a}) = -m\underline{a}$, m GKw⁺ t⁻ j vi |

(M) $\frac{\underline{a}}{|\underline{a}|}$ GKw⁺ GKK tf±i, hLb $\underline{a} \neq \underline{0}$

mgvavb t (K) w⁺cixZ tf±i i ag⁺ Ab⁺hvqx $\underline{a} + (-\underline{a}) = \underline{0}$

Avevi $(-\underline{a}) + (-(-\underline{a})) = \underline{0}$

$\therefore -(-\underline{a}) + (-\underline{a}) = \underline{a} + (-\underline{a})$

$\therefore -(-\underline{a}) = \underline{a}$ [tf±i thv⁺ Mi eR⁺ w⁺va]

(L) $m\underline{a} + (-m)\underline{a} = \{m + (-m)\}\underline{a} = 0\underline{a} = \underline{0}$

$\therefore (-m)\underline{a} = -m\underline{a}$ (1)

Avevi $m\underline{a} + m(-\underline{a}) = m[\underline{a} + (-\underline{a})] = m\underline{0} = \underline{0}$

$\therefore m(-\underline{a}) = -m\underline{a}$ (2)

(1) Ges (2) $\dagger_{\dagger}K (-m)\underline{a} = m(-\underline{a}) = -m\underline{a}$

(M) gtb Kwii \underline{a} Akb" \hat{a} nq| $\dagger f_{\dagger}i i w`K$ eivei \hat{a} GKwU GKK $\dagger f_{\dagger}i$ Ges \underline{a} $\dagger f_{\dagger}i i \hat{a} \hat{a} N^{\circ} a A_{\dagger}$
 $|\underline{a}| = a$

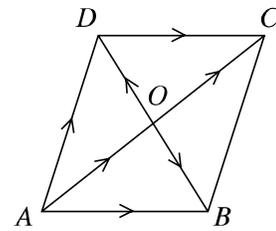
Zvntj $\underline{a} = (a) \hat{a} = |\underline{a}| \hat{a}$; GLvfb $|\underline{a}| = a$ GKwU $\dagger_{\dagger}j$ vi hv Akb" Kwii $\underline{a} \neq \underline{0}$

$\therefore \frac{\underline{a}}{|\underline{a}|} = \frac{|\underline{a}| \hat{a}}{|\underline{a}|} = \hat{a}$ GKwU GKK $\dagger f_{\dagger}i$ |

D`vniY 2| ABCD GKwU mvgvšwi K hvi KY0q AC | BD |

(K) $\overrightarrow{AC}, \overrightarrow{BD}$ $\dagger f_{\dagger}i \theta q \dagger K \overrightarrow{AB}$ Ges \overrightarrow{AD} $\dagger f_{\dagger}i \theta q i$ gva`tg cKvk Ki |

(L) \overrightarrow{AB} Ges \overrightarrow{AD} $\dagger f_{\dagger}i \theta q \dagger K \overrightarrow{AC} | \overrightarrow{BD}$ $\dagger f_{\dagger}i \theta q i$ gva`tg cKvk Ki |



mgravb t (K) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AD} + \overrightarrow{AB}$

Avevi, $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$ ev $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

(L) thtnZimvgvšwi $\dagger K i$ KY0q ci`úi mglwLWZ nq|

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{DB} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{BD}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{BD}$.

D`vniY 3| $\dagger f_{\dagger}i i$ mrvn`th" c0vY Ki th, wlf`fRi th`Kv`bv `β evûi ga`we` \dagger qti msthvRK ti Lvsk H wlf`fRi ZZxq evûi mvgvšvj | Zvi A`taR |

mgravb t gtb Kwii, ABC wlf`fRi AB | AC evû0qti ga`we` \dagger h_vµtg D | E. D, E thvM Kwii |

c0vY Ki $\dagger Z$ nte th, $DE \parallel BC$ Ges $DE = \frac{1}{2}BC$

$\dagger f_{\dagger}i$ wetqv`Mi wlf`fRi eia Abjvnti,

$\overrightarrow{AE} - \overrightarrow{AD} = \overrightarrow{DE} \dots \dots \dots (1)$

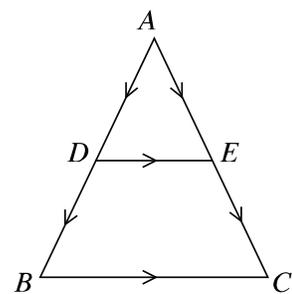
Ges $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$

wKš' $\overrightarrow{AC} = 2\overrightarrow{AE}, \overrightarrow{AB} = 2\overrightarrow{AD}$

[$\therefore D | E$ h_vµtg AB | AC evûi ga`we` \dagger]

$\therefore \overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC} \dagger_{\dagger}K$ cvB

$2\overrightarrow{AE} - 2\overrightarrow{AD} = \overrightarrow{BC} A_{\dagger} 2(\overrightarrow{AE} - \overrightarrow{AD}) = \overrightarrow{BC}$



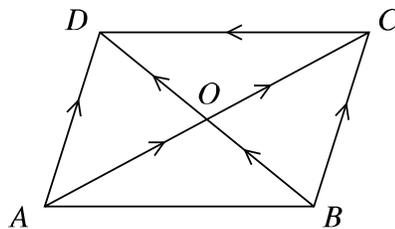
ev, $2\overrightarrow{DE} = \overrightarrow{BC}$, [(1) n'Z]

$$\therefore \overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$$

Avevi $|\overrightarrow{DE}| = \frac{1}{2}|\overrightarrow{BC}|$ ev $DE = \frac{1}{2}BC$ m'Zivs $\overrightarrow{DE} \parallel \overrightarrow{BC}$ t'f'±i'θ'q'i aviK tiLv GKB ev m'gvs'ij |

i'v j | w'Kš' GLv'tb aviK tiLv GK bq | m'Zivs $\overrightarrow{DE} \parallel \overrightarrow{BC}$ t'f'±i'θ'q'i aviK tiLv'θ'q A_# DE Ges BC m'gvs'ij |

D`vniY 4 | t'f'±i c'x'w'Z'Z c'g'vY Ki th, m'gvs'ij t'Ki KY'θ'q ci`'ú'i t'K m'g'w'θ'w'Z K'ti |



m'g'v'ab t' g'tb K'wi, ABCD m'gvs'ij t'Ki $AC \parallel BD$ KY'θ'q ci`'ú'i t'K O w'e`'t'Z t'Q` K'ti t'Q |

g'tb K'wi, $\overrightarrow{AO} = \underline{a}$, $\overrightarrow{BO} = \underline{b}$, $\overrightarrow{OC} = \underline{c}$, $\overrightarrow{OD} = \underline{d}$

c'g'vY Ki t'Z n'te th, $|\underline{a}| = |\underline{c}|$, $|\underline{b}| = |\underline{d}|$

c'g'vY t' $\overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{AD}$ Ges $\overrightarrow{BO} + \overrightarrow{OC} = \overrightarrow{BC}$

m'gvs'ij t'Ki w'eci xZ ev'θ'q ci`'ú'i m'g'v' | m'gvs'ij | $\therefore \overrightarrow{AD} = \overrightarrow{BC}$

$$A_{\#} \overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{BO} + \overrightarrow{OC}$$

$$\text{ev, } \underline{a} + \underline{d} = \underline{b} + \underline{c}$$

$$A_{\#} \underline{a} - \underline{c} = \underline{b} - \underline{d} \text{ [D'f'q c't'q] } - \underline{c} - \underline{d} \text{ t'hwM K'ti |}$$

GLv'tb $\underline{a} \parallel \underline{c}$ Gi aviK AC, $\therefore \underline{a} - \underline{c}$ Gi aviK AC.

$\underline{b} \parallel \underline{d}$ Gi aviK BD, $\therefore \underline{b} - \underline{d}$ Gi aviK BD.

$\underline{a} - \underline{c} \parallel \underline{b} - \underline{d}$ `B'u m'g'v' m'g'v' Akb' t'f'±i n'tj Zv't` i aviK tiLv GKB A_ev m'gvs'ij n'te | w'Kš' AC \parallel BD `B'u ci`'ú'i t'Q` x Am'gvs'ij mij t'Lv | m'Zivs $\underline{a} - \underline{c} \parallel \underline{b} - \underline{d}$ t'f'±i'θ'q Akb' n'tj cv'ti bv w'ev'q G't` i g'v' kb' n'te |

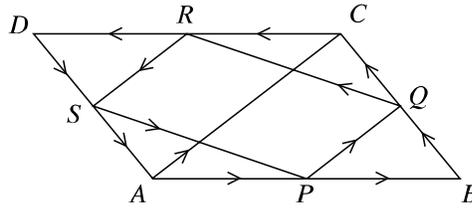
$$\therefore \underline{a} - \underline{c} = \underline{0} \text{ ev } \underline{a} = \underline{c} \text{ Ges } \underline{b} - \underline{d} = \underline{0} \text{ ev } \underline{b} = \underline{d}$$

$$\therefore |\underline{a}| = |\underline{c}| \text{ Ges } |\underline{b}| = |\underline{d}|$$

A_# m'gvs'ij t'Ki KY'θ'q ci`'ú'i t'K m'g'w'θ'w'Z K'ti |

D`vniY 5 | t'f'±i c'x'w'Z'Z c'g'vY Ki th, t'Kv't'bv PZ'f'P'ri m'w'bu'Z ev n'tj vi ga'w'e`'j m'st'hw'RK ti Lv m'g'v' GK'w' m'gvs'ij K Drc'ba K'ti |

mgvavb t gtb Kwī, ABCD PZfRi
 evū, tj vi ga'we> y P, Q, R, S | P | Q, Q
 | R, R | S Ges S | P thvM Kwī | cōvY
 Ki tZ nte th, PQRS GKwU mgvšwi K |



cōvY t gtb Kwī, $\overline{AB} = \underline{a}$, $\overline{BC} = \underline{b}$, $\overline{CD} = \underline{c}$, $\overline{DA} = \underline{d}$

$$\text{Zvntj, } \overline{PQ} = \overline{PB} + \overline{BQ} = \frac{1}{2} \overline{AB} + \frac{1}{2} \overline{BC} = \frac{1}{2} (\underline{a} + \underline{b})$$

$$\text{Abjfcvte, } \overline{QR} = \frac{1}{2} (\underline{b} + \underline{c}), \quad \overline{RS} = \frac{1}{2} (\underline{c} + \underline{d}) \text{ Ges } \overline{SP} = \frac{1}{2} (\underline{d} + \underline{a})$$

$$\text{Kš' } (\underline{a} + \underline{b}) + (\underline{c} + \underline{d}) = \overline{AC} + \overline{CA} = \overline{AC} - \overline{AC} = \vec{0}$$

$$\text{A_ŕ } \underline{a} + \underline{b} = -(\underline{c} + \underline{d})$$

$$\overline{PQ} = \frac{1}{2} (\underline{a} + \underline{b}) = -\frac{1}{2} (\underline{c} + \underline{d}) = -\overline{RS} = \overline{SR}$$

∴ PQ Ges SR mgvb l mgvšivj |

Abjfcvte, QR Ges PS mgvb l mgvšivj |

∴ PQRS GKwU mgvšwi K |

Abkxj bxÑ12

1| $AB \parallel DC$ ntj

i $\overline{AB} = m \cdot \overline{DC}$, thLvfb m GKwU t'j vi i vk

ii $\overline{AB} = \overline{DC}$

iii $\overline{AB} = \overline{CD}$

I cti i evK' tj vi gta' tKvbwU mwVK?

- | | |
|-----------|----------------|
| K. i | L. ii |
| M. i l ii | N. i, ii l iii |

2| ũU t'±i mgvšivj ntj -

i Gt' i thvM i t'ŕt'ŕ mgvšwi K vewa cōhvR'

ii Gt' i thvM i t'ŕt'ŕ vŕ fR vewa cōhvR'

iii Gt' i ũN' me'v mgvb

I cti i evK' tj vi gta' tKvbwU mwVK?

- | | |
|-----------|----------------|
| K. i | L. ii |
| M. i l ii | N. i, ii l iii |

3| $AB = CD$ Ges $AB \parallel CD$ ntj tKvbwU mwVK?

K. $\overline{AB} = \overline{CD}$

L. $\overline{AB} = m \cdot \overline{CD}$ thLvfb $m > 1$

M. $\overline{AB} + \overline{DC} < O$

N. $\overline{AB} + m \cdot \overline{CD} = O$ thLvfb $m > 1$

Wbtpi Zt_i Avtj vtK 4 | 5 baf c0k0 DEi `vl :

AB ti Lvstki Dci thtKvtrv we`y C Ges tKvtrv tfti gj we`j mvtct¶ A, B | C we`j Ae`vb tfti h_vµtg a, b, | c |

4 | C we`j AB ti LvstK 2:3 AbvrtZ Ašief© Ki tj wbtPi tKvtrv mWk?

K. $\underline{c} = \frac{a+2b}{5}$

L. $\underline{c} = \frac{2a+b}{5}$

M. $\underline{c} = \frac{3a+2b}{5}$

N. $\underline{c} = \frac{2a+3b}{5}$

5 | tfti gj we`j O ntj wbtPi tKvtrv mWk?

K. $\overrightarrow{OA} = \underline{a} - \underline{b}$

L. $\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{AC}$

M. $\overrightarrow{AB} = \underline{b} - \underline{a}$

N. $\overrightarrow{OC} = \underline{c} - \underline{b}$

6 | ABC wlfri BC, CA, AB evutqi ga`we`yh_vµtg D, E, F ntj ,

(K) \overrightarrow{BC} , \overrightarrow{AD} , \overrightarrow{BE} , \overrightarrow{CF} tfti ,tj vtK \overrightarrow{AB} Ges \overrightarrow{AC} tfti i gva`tg cKvk Ki |

(L) \overrightarrow{BC} , \overrightarrow{CA} , \overrightarrow{AD} tfti ,tj vtK \overrightarrow{BE} Ges \overrightarrow{CF} tfti i gva`tg cKvk Ki |

7 | ABCD mgvšwi tKi KY0q \overrightarrow{AC} | \overrightarrow{BD} ntj \overrightarrow{AB} | \overrightarrow{AC} tfti 0qtK \overrightarrow{AD} | \overrightarrow{BD}

tfti 0tqi gva`tg cKvk Ki Ges t`Lvl th, $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$ Ges $\overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$

8 | t`Lvl th, (K) $-(\underline{a} + \underline{b}) = -\underline{a} - \underline{b}$

(L) $\underline{a} + \underline{b} = \underline{c}$ ntj $\underline{a} = \underline{c} - \underline{b}$

9 | t`Lvl th (K) $\underline{a} + \underline{a} = 2\underline{a}$ (L) $(m - n)\underline{a} = m\underline{a} - n\underline{a}$

(M) $m(\underline{a} - \underline{b}) = m\underline{a} - m\underline{b}$

10 | (K) \underline{a} , \underline{b} c0Z tK Akb` tfti ntj t`Lvl th, $\underline{a} = m\underline{b}$ ntZ cvti tKej gvI hw` \underline{a} , \underline{b} Gi mgvšivj nq |

(L) \underline{a} , \underline{b} Akb` Amgvšivj tfti Ges $m\underline{a} + n\underline{b} = 0$ ntj t`Lvl th, $m = n = 0$

11 | A, B, C, D we`j ,tj vi Ae`vb tfti h_vµtg a, b, c, d ntj t`Lvl th, ABCD mgvšwi K nte

hw` Ges tKej gvI hw` $\underline{b} - \underline{a} = \underline{c} - \underline{d}$ nq |

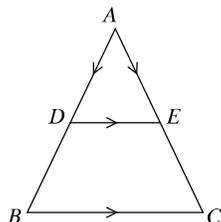
12 | tfti i mrvth` c0vY Ki th, wlfri GK evui ga`we`yt_tK Aw4Z Aci evui mgvšivj ti Lv ZZxq evui ga`we`Mvgx |

13 | c0vY Ki th, tKvtrv PZfRi KY0q ci`ui tK mgvšivj tK ntj Zv GKw mgvšwi K nq |

14 | tfti i mrvth` c0vY Ki th, UmcwRqvtgi Amgvšivj evu0tqi ga`we`j msthvRK mij ti Lv mgvšivj evu0tqi mgvšivj | Zv`i thvMd tji AfaR |

15 | $\vec{AD} + \vec{DE} = \vec{AE}$ Ki th, $\vec{AD} + \vec{DE} = \vec{AC}$ ga'we' y msthrRK mij ti Lv mgvš+vj evú0tqi mgvš+vj Ges Zv' i we'qVMDtj i AtAR |

16 |



ΔABC Gi $AB \parallel AC$ evú i ga'we' y h_vμtg D | E

K. $(\vec{AD} + \vec{DE})$ tK \vec{AC} t'f±ti i gva'tg cKvk Ki |

L. t'f±ti i m'vnt'h' c0vY Ki th, $AB \parallel DC$ Ges $DE = \frac{1}{2} BC$

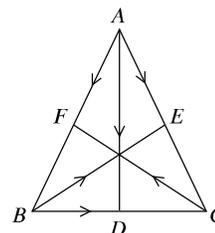
M. ABCD U'ncwRqvtgi KY0tqi ga'we' y h_vμtg M | N ntj t'f±ti i m'vnt'h' c0vY Ki th, $MN \parallel DE \parallel BC$ Ges $MN = \frac{1}{2} (BC - DC)$

17 | ΔABC Gi $BC, CA \parallel AB$ evú i ga'we' y h_vμtg D, E | F

K. \vec{AB} t'f±ti tK $\vec{BE} \parallel \vec{CF}$ t'f±ti i gva'tg cKvk Ki |

L. c0vY Ki th, $\vec{AD} + \vec{BE} + \vec{CF} = \vec{O}$

M. t'f±ti i m'vnt'h' c0vY Ki th, F we' yw' tq A w' Z BC Gi mgvš+vj ti Lv Aek' B E we' ylvngx nte |



Îtqv`k Aa`vq Nb R`wguwZ

Avqv`i ev`e Rxe`tb we`wfbœAvKv`i i Nbe`i c`0qvRb I Zvi e`envi me`vB n`tq Zv`K | Gi gta` m`j g I
we`l g AvKv`i i Nbe`i Av`Q | m`j g AvKv`i i Nbe`i Ges `Bw m`j g Nbe`i mg`stq MwZ th`šMK
Nbe`i AvqZb I c`pZ`j i t`qI`dj w`bY`q c`xwZ GB Aa`vtq Av`j vPbv Kiv n`te |

Aa`vq tk`I w`k`qIv`xPv

- Nbe`i c`0Z`Kxq w`P`T A`¼b Ki`Z cvi`te |
- w`cRg, w`ciw`gW AvKw`Zi e`i; tMvj K I mge`E`f`w`gK tKiv`tKi AvqZb Ges c`pZ`j i t`qI`dj w`bY`q Ki`Z
cvi`te |
- Nb R`wguwZi avi Yv c`0qvM K`ti m`gm`v m`gvarb Ki`Z cvi`te |
- th`šMK Nbe`i AvqZb I c`pZ`j i t`qI`dj cwi`gvc Ki`Z cvi`te |
- Nb R`wguwZi avi Yv e`envi K t`qI`t c`0qvM Ki`Z cvi`te |

13.1 tg`šwj K avi Yv

gva`w`gK R`wguwZ`Z w`e`y ti Lv I Z`j i tg`šwj K avi Yv Av`j w`PZ n`tq`Q | Nb R`wguwZ`Z w`e`y ti Lv I
Zj tg`šwj K avi Yv w`ntm`te M`thY Kiv nq |

- 1 | e`i `N`0, c`0`I D`PZv c`0Z`Kw`tK H e`i gv`Iv (dimension) ej v nq |
- 2 | w`e`y `N`0, c`0`I D`PZv t`b | GwU GKwU avi Yv | ev`te tevSvi R`tb` Avgiv GKwU WU (.) e`envi
Kwi | G`K Ae`v`tbi c`0Z`i`c ej v th`Z cvi`te | m`Zi`vs w`e`y tKvb gv`Iv t`b | ZvB w`e`y`k`b` gw`I`K |
- 3 | ti Lvi tKej gv`I `N`0 Av`Q, c`0`I D`PZv t`b | ZvB ti Lv GKgw`I`K |
- 4 | Z`j i `N`0 I c`0`I Av`Q, D`PZv t`b | ZvB Zj w`0gw`I`K |
- 5 | th e`i `N`0, c`0`I D`PZv Av`Q, Zv`K Nbe`i`ej v nq | m`Zi`vs Nbe`i`w`I gw`I`K |

13.2 Kw`Zcq c`0`w`gK ms`Av

- 1 | mgZj (Plane surface) t tKv`tbv Z`j i Dci`th tKv`tbv `BwU w`e`y m`sthvRK mij`ti Lv m`u`Y`P`f`c
H Z`j i Dci Aew`Z n`j, H Zj`K mgZj ej v nq | c`K`ti i c`w`b w`i`_vK`j H c`w`bi Dcwi`f`w`M
GKwU mgZj | w`ntg`U w`tq w`b`w`g`Z ev`tgvRvBKZ N`ti i tg`st`K Avgiv mgZj e`tj`_w`K | w`K`S`
R`wguwZK`f`ite Zv mgZj bq, Kvi Y N`ti i tg`st`Z w`KQyD`P`z`w`b`P`z`v`t`KB |
`b`e` t Ab` w`KQyD`t`j`L bv`_vK`j Nb R`wguwZ`Z ti Lv ev` `N`0 Ges Z`j i w`e`vi Am`xg (infinite) ev
Aw`b`w`0`g`tb Kiv nq | m`Zi`vs Z`j i ms`Av t`_t`K Ab`g`vb Kiv hvq th, tKv`tbv mij`ti Lv GKwU Ask tKv`tbv
Z`j i Dci`_vK`j Aci tKv`tbv Ask H Z`j i ev`B`ti`_vK`Z cvi`te bv |
- 2 | eµZj (Curved surface) t tKv`tbv Z`j i Dci Aew`Z th`tKv`tbv `BwU w`e`y m`sthvRK mij`ti Lv
m`u`Y`P`f`c H Z`j i Dci Aew`Z bv n`j, H Zj`K eµZj ej v nq | tMvj`tKi c`pZ`j GKwU eµZj |

3| Nb R'wguZ (Solid geometry) t MmYZ kvf`j th kvLvi mrvvth` Nbe` Ges Zj , ti Lv l we`j ag®Rvbr hvq, ZvfK Nb R'wguZ ejv nq| KLbl KLbl GtK RvMmZK R'wguZ (Geometry of space) ev wlgvwlK R'wguZI (Geometry of three dimensions) ejv nq|

4| GKZj xq ti Lv (Coplanar straight lines) t GKwaK mij ti Lv GKB mgZtj Aew`Z ntj , ev Zvf` i mKtj i ga` w`tq GKwU mgZj A¼b mæe ntj H mij ti Lv ,ti vtfK GKZj xq ejv nq|

5| `bKZj xq ti Lv (Skew or non coplanar lines) t GKwaK mij ti Lv GKB mgZtj Aew`Z bv ntj ev Zvf` i ga` w`tq GKwU mgZj A¼b Kiv mæe bv ntj G`tj vtfK `bKZj xq mij ti Lv ejv nq| `BwU tcvYj tK GKwU Dci Avi GKwU w`tq thvM ev `YwPy AvKwZi GKwU e`%Zwi Kijtj B `BwU `bKZj xq mij ti Lv Drcbæte|

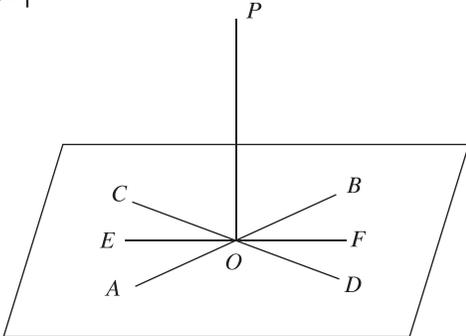
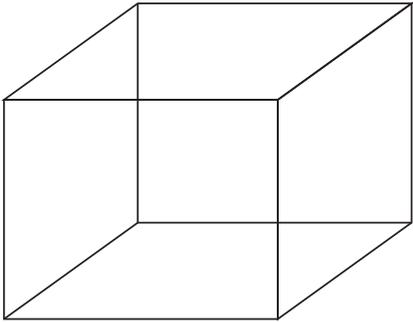
6| mgvš+vj mij ti Lv (Parallel line) t `BwU GKZj xq mij ti Lv hw` ci`úi tQ` bv Kti A_® hw` Zvf` i tKvfbv mrvviY we`y bv _vtfK, Zte Zvf` i mgvš+vj mij ti Lv ejv nq|

7| mgvš+vj Zj (Parallel planes) t `BwU mgZj hw` ci`úi tQ` bv Kti A_® hw` Zvf` i tKvfbv mrvviY ti Lv bv _vtfK Zte H Zj ØqtK mgvš+vj Zj ejv nq|

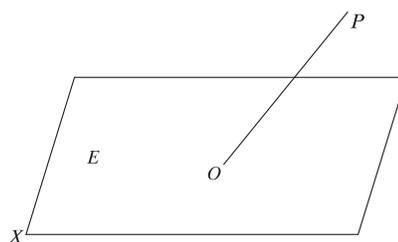
8| mgZtj i mgvš+vj ti Lv t GKwU mij ti Lv l GKwU mgZj tK Awbw` Øfvte ewæZ Kijtj l hw` Zviv ci`úi tQ` bv Kti, Zte H mij ti Lv tK D³ Ztj i mgvš+vj ti Lv ejv nq|

`be` t mrvviY wlgvwlK e`i Qwe wlgvwlK KvMR ev tevW®A¼b wKQjlv RvUj | ZvB tKvYKt¶ cvV` vbKvtj cØZ`KwU msÁvi e`vL`vi m½ Zvi GKwU wPÎ A¼b Kti t`wL`q w`tj we l qwU wK¶v_x® i ct¶ tevSv l gtb ivLv mnRZi nte|

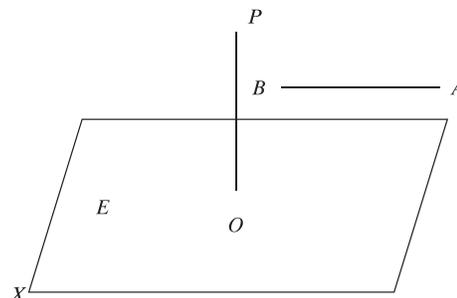
9| Ztj i j æ^ti Lv (Normal or perpendicular to a plane) t tKvfbv mij ti Lv GKwU mgZtj i Dci ` tKvfbv we`y t_ tK H mgZtj i Dci Aw¼Z tKvfbv we`y t_ tK H mgZtj i Dci Aw¼Z th tKvfbv ti Lvi Dci j æ^ntj , D³ mij ti Lv tK H mgZtj i Dci j æ^ejv nq|



10| wZhR (Oblique) tiLv t tKvb mij tiLv GKwU mgZtj i mvt_ mgvš+vj ev j^α bv ntj, H mij tiLv tK mgZtj i wZhR tiLv ejv nq|



11| Dj^α (Vertical) tiLv ev Zj t wⁱ Ae⁻vq Sj š-l j tbi mZvi m½ mgvš+vj tKt^{bv} tiLv ev Zj tK Lvov ev Dj^αZj etj |

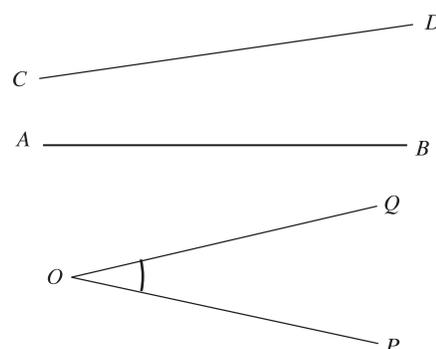


12| Abf^wgK (Horizontal) Zj I tiLv t tKt^{bv} mgZj GKwU Lvov mij tiLvi mvt_ j^α ntj, ZvtK kvvb ev Abf^wgK Zj ejv nq| Avevi tKt^{bv} Abf^wgK Ztj Aew⁻Z th tKvb mij tiLv tK Abf^wgK mij tiLv ejv nq|

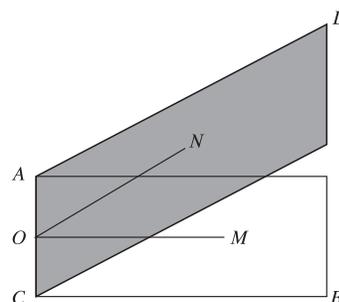
13| mgZj I bKZj xq PZ^fR t tKt^{bv} PZ^fRi ev^u, tjv mKtj GB Ztj Aew⁻Z ntj, ZvtK mgZj PZ^fR ejv nq| Avevi tKt^{bv} PZ^fRi ev^u, tjv mKtj GKB Ztj Aew⁻Z bv ntj, H PZ^fR tK bKZj xq PZ^fR ejv nq| bKZj xq PZ^fRi b^wU m^wuⁿZ ev^u GKZtj Ges Aci b^wU Ab^o Ztj Aew⁻Z| mZivs tKt^{bv} bKZj xq PZ^fRi w^ecixZ ev^uθq bKZj xq|

14| bKZj xq tiLvi AšM^Z tKvY t b^wU bKZj xq tiLvi AšM^Z tKvY Zvtⁱ i th tKt^{bv} GKwU I Zvi Dci⁻ th tKt^{bv} w^ey^t tK Aw⁴Z Aci^wU mgvš+vj tiLvi AšM^Z tKvYi mgvb| Avevi b^wU bKZj xq tiLvi c⁰Z tKi mgvš+vj b^wU tiLv tKt^{bv} w^ey^t Z A⁴b Ki tj H w^ey^t Z Drcb^etKvYi c^wigvYI bKZj xq tiLv t^qi AšM^Z tKvYi mgvb|

gtb Kwⁱ, AB I CD b^wU bKZj xq tiLv| th tKt^{bv} O w^ey^t Z AB I CD Gi mgvš+vj h^vμtg OP Ges OQ tiLvθq A⁴b Ki tj ∠POQ B AB I CD Gi AšM^Z tKvY w^bt⁻ R Ki te|

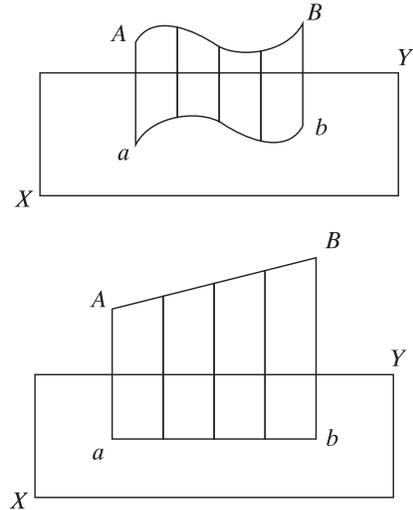


15| w⁰Zj tKvY (Dihedral angle) t b^wU mgZj mij tiLvq t⁰ Ki tj Zvtⁱ i t⁰ tiLv⁻ th tKt^{bv} w^ey^t tK H mgZj t^qi c⁰Z tKi Dci H t⁰ tiLvi mvt_ j^α Gi c GKwU Kti tiLv A⁴b Ki tj Drcb^etKvYB H mgZj t^qi AšM^Z w⁰Zj tKvY|



AB I CD mgZj Øq AC tiLvq ci`úi tQ` Kti:tQ| AC tiLv` O we`jZ AB mgZtj OM Ges CD mgZtj ON Gifc `BmW mij tiLv A¼b Kiv ntjv thb Zviv DfqB AC Gi m½ O we`jZ j`nq| Zvntj $\angle MON$ B AB I CD mgZj Øtqi AŠMZ wØZj tKvY mWPZ Kti | `BmW ci`úf`Q`x mgZtj i AŠMZ wØZj tKvYi cwi gvY GK mgtKvY ntj , H mgZj Øq ci`úi j`nq|

16| Awf`f`c t tKvfbv we`y t`f`K GKwW wbw`Ø mij tiLvi Dci ev tKvfbv mgZtj i Dci Aw¼Z j`nq| Lvi cv`we`jK H tiLv ev mgZtj i Dci D³ we`j cvZb ev Awf`f`c (Projection) ejv nq| tKvfbv mij tiLv ev eµtiLvi mKj we`y t`f`K tKvfbv wbw`Ø mgZtj i Dci Aw¼Z j`nq| tvi cv`we`yngf`ni tmUf`K H mgZtj i Dci D³ mij tiLv ev eµtiLvi Awf`f`c ejv nq| GB Awf`f`c tK j`nq| Awf`f`c i (Orthogonal Projection) ejv nq|



wP`f` XY mgZtj i Dci GKwW eµtiLvi I GKwW mij tiLvi Awf`f`c t` Lvfbv ntqtQ|

13.3 `BmW mij tiLvi gfa` m`uK©

(K) `BmW mij tiLv GKZj xq ntZ cvti , tmf`f`f` Zviv Aek`B mgvš+vj nte ev tKvfbv GK we`jZ ci`úi tQ` Kite|

(L) `BmW mij tiLv `bKZj xh ntZ cvti , tmf`f`f` Zviv mgvš+vj I nte bv wKsev tKvfbv we`jZ tQ` I Kite bv|

13.4 `Ztwx

(K) tKvfbv mgZtj i Dci `BmW we`j msthvRK mij tiLv tK Awbw`Øfvte euaZ Kiti I Zv m`uYfvte H mgZtj Aew`Z _vKte| mZivs GKwW mij tiLv I GKwW mgZtj i gfa` `BmW mvaviY we`y _vKte , H mij tiLv eivei Zv` i gfa` AmsL` mvaviY we`y _vKte|

(L) `BmW wbw`Ø we`yev GKwW mij tiLvi ga` w`fq AmsL` mgZj A¼b Kiv hvq|

13.5 mij tiLv I mgZtj i gfa` m`uK©

(K) GKwW mij tiLv GKwW mgZtj i m½ mgvš+vj ntj Zv` i gfa` tKvfbv mvaviY we`y _vKte bv|

(L) GKwW mij tiLv tKvfbv mgZtj tK tQ` Kiti Zv` i gfa` gv` GKwW mvaviY we`y _vKte|

(M) hw` tKvfbv mij tiLv I mgZtj i `BmW mvaviY we`y _vKte , Zvntj m`uY©mij tiLwW H mgZtj Aew`Z nte|

13.6 `BmW mgZtj i gfa` m`uK©

(K) `BmW mgZj ci`úi mgvš+vj ntj Zv` i gfa` tKvfbv mvaviY we`y _vKte bv|

(L) `BmW mgZj ci`úi tQ`x ntj Zviv GKwW mij tiLvq tQ` Kite Ges Zv` i AmsL` mvaviY we`y _vKte|

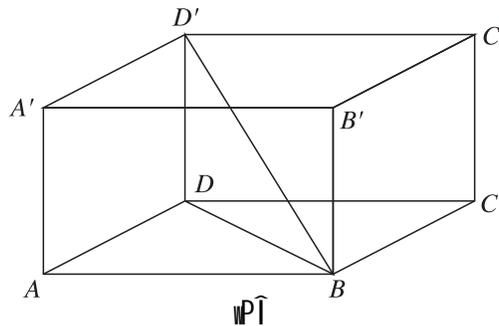
13.7 Nbe⁻

Avgiv Rmb, GKlvb eB ev GKlvb BU ev GKwU ev ev GKwU tMvj vKvi ej meB Nbe⁻ Ges Zviv cØZ⁻KB wKQy cwi gvb ⁻vb (Space) `Lj Kti ⁻vFK Avevi GKLU cv_i ev KvV, BtUi GKwU LU, Kqjvi UKiv, GtUj gwUi i Kbv LU BZ⁻wi I Nbe⁻i D⁻vni Y| Zte G₃ t_j v weig Nbe⁻ | mgZj A_{ev} eμZj Øviv tewóZ kþi wKQy ⁻vb `Lj Kti ⁻vFK Gi e⁻tK Nbe⁻ (Solid) ej v nq | mgZj ⁻ tKvþv ⁻vb tK teób Ki tZ ntj thgb, Kgc t³ wZbwU mij t_i Lv ⁻ i Kvi tZgwB RvMwZK tKvþv ⁻vb tK teób Ki tZ ntj AšZ PviwU mgZj ⁻ i Kvi | GB Zj ₃ t_j v Nbe⁻i Zj ev cØZj (Surface) Ges Gt⁻ i ⁻ wU mgZj th t_i Lvq tQ⁻ Kti, ZvFK H Nbe⁻i avi (Edge) ej v nq | GKwU evt⁻ i ev GKlvb BtUi QquU cØZj AvtQ Ges eviwU avi AvtQ | GKwU wμtKU ej gv¹ GKwU eμZj Øviv Ave^x |

- KvR: 1 | tZvgiv cØZ⁻KB GKwU Kti mlg Nbe⁻ I weig Nbe⁻i bvg wj L |
 2 | tZvgvi D₃ wLZ Nbe⁻ t_j vi KtqKwU e⁻envi wj L |

13.8 mlg Nbe⁻i AvqZb I Ztj i t³ d_j

1 | AvqWZK Nb ev AvqZvKvi Nbe⁻ (Rectangular Parallelopiped)



wZbtRvov mgvš³ vj mgZj Øviv Ave^x Nbe⁻tK mgvš³ K Nbe⁻ ej v nq | GB QquU mgZt_j i cØZ⁻KwU GKwU mgvš³ K Ges weci xZ cØ₃ t_j v me³ Zvfvte mgvb | mgvš³ K Nbe⁻i wZbwU ⁻ t_j wef³ eviwU avi AvtQ |

th mgvš³ K Nbe⁻i cØZj ₃ t_j v AvqZt³ d_j, ZvFK AvqZvKvi Nbe⁻ ej v nq | th AvqZvKvi Nbe⁻i cØZj ₃ t_j v eM³ d_j, ZvFK NbK (Cube) ej v nq | Dctiv³ w³ d_j AvqZvKvi Nbe⁻ i Ges Nb³ Ki cØ₃ t_j v ABCD, A'B'C'D', BCC'B', ADD'A', ABB'A', DCC'D' Ges avi ₃ t_j v AB, A'B', CD, C'D', BC, B'C', AD, A'D', AA', BB', CC', DD' Ges GKwU KY³ BD'.

g³ b Kw³, AvqZvKvi Nbe⁻i ³ N³, c³ -I D³ PZv h₃ v μt³ g AB = a GKK AD = b GKK Ges AA' = c GKK |

(K) AvqZvKvi Nbe⁻i mgM³ Zt_j i t³ d_j (Area of the whole surface)

= QquU c³ t_j i t³ d_j i mgwó

$$\begin{aligned}
 &= 2(ABCD \text{ Ztj i t}\hat{\eta}\hat{\Gamma} \text{ dj} + ABB'A' \text{ Ztj i t}\hat{\eta}\hat{\Gamma} \text{ dj} + ADD'A' \text{ Ztj i t}\hat{\eta}\hat{\Gamma} \text{ dj}) \\
 &= 2(ab + ac + bc) \text{ eM\textcircled{G}KK} \\
 &= 2(ab + bc + ca) \text{ eM\textcircled{G}KK}
 \end{aligned}$$

(L) AvqZb (Volume) = AB × AD × AA' NbGKK = abc NbGKK

(M) $KY\textcircled{B}D' = \sqrt{BD^2 + DD'^2} = \sqrt{AB^2 + AD^2 + DD'^2} = \sqrt{a^2 + b^2 + c^2}$ GKK

2 | Nbtki t}\hat{\eta}\hat{\Gamma}, a = b = c. AZGe

(K) $mgM\textcircled{Z}tj \text{ i t}\hat{\eta}\hat{\Gamma} \text{ dj} = 2(a^2 + a^2 + a^2) = 6a^2$ eM\textcircled{G}KK

(L) AvqZb = a. a. a = a³ NbGKK

(M) $KY\textcircled{=} \sqrt{a^2 + a^2 + a^2} = \sqrt{3} a$ GKK |

D`vniY 1 | GKwU AvqZvKvi Nbe`i ^`N\textcircled{,} c\textcircled{'} I D"pZvi AbjcvZ 4: 3: 2 Ges Zvi mgM\textcircled{Z}tj \text{ i t}\hat{\eta}\hat{\Gamma} \text{ dj} 468 eM\textcircled{g}Uvi ntj , Zvi KY\textcircled{=} AvqZb wby\textcircled{=} Ki |

mgvavb : gtb Kwi , ^`N\textcircled{,} c\textcircled{'} I D"pZv h_v\textcircled{=}tj 4x, 3x, 2x w\textcircled{=}Uvi |

Zvntj , $2(4x \cdot 3x + 3x \cdot 2x + 2x \cdot 4x) = 468$

ev, $52x^2 = 468$ ev, $x^2 = 9$ ∴ $x = 3$

∴ Nbe`i ^`N\textcircled{,} 12 w\textcircled{,}, c\textcircled{'} 9 w\textcircled{,}. Ges D"pZv 6 w\textcircled{,}.

Bnvi KtY\textcircled{=} ^`N\textcircled{=} = $\sqrt{12^2 + 9^2 + 6^2} = \sqrt{144 + 81 + 36} = \sqrt{261}$ w\textcircled{=}Uvi = 16.16 w\textcircled{=}Uvi (c\textcircled{=}q)

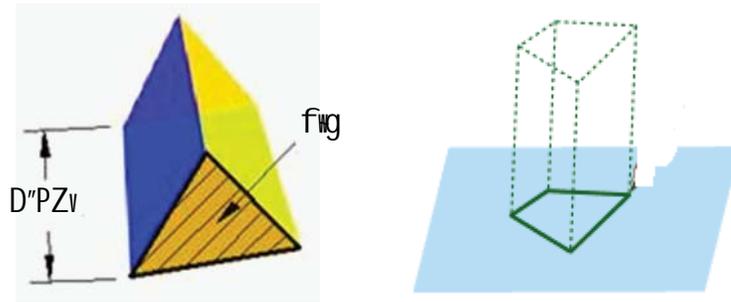
Ges AvqZb = $12 \times 9 \times 6 = 648$ Nb\textcircled{=}Uvi |

KvR : 1 | w\textcircled{=}Rt\textcircled{=}w\textcircled{=} GKwU t\textcircled{=}U ev. (KvU\textcircled{=} A_ev JI\textcircled{=}tai tevZtj i c`v\textcircled{=}KU) Gi ^`N\textcircled{,} c\textcircled{'} I D"pZv t\textcircled{=}c Zvi AvqZb, Q\textcircled{=}U Ztj i t}\hat{\eta}\hat{\Gamma} \text{ dj I KtY\textcircled{=} ^`N\textcircled{=}wby\textcircled{=} Ki |

3 | w\textcircled{=}Rg (Prism)

th Nbe`i `B c\textcircled{=}S-me\textcircled{=}g I mgv\textcircled{=}i+j euf\textcircled{=}R \textcircled{=}v Avex Ges Ab`vb` Zj ,tj v mgv\textcircled{=}wi K Zv\textcircled{=}K w\textcircled{=}Rg etj | w\textcircled{=}Rt\textcircled{=}gi `B c\textcircled{=}S\textcircled{=}K Bnvi f\textcircled{=}g Ges Ab`vb` Zj ,tj v\textcircled{=}K cvk\textcircled{=}Zj etj | me ,tj v cvk\textcircled{=}Zj AvqZvKvi ntj w\textcircled{=}RgU\textcircled{=}K Lvov w\textcircled{=}Rg Ges Ab`t}\hat{\eta}\hat{\Gamma} w\textcircled{=}RgU\textcircled{=}K Zxh\textcircled{=}R w\textcircled{=}Rg ejv nq | ev`e t}\hat{\eta}\hat{\Gamma} Lvov w\textcircled{=}RgB A\textcircled{=}K e`enZ nq | f\textcircled{=}g Ztj i bv\textcircled{=}gi Dci w\textcircled{=}f\textcircled{=} Kti w\textcircled{=}Rt\textcircled{=}gi bvgKiY Kiv nq | thgb, w\textcircled{=}f\textcircled{=}RvKvi w\textcircled{=}Rg, PZf\textcircled{=}RvKvi w\textcircled{=}Rg, c\textcircled{=}f\textcircled{=}RvKvi w\textcircled{=}Rg BZ`w` |

f\textcircled{=}g m\textcircled{=}g euf\textcircled{=}R ntj w\textcircled{=}Rgt\textcircled{=}K m\textcircled{=}g w\textcircled{=}Rg (Regular prism) etj | f\textcircled{=}g m\textcircled{=}g bv ntj Bnvt\textcircled{=}K w\textcircled{=}l g w\textcircled{=}Rg (Irregular prism) ejv nq | ms\textcircled{=}vbmvti AvqZvKvi Nbe`i I NbK Df\textcircled{=}q\textcircled{=}KB w\textcircled{=}Rg ejv nq | Kv\textcircled{=}Pi `Zwi Lvov w\textcircled{=}f\textcircled{=}RvKvi w\textcircled{=}Rg Av\textcircled{=}tj vKi w\textcircled{=}k\textcircled{=}i w\textcircled{=}Q\textcircled{=}i t}\hat{\eta}\hat{\Gamma} Rb` e`enZ nq |



β ai tbi wRg

K) wRtgi mgMZtj i tñĀdj
 = 2 (fwi tñĀdj) + cvkZj ,tjvi tñĀdj
 = 2 (fwi tñĀdj) + fwi cwi mxgv × D" PZv
 L) AvqZb = fwi tñĀdj × D" PZv

D`vni Y 2 | GKw wĪ fRvKvi wRtgi fwi evu ,tjvi N`h_vµtg 3, 4 l 5 tm. wg. Ges D" PZv 8 tm. wg. | Bnvi mgMZtj i tñĀdj l AvqZb wbyĒ Ki |
 mgvavb : wRtgi fwi evu ,tjvi N`h_vµtg 3, 4 l 5 tm. wg. |

thtnZi $3^2 + 4^2 = 5^2$, Bnvi fwi GKw mgkVyx wĪ fR hvi tñĀdj = $\frac{1}{2} \times 4 \times 3 = 6$ eMm. wg.

∴ wRguĪ mgMZtj i tñĀdj = $2 \times 6 + \frac{1}{2} (3 + 4 + 5) \times 8 = 12 + 48 = 60$ eMm. wg.

Ges Bnvi AvqZb = $6 \times 8 = 48$ Nb tm. wg.

AZGe wRguĪ mgMZtj i tñĀdj 60 eMm. wg. Ges AvqZb 48 Nb tm. wg. |

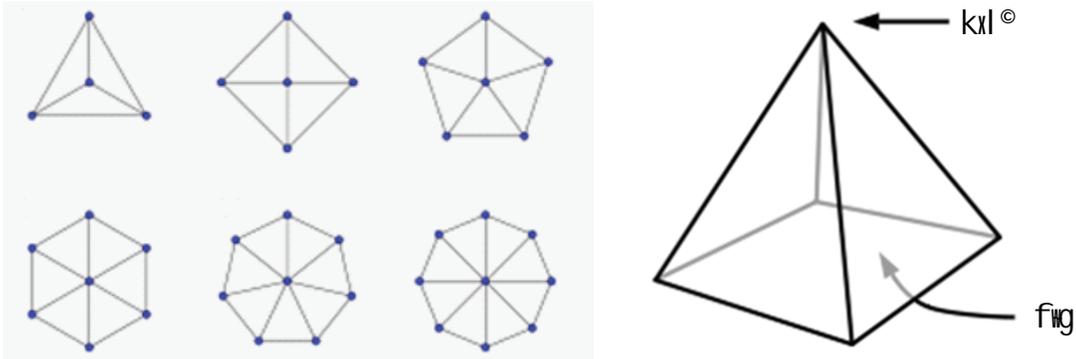
4. wciwgW (Pyramid)

eufRi Dci Aew`Z th Nbe`i GKw kxl e`y_vtK Ges hvi cvkZj ,tjvi cĀZ`Kw wĪ fRvKvi ZvtK wciwgW etj |

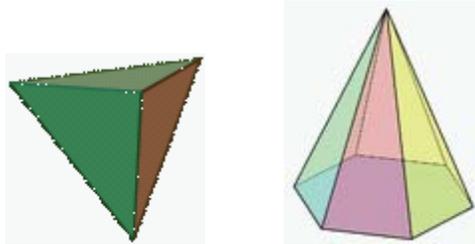
wciwgtWi fwi thtKvtr AvKvti eufR Ges Zvi cvkZj ,tjvi thtKvtr ai tbi wĪ fR ntZ cvti | Zte fwi mlg eufR Ges cvkZj ,tjv mefġ wĪ fR ntj ZvtK mlg wciwgW ej v nq | mlg wciwgW ,tjv LpB `wob`b | kxl e`y l fwi thtKvtr tKŠYK we`j msthvRK ti LvK wciwgtWi avi etj | kxl ntZ fwi Dci AswKZ j % NtK wciwgtWi D" PZv ej v nq |

Zte Avgiv wciwgW etj mPivPi eMkvi fwi Dci Aew`Z Pviw mefġ wĪ fR ōviv tewZ Nbe`tKB ejs | GB ai tbi wciwgtWi euj e`envi AvtQ |

Pri wU mgevü wî fR ðviv tewóZ Nbe`tk m|g PZi`j K (Regular tetrahedron) etj hv GKwU wci wgw | GB wci wgwWi 3 + 3 = 6 wU avi | 4 wU `KšwYK we`yAvtQ | Bnvi kxl`htZ fvgi Dci AwZ j`fvgi fi`tk`cwwZ nq |



wefbæaitbi wci wgwWi fvgi bKkv



wci wgw

K) wci wgwWi mgMz`tj i t`q`d`j
 = fvgi t`q`d`j + cvkZj`_tj vi t`q`d`j
 wKš`cvkZj`_tj v mef`g wî fR ntj ,

$$\text{wci wgwWi mgMz`tj i t`q`d`j} = \text{fvgi t`q`d`j} + \frac{1}{2} (\text{fvgi cwi wa} \times \text{tnj v`bv D`PZv})$$

wci wgwWi D`PZv h, fvg t`q`d`i Aš`e`i e`vma`r Ges tnj v`bv D`PZv l ntj , $l = \sqrt{h^2 + r^2}$

$$\text{L) AvqZb} = \frac{1}{3} \times \text{fvgi t`q`d`j} \times \text{D`PZv}$$

D`vniY 3 | 10 tm. wg. evüwewkó eMfKvi fvgi Dci Aew`Z GKwU wci wgwWi D`PZv 12 tm. wg. | Bnvi mgMz`tj i t`q`d`j | AvqZb wby`q Ki |

mgvarb: wci wgwWi fvgi tK`we`yntZ thtKv`bv evüi j`f`Zi $r = \frac{10}{2}$ tm. wg. = 5 tm. wg. ,

WciWgtWi D°PZv 12 tm. Wg. | AZGe

Bnvi thtKvttbv cvkZtj i tnj vttbv D°PZv = $\sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$ tm. Wg.

WciWgtWi mgMztj i t°tt dj = $10 \times 10 + \frac{1}{2}(4 \times 10) \times 13 = 100 + 260 = 360$ eM°m. Wg.

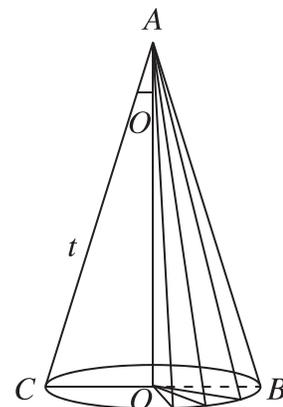
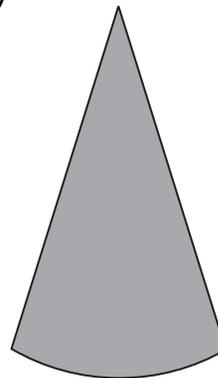
Ges Bnvi AvqZb = $\frac{1}{3} \times (10 \times 10) \times 12 = 10 \times 10 \times 4 = 400$ Nb tm. Wg.

AZGe WciWgtWi mgMztj i t°tt dj 360 eM°m. Wg. Ges AvqZb 400 Nb tm. Wg. |

KvR : 1| tZvgiv cttZttK GKw Kti mlg | GKw Kti welg (K) WcRg | (L) WciWgtW AvK |
 2| tht°tt mæ, tZvgiv AwZ NbeWUi mgMztj i t°tt dj Ges AvqZb WbYq Ki |

4| mgeffwgK tKvYK (Right circular cone)

tKvttbv mgttKvYx wtt fttRi mgttKvY msj Mæ GKwU evttK A° (axis) atj Zvi PZtt K wtt fttRttK GKevi Ngtttq Avttj th NbeW Drcbæng, ZttK mgeffwgK tKvYK ej v nq|



Wtttt, OAC mgttKvYx wtt fttRttK OA Gi PZtt K tNvttvttvtti dtj ABC mgeffwgK tKvYK Drcbæ nttttQ| Gtttttt wtt fttRi kxl KkY θ nttj, θ tK tKvYttKi Aaxl KkY (Semi- vertical angle) ej v nq|

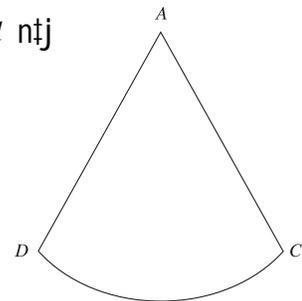
tKvYttKi D°PZv $OA = h$, fttgi eWmva° $OC = r$ Ges tnj vttbv D°PZv $AC = l$ nttj

(K) eµZttj i t°tt dj = $\frac{1}{2} \times$ fttgi cwi wa \times tnj vttbv D°PZv

$$= \frac{1}{2} \times 2\pi r \times l = \pi r l \text{ eM°KK}$$

(L) mgMztj i t°tt dj = eµZttj i t°tt dj + fttgZttj i t°tt dj

$$= \pi r l + \pi r^2 = \pi r (r + l) \text{ eM°KK}$$



$$(M) \text{AvqZb} = \frac{1}{3} \times \text{fwi} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} \times \text{D}^{\text{PZv}} = \frac{1}{3} \pi r^2 h \text{ NbGKK} |$$

[AvqZtbi GB m\hat{\text{f}} \hat{\text{u}} \text{ wby} \hat{\text{q}} \text{ c} \times \hat{\text{u}} \text{Z D}^{\text{PZi}} \text{ tk} \hat{\text{u}} \hat{\text{y}} \hat{\text{t}} \text{Z} \text{ wkLv} \hat{\text{t}} \text{bv} \text{ nte} |]

D`vniY 4 | GK\hat{\text{u}} \text{ mge} \hat{\text{E}} \text{ fwi} \text{GK} \text{ tKvY} \hat{\text{t}} \text{Ki} \text{ D}^{\text{PZv}} \text{ 12 tm. wg. Ges fwi} \text{ e}^{\text{vm}} \text{ 10 tm. wg. ntj Zvi} \text{ t} \hat{\text{n}} \hat{\text{j}} \text{ v} \hat{\text{t}} \text{bv} \text{ D}^{\text{PZv}} \text{, e} \hat{\text{u}} \text{Z} \hat{\text{t}} \text{j} \text{ i} \text{ l} \text{ mgM} \hat{\text{Z}} \hat{\text{t}} \text{j} \text{ i} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} \text{ Ges AvqZb} \text{ wby} \hat{\text{q}} \text{ Ki} |

$$\text{mgvavb} : \text{fwi} \text{ e}^{\text{vm}} \text{v} \hat{\text{a}} \text{ }^{\circ} r = \frac{10}{2} \text{ tm. wg.} = 5 \text{ tm. wg.}$$

$$\text{t} \hat{\text{n}} \hat{\text{j}} \text{ v} \hat{\text{t}} \text{bv} \text{ D}^{\text{PZv}} \text{ } l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13 \text{ tm. wg.}$$

$$\text{e} \hat{\text{u}} \text{Z} \hat{\text{t}} \text{j} \text{ i} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} = \pi r l = \pi \times 5 \times 13 = 204.2035 \text{ e. tm. wg.}$$

$$\text{mgM} \hat{\text{Z}} \hat{\text{t}} \text{j} \text{ i} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} = \pi r (l + r) = \pi \times 5 (13 + 5) = 282.7433 \text{ e. tm. wg.}$$

$$\text{AvqZb} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 5^2 \times 12 = 314.1593 \text{ N. tm. wg.} |$$

KvR: Rb\hat{\text{w}} \hat{\text{t}} \text{b} \text{ e} \text{v} \text{ A} \hat{\text{b}} \hat{\text{v}} \hat{\text{b}} \hat{\text{v}} \text{ A} \text{v} \hat{\text{b}} \hat{\text{v}} \text{ } \text{D} \text{r} \text{m} \hat{\text{t}} \text{e} \text{ e}^{\text{en}} \text{Z} \text{ tKvYK} \text{ A} \text{v} \text{K} \hat{\text{u}} \text{Zi} \text{ GK} \hat{\text{u}} \text{ K}^{\text{vc}} \text{ m} \text{s} \text{M} \hat{\text{h}} \text{ K} \hat{\text{t}} \text{i} \text{ Zvi} \text{ e} \hat{\text{u}} \text{Z} \hat{\text{t}} \text{j} \text{ i} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} \text{ |}

AvqZb wby\hat{\text{q}} \text{ Ki} |

5 | tMvj K (Sphere)

tKv\hat{\text{t}} \text{bv} \text{ A} \hat{\text{a}} \hat{\text{E}} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ i} \text{ e}^{\text{vm}} \hat{\text{t}} \text{K} \text{ A} \hat{\text{q}} \text{ a} \text{t} \text{i} \text{ H} \text{ e}^{\text{v}} \hat{\text{t}} \text{mi} \text{ PZ} \hat{\text{v}} \text{ } \hat{\text{K}} \text{ A} \hat{\text{a}} \hat{\text{E}} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ t} \hat{\text{K}} \text{ G} \text{K} \text{e} \text{v} \text{i} \text{ N} \text{y} \text{i} \text{ t} \hat{\text{q}} \text{ A} \text{v} \hat{\text{t}} \text{j} \text{ t} \text{h} \text{ N} \text{b} \hat{\text{e}} \hat{\text{v}} \text{ } \text{D} \text{r} \text{c} \text{b} \hat{\text{a}} \text{e} \text{q} \text{ Z} \hat{\text{v}} \hat{\text{t}} \text{K} \text{ t} \text{M} \text{v} \text{j} \text{ K} \text{ e} \hat{\text{t}} \text{j} | \text{ A} \hat{\text{a}} \hat{\text{E}} \hat{\text{u}} \text{i} \text{ t} \text{K} \hat{\text{v}} \text{ } \text{B} \text{ t} \text{M} \text{v} \text{j} \text{ t} \text{K} \text{i} \text{ t} \text{K} \hat{\text{v}} \text{ } \hat{\text{q}} \text{ G} \text{B} \text{ N} \text{Y} \hat{\text{f}} \text{b} \text{i} \text{ d} \hat{\text{t}} \text{j} \text{ A} \hat{\text{a}} \hat{\text{E}} \text{ t} \text{h} \text{ Z} \text{j} \text{ D} \text{r} \text{c} \text{b} \hat{\text{a}} \text{K} \hat{\text{t}} \text{i} \text{ Z} \text{v} \text{B} \text{ n} \text{j} \text{ t} \text{M} \text{v} \text{j} \text{ t} \text{K} \text{i} \text{ Z} \text{j} | \text{ t} \text{M} \text{v} \text{j} \text{ t} \text{K} \text{i} \text{ t} \text{K} \hat{\text{v}} \text{ } \hat{\text{a}} \text{e} \text{j} \text{ t} \text{Z} \text{ g} \hat{\text{j}} \text{ e} \hat{\text{t}} \hat{\text{E}} \text{i} \text{ t} \text{K} \hat{\text{v}} \text{ } \hat{\text{t}} \text{K} \text{B} \text{ e} \hat{\text{f}} \text{v} \hat{\text{q}} |

$CQAR$ tMvj tKi tK\hat{\text{v}} \hat{\text{a}} O , e\text{vm} \hat{\text{v}} \hat{\text{a}} \text{ }^{\circ}

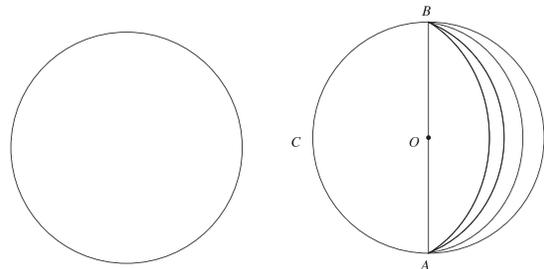
$OA = OB = OC$ Ges tK\hat{\text{v}} \hat{\text{a}} \text{ t} \hat{\text{t}} \text{K} \text{ } h \text{ } \hat{\text{t}} \hat{\text{t}} \text{Z} \text{i} \text{ } P

w\hat{\text{e}} \hat{\text{v}} \text{ } \hat{\text{j}} \text{ g} \hat{\text{a}} \hat{\text{v}} \text{ } \hat{\text{t}} \hat{\text{q}} \text{ } OA \text{ t} \text{i} \text{ L} \text{v} \text{i} \text{ m} \hat{\text{v}} \hat{\text{t}} \text{ } \text{ } \hat{\text{j}} \text{ } \hat{\text{a}} \text{ } \hat{\text{n}} \text{q} \text{ G} \text{i} \text{f} \text{c} \text{ G} \text{K} \hat{\text{u}} \text{w}

mgZj tMvj K\hat{\text{u}} \hat{\text{t}} \text{K} \text{ t} \hat{\text{Q}} \hat{\text{v}} \text{ K} \hat{\text{t}} \text{i} \text{ } QBR \text{ e} \hat{\text{E}} \hat{\text{u}} \text{ D} \text{r} \text{c} \text{b} \hat{\text{a}} \text{e}

K\hat{\text{t}} \text{i} \text{ t} \hat{\text{Q}} | \text{ G} \text{B} \text{ e} \hat{\text{t}} \hat{\text{E}} \text{i} \text{ t} \text{K} \hat{\text{v}} \text{ } \hat{\text{a}} \text{ } P \text{ Ges e}^{\text{vm}} \hat{\text{v}} \hat{\text{a}} \text{ }^{\circ} \text{ } PB |

Zvntj \text{ } PB \text{ Ges } OP \text{ c} \text{i} \hat{\text{v}} \text{i} \text{ m} \text{g} \text{v} \text{b} |



$$\therefore OB^2 = OP^2 + PB^2$$

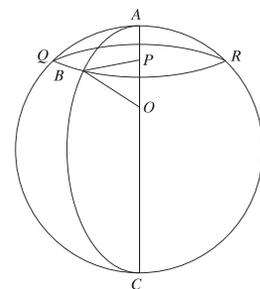
$$\therefore PB^2 = OB^2 - OP^2 = r^2 - h^2$$

tMvj tKi e\text{vm} \hat{\text{v}} \hat{\text{a}} \text{ }^{\circ} r \text{ ntj } ,

$$(K) \text{tMvj tKi} \text{ c} \hat{\rho} \text{Z} \hat{\text{t}} \text{j} \text{ i} \text{ t} \hat{\text{q}} \hat{\text{I}} \text{ dj} = 4\pi r^2 \text{ e} \text{M} \hat{\text{E}} \text{K} \text{K} |$$

$$(L) \text{AvqZb} = \frac{4}{3} \pi r^3 \text{ NbGKK} |$$

$$(M) \text{ } h \text{ D}^{\text{PZv}} \text{q} \text{ Z} \text{j} \text{ t} \hat{\text{P}} \hat{\text{Q}} \hat{\text{v}} \text{ } \text{D} \text{r} \text{c} \text{b} \hat{\text{a}} \text{e} \hat{\text{t}} \hat{\text{E}} \text{i} \text{ e}^{\text{vm}} \hat{\text{v}} \hat{\text{a}} \text{ }^{\circ} = \sqrt{r^2 - h^2} \text{ GKK} |$$



KvR: GKwU tLj br ej ev dblej wbtq Zvi e'vmva^QbY^Q Ki | AZtci Gi AvqZbl tci Ki |

D`vniY 5 | 4 tm. wg. e'vtmi GKwU tj Sn tMvj KtK wciUtq $\frac{3}{2}$ tm. wg. cij " GKwU e'EvKvi tj SnvZ cOZ Kiv nj | H cvtZi e'vmva^QKZ?

mgvavb : tj Sn tMvj tKi e'vmva^Q = $\frac{4}{2} = 2$ tm. wg. | \therefore Zvi AvqZb = $\frac{4}{3}\pi \cdot 2^3 = \frac{32}{3}\pi$ Nb tm. wg.

gtb Kwi, cvtZi e'vmva^Q = r tm. wg. | cvZwU $\frac{2}{3}$ tm. wg. cij "

\therefore cvtZi AvqZb = $\pi r^2 \times \frac{2}{3}$ N. tm. wg. = $\frac{2}{3}\pi r^2$ N. tm. wg. |

kZ^Qbnv^{ti}, $\frac{2}{3}\pi r^2 = \frac{32}{3}\pi$ ev, $r^2 = 16$ ev, $r = 4$

\therefore cvtZi e'vmva^Q = 4 tm. wg.

D`vniY 6 | mgvb D'PZv wnkó GKwU mge'FwgK tKvYK, GKwU Aa^QMvj K I GKwU wmwj Úvi mgvb mgvb fvgi Dci Aew⁻Z | t⁻Lvl th, Zvt⁻ i AvqZtbi AbcvZ 1: 2: 3

mgvavb : gtb Kwi, mvariY D'PZv I fvgi e'vmva^Qh_vμtg h Ges r GKK | thtZi Aa^QMvj tKi D'PZv I e'vmva^Qmgvb | $\therefore h = r$

Zvntj tKvYtKi AvqZb = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ NbGKK

Aa^QMvj tKi AvqZb = $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$ NbGKK Ges wmwj Úvti i AvqZb = $\pi r^2 h = \pi r^3$

\therefore wbtY^Q AbcvZ = $\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

D`vniY 7 | GKwU AvqZvKvi tj Sn dj tKi ⁻N^Q, cO' I D'PZv h_vμtg 10, 8 I $5\frac{1}{2}$ tm. wg. | GB

dj KwU⁺K Mvj t^q $\frac{1}{2}$ tm. wg. e'vmva^Qwnkó KZ_v t^v tMvj vKvi_v wj cOZ Kiv hvte?

mgvavb : tj Sn dj tKi AvqZb = $10 \times 8 \times 5\frac{1}{2}$ N. tm. wg. = 440 N. tm. wg.

gtb Kwi, wj i msL⁻v = n

\therefore n msL⁻K_v wj i AvqZb = $n \times \frac{4}{3}\pi \left(\frac{1}{2}\right)^3 = \frac{n\pi}{6}$ N. tm. wg.

cK^Qbnv^{ti}, $\frac{n\pi}{6} = 440$ $\therefore n = \frac{440 \times 6}{\pi} = 840 \cdot 3$

∴ wbtY@ , wj i msL"v 840 wU|

D`vni Y 8| GKwU mgeEfwgK tKvYtKi AvqZb V, eµZtj i tñl dj S, fwi e"vma®r, D" PZv h Ges Aa®kxl fKvY α ntj t`LvI th,

$$(i) S = \frac{\pi h^2 \tan \alpha}{\cos \alpha} = \frac{\pi r^2}{\sin \alpha} \text{ eM®KK}$$

$$(ii) V = \frac{1}{3} \pi h^3 \tan^2 \alpha = \frac{\pi r^3}{3 \tan \alpha} \text{ NbGKK}$$

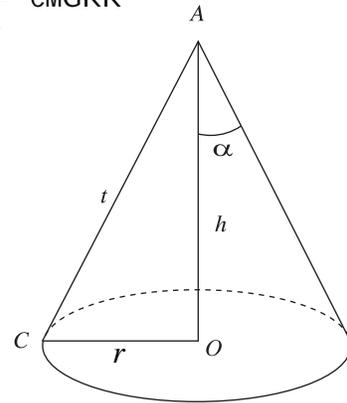
mgvavb : cvtKi wPt, tKvYtKi D" PZv OA = h, tnj vtbv D" PZv AC = l, fwi e"vma® OC = r Ges Aa®kxl fKvY ∠OAC = α .

tnj vtbv D" PZv $l = \sqrt{h^2 + r^2}$.

$$\text{wPt ntZ t`Lv hvq th, } \tan \alpha = \frac{r}{h} \quad \therefore r = h \tan \alpha \text{ ev, } h = \frac{r}{\tan \alpha} = r \cot \alpha$$

$$\begin{aligned} \text{GLb (i) } S &= \pi r l = \pi r \sqrt{h^2 + h^2 \tan^2 \alpha} = \pi r h \sqrt{1 + \tan^2 \alpha} = \pi r h \sqrt{\sec^2 \alpha} \\ &= \pi r h \sec \alpha = \frac{\pi r}{\cos \alpha} \cdot r \cot \alpha = \frac{\pi r^2}{\cos \alpha} \cdot \frac{\cos \alpha}{\sin \alpha} = \frac{\pi r^2}{\sin \alpha} \text{ eM®KK} \end{aligned}$$

$$\begin{aligned} (ii) V &= \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (h \tan \alpha)^2 h = \frac{1}{3} \pi h^3 \tan^2 \alpha \\ &= \frac{1}{3} \pi \left(\frac{r}{\tan \alpha} \right)^3 \tan^2 \alpha = \frac{\pi r^3}{3 \tan \alpha} \text{ NbGKK} \end{aligned}$$



5| thŠMK Nbe" (Compound solid)

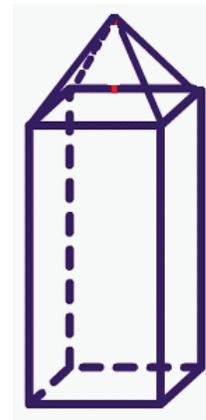
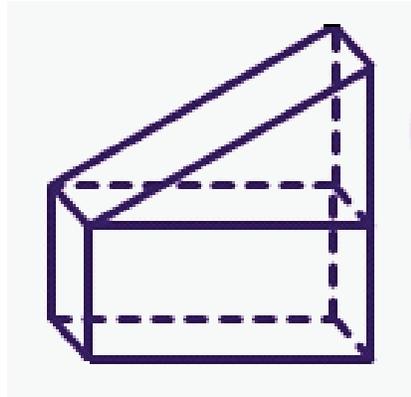
`Bw Nbe"i mgštq MwZ Nbe"tK thŠMK Nbe"etj | thŠMK Nbe"i KtqKwU D`vni Y t

(1) GKwU AvqZvKvi Nbe"i Dctii Zj hw` GKwU Lvov w®Rtgi tKvbl GKwU Ztj i mgvb nq Zte Nbe"i Dci wgvj tq w®RwU emvtj GKwU thŠMK Nbe"nq|

(2) GKwU w®Rtgi fwi I GKwU PZl j tKi fwi mefwg ntj Ges PZl j KwU tK w®Rtgi Dci emvtj GKwU thŠMK Nbe"nq|

(3) GKwU tMvj tKi e"vma®I GKwU mgeEfwgK tKvYtKi fwi e"vma®mgvb ntj Ges tKvYKwU tK tMvj tKi Dci emvtj GKwU bZb Nbe"mjo nq|

(4) `Bw Aa®Mvj K I GKwU mgeEfwgK wmwj Utvi i mgštq MwZ thŠMK Nbe"tK K"vcmj ej v thtZ cvti |



wewfbaAvKv̄i i thšMK Nbe⁻

Gf̄vte `β ev `βtqi AwaK Nbe⁻i mgštq wewfbaAcKv̄i i thšMK Nbe⁻i Zwi Kiv hvq| AtbK `wob>`b
⁻vcbvl thšMK Nbe⁻| e⁻vqvg Kivi AtbK DcKiYl GKwaK Nbe⁻i mgštq %Zwi Kiv nq|

KvR: †Zvgiv cōZ⁻†K GKw̄ K†i thšMK Nbe⁻Av̄b Ki | Bnvi eYθ⁻v̄l | m[⊕]e ntj Bnvi Zj mg†ni †¶†dj | AvqZb w̄bYq̄i m† tj L|

D⁻vniY 9| GKw̄ K⁻vcm̄tj i %N[⊙] 15 tm. wg. | Bnvi w̄w̄j Ūvi AvKw̄Zi Astki e⁻vmva[⊙]3 tm. wg. ntj , mgM̄Ztj i †¶†dj | AvqZb w̄bYq̄ Ki |

mgvavb: K⁻vcm̄tj i m[⊕]úY[⊙]%N[⊙] 15 tm. wg. | ††nZi K⁻vcm̄tj i `β cōš-Aa¶M̄j Kv̄Kw̄Zi , ††nZi Bnvi w̄w̄j Ūvi AvKw̄Zi Astki %N[⊙]l = 15 - (3 + 3) = 9 tm. wg. |

m̄Zi vs K⁻vcm̄tj i mgM̄Ztj i †¶†dj
 = `β cōš† Aa¶M̄j vKw̄Z Astki cōZtj i †¶†dj + w̄w̄j Ūvi AvKw̄Zi Astki cōZtj i †¶†dj

$$= 2 \times \frac{1}{2} \times 4\pi r^2 + 2\pi r l = 4\pi (3)^2 + 2\pi \times 3 \times 9 \quad [\because r = 3 \text{ tm. wg.}]$$

$$= 90\pi = 282.74 \text{ eM}^{\oplus}\text{tm. wg.}$$

$$\text{Ges K⁻vcm̄tj w̄i AvqZb} = 2 \times \frac{1}{2} \times \frac{4}{3}\pi r^3 + \pi r^2 l = \frac{4}{3}\pi (3)^3 + \pi (3)^2 \times 9 = 117\pi = 367.57 \text{ Nb tm. wg. |}$$

Abjxj bñ 13

- 1| GKw̄ AvqZvKvi Nbe⁻i ^ˆN[⊗] 8 tm.wg., cō⁻4 tm.wg Ges D'PZv 3 tm.wg. ntj Gi KY[⊙]KZ?
 - K. $5\sqrt{2}$ tm.wg.
 - L. 25 tm.wg
 - M. $25\sqrt{2}$ tm.wg
 - N. 50 tm.wg
- 2| †Kv̄t̄bv mg†KvYx w̄l f̄Ri AwZfR w̄fbaAc̄i ev̄ú†tqi ^ˆN[⊙]4 tm.wg. Ges 3 tm.wg | w̄l f̄Rw̄†K ep̄Ēi ev̄úi PZ††K †Nvi v̄tj -
 - i DrcbaNbe⁻w̄ GKw̄ mgeĒf̄wgK †KvYK n̄te
 - ii Nbe⁻w̄ GKw̄ mgeĒf̄wgK w̄w̄w̄j Ūvi n̄te
 - iii DrcbaNbe⁻w̄i f̄w̄gi †¶†dj n̄te $9\pi \text{ eM}^{\oplus}\text{tm.wg.}$

I cti i evK" ,tj vi gta" tKvbU mW/K?

- K. i
- L. ii
- M. i I iii
- N. ii I iii

wbtgè Zt_ i Avtj vtK 3 I 4 baf cOkè DEi `vl |

2 tm.wg. e'vmvènkó GKwU tMvj K AvKwZi ej GKwU vmvj Úvi AvKwZi evf. wK frte GtU hvq |

3 | vmvj Úvti i AvqZb KZ?

- K. 2π Nb tm.wg.
- L. 4π Nb tm.wg.
- M. 6π Nb tm.wg.
- N. 8π Nb tm.wg.

4 | vmvj Úvi wU AbwaKZ Astki AvqZb KZ?

- K. $\frac{\pi}{3}$ Nb tm.wg.
- L. $\frac{2\pi}{3}$ Nb tm.wg.
- M. $\frac{4\pi}{3}$ Nb tm.wg.
- N. $\frac{3\pi}{3}$ Nb tm.wg.

wbtgè Zt_ i wfvÉtZ 5 I 6 baf cOkè DEi `vl :

6 tm.wg. e'vmvènkó GKwU avZe KwB tMvj KtK Mvj tq 3 tm.wg. e'vmvènkó GKwU mgeÉfwgK vmvj Úvi `Zwi Kiv ntj v |

5 | Drcbævmvj Úvi wU D" PZv KZ?

- K. 4 tm. wg.
- L. 6 tm. wg.
- M. 8 tm. wg.
- N. 12 tm. wg.

6 | vmvj Úvi wU eµZtj i t¶¶dj KZ eM¶tm.wg.?

- K. 24π
- L. 42π
- M. 72π
- N. 96π

7 | GKwU AvqZvKvi Nbe`i ^^N°, cŕ' I D" PZv h_vµtg 16 wg., 12 wg. I 4.5 wg. | Gi cŕZtj i t¶¶dj , KtY¶ ^^N°Ges AvqZb wby¶ Ki |

8 | fvgi Dci Aew`Z 2.5 wg. ^^N° I 1.0 wg. cŕ' wènkó (Af`š+xY cwi gvc) GKwU AvqZvKvi Rj vavti i D" PZv 0.4 wguvi ntj , Gi AvqZb Ges Af`š+xY Ztj i t¶¶dj wby¶ Ki |

9 | GKwU AvqZvKvi Nbe`i gvŕv ,tj v 5 tm. wg., 4 tm. wg. I 3 tm. wg. ntj , Gi KtY¶ mgvb aviwènkó Nbtki mgMŕZtj i t¶¶dj wby¶ Ki |

10 | 70 Rb Qrtŕi Rb` Gifc GKwU trvf =j wbg¶y Ki tZ nte hvZ cŕZ`K Qrtŕi Rb` 4.25 eM¶guvi tgs I 13.6 Nbvguvi kb`vb _vtK | NiwU 3.4 wguvi j ¶¶ntj , Gi cŕ' I D" PZv KZ nte?

11 | GKwU mgeÉfwgK tKvYtki D" PZv 8 tm. wg. Ges fvgi e'vmva¶ 6 tm. wg. ntj , mgMŕZtj i t¶¶dj I AvqZb wby¶ Ki |

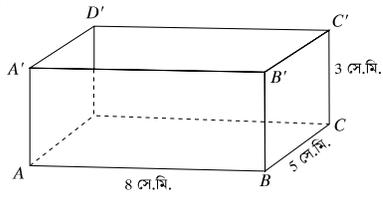
12 | GKwU mgeÉfwgK tKvYtki D" PZv 24 tm. wg. Ges AvqZb 1232 Nb tm. wg. | Gi tnj vtbv D" PZv KZ?

13 | tKvYtki mgtkvYx wŕ f¶Ri `BwU evûi ^^N°5 tm. wg. Ges 3.5 tm. wg. | GtK epÈi evûi PZw`¶K tNvvtj th Nbe`' Drcbæng, Zvi AvqZb wby¶ Ki |

14 | 6 tm. wg. e'vmvènkó GKwU tMvj tki cŕZj I AvqZb wby¶ Ki |

- 15| 6, 8, r tm. wg. e`vma^oweikó wZbW KwB KutPi ej Mvj tq 9 tm. wg. e`vma^oweikó GKw KwB tMvj tK cwi YZ Kiv nj | r Gi gvb wBY^o Ki |
- 16| GKw ducv tj vnvi tMvj tKi evBti e`vm 13 tm. wg. Ges tj vnvi tea 2 tm. wg. | H tMvj tK e`eüZ tj vnv w`tq GKw wbtU tMvj K`Zwi Kiv ntj v | Zvi e`vm KZ nte?
- 17| 4 tm. wg. e`vmvta^p GKw wbtU tMvj tK Mvj tq 5 tm. wg. e`vma^oweikó I mgfvte cij“ GKw ducv tMvj K cÖZ Kiv ntj v | wZxq tMvj Kw KZ cij“?
- 18| GKw tj vnvi wbtU tMvj tKi e`vma^o6 tm. wg. | Gi tj vnv t`tK 8 tm. wg. `N^o I 6 tm. wg. e`vtmi KqW wbtU wmvj Üvi cÖZ Kiv hvte?
- 19| $\frac{22}{\pi}$ tm. wg. e`vma^oweikó GKw tMvj K AvKwZi ej GKw NbK AvKwZi evt. wKfvte GtU hvq | ev. wJi AbwaKZ Astki AvqZb wBY^o Ki |
- 20| 13 tm. wg. e`vma^oweikó GKw tMvj tKi tK`^t`_tK 12 tm. wg. `ieZ^ptKv^tbv wex`y ga` w`tq e`vtmi Dci j`^mgZj tMvj Kw tK tÖ` Kti | DrcbrZj wJi t^qt dj wBY^o Ki |
- 21| GKw XvKbvhy^p Kv^tVi evt. i evBti `N^o I cÖ`h_v^utg 1.6 wg. I 1.2 wg., D"pZv 0.8 wg. Ges Gi Kw 3 tm. wg. cij“ | ev. wJi wFZti Ztj i t^qt dj KZ? cÖZ eM^oUvi 14.44 UvKv wmvte evt. i wFZi is Ki tZ KZ LiP nte?
- 22| 120 wg. `N^o I 90 wg. cÖweikó (eing^o) AvqZvKvi evM^tbi PZw`K 2 wg. D"p I 25 tm. wg. cij“ cÖPxi wbg^oY Ki tZ 25 tm. wg. `N^o 12.5 tm. wg. cÖ`Ges 8 tm. wg. teaweikó KZ`_tj v BU j vMte?
- 23| GKw AvqZvKvi Nbe`i `N^o I cÖ`i AbcvZ 4:3 Ges Gi AvqZb 2304 Nb tm. wg. | cÖZ eM^oUvi 10 UvKv wntmte H e`i Zj vq mxmvi cÖj c w`tZ 1920 UvKv LiP ntj , H e`i gv^tv`_tj v wBY^o Ki |
- 24| tKvYK AvK^ti GKw Zuej D"pZv 7.5 wgUvi | GB Zuey^ovi 2000 eM^oUvi Rwg wNi tZ PvBtj wK cwi gvY K`vb^tvm j vMte?
- 25| GKw cÄfRvKvi w^tRtgi `Bw evüi `N^o h_v^utg 6 tm. wg. I 8 tm. wg. Ges Aci wZbW evüi cÖZ`KwI `N^o 12.5 tm. wg. | w^tRgwI mgM^ttj i t^qt dj I AvqZb wBY^o Ki |
- 26| 4 tm. wg. evüweikó GKw mlg l ofRvKvi w^tRtgi D"pZv 5 tm. wg. | Bnvi mgM^ttj i t^qt dj I AvqZb tei Ki |
- 27| 6 tm. wg. evüweikó mlg l of^tRi Dci Aew`Z GKw wci wgtWi D"pZv 10 tm. wg. | Bnvi mgM^ttj i t^qt dj I AvqZb wBY^o Ki |
- 28| GKw mlg PZ`_tj tKi th^tKv^tbv avti i `N^o 8 tm. wg. ntj , Bnvi mgM^ttj i t^qt dj I AvqZb wBY^o Ki |
- 29| GKw `vcbvi wbtPi Ask 3 wg. `N^o AvqZvKvi Nbe`I Dcti Ask mlg wci wgtW | wci wgtWi fngi evüi `N^o 2 wg. Ges D"pZv 3 wg. ntj `vcbwI mgM^ttj i t^qt dj I AvqZb wBY^o Ki |
- 30| 25 wg. `N^o 18 wg. cÖweikó fngi Dci Aew`Z t`vPj v`_vg Nti i t`qv^t i D"pZv 5 wg. | cÖZwU Pj vi cÖ` 14 wg. ntj`_vg Ni wJi AvqZb wBY^o Ki |

31|



সৃজনশীল

- K. $\pi P \hat{t} i$ Nbe $\bar{u} i$ mgM $\hat{z} t j$ i $\dagger \hat{q} \hat{t} d j$ $\bar{w} b Y \hat{q}$ Ki |
 - L. Nbe $\bar{u} i$ K $\dagger Y \hat{p}$ mgvb avi $\bar{w} e \bar{w} k \acute{o}$ GK \bar{u} avZe NbK $\dagger K$ Mvj $\dagger q$ 18 $\bar{t} m. \bar{w} g.$ e $\bar{v} m \bar{w} e \bar{w} k \acute{o}$ KZ $\bar{z} t j$ v $\bar{w} b \dagger i U$ $\dagger M v j$ K Drcb $\bar{a} K i v$ hv $\bar{t} e$ Zv $\bar{w} b K U Z g$ cY $\bar{m} s L \bar{v} q$ $\bar{w} b Y \hat{q}$ Ki |
 - M. Nbe $\bar{u} i$ ABCD Z $\dagger j$ i mgvb GK \bar{u} AvqZ $\dagger \hat{q} \hat{t} \dagger K$ en $\bar{E} i$ ev $\bar{u} i$ PZ \bar{v} $\dagger K$ $\dagger N v i v \dagger j$ th Nbe \bar{s} ' Drcb \bar{a} nq, Zvi mgM $\hat{z} t j$ i $\dagger \hat{q} \hat{t} d j$ Ges AvqZb $\bar{w} b Y \hat{q}$ Ki |
- 32| GK \bar{u} mge $\bar{E} \bar{f} \bar{w} g K$ Zve \bar{j} D'PZv 8 $\bar{w} g U v i$ Ges Gi $\bar{f} \bar{w} g i$ e $\bar{v} m$ 50 $\bar{w} g U v i$
- K. Zve $\bar{j} U i$ $\bar{t} n j$ v $\dagger b v$ D'PZv $\bar{w} b Y \hat{q}$ Ki |
 - L. Zve $\bar{j} U$ $\bar{v} c b$ Ki $\dagger Z$ KZ eM $\hat{g} U v i$ R $\bar{w} g i$ c $\bar{a} q R b$ n $\bar{t} e$? Zve $\bar{j} U i$ $\bar{w} f Z \dagger i$ i kb $\bar{v} \bar{v} \bar{t} b i$ c $\bar{w} i g v Y$ $\bar{w} b Y \hat{q}$ Ki |
 - M. Zve $\bar{j} U i$ c $\bar{a} Z$ eM $\hat{g} U v i$ K $\bar{v} b f v \bar{t} m i$ g \bar{j} 125 $\bar{U} v K v$ n $\dagger j$ K $\bar{v} b f v m$ eve \bar{v} KZ LiP n $\bar{t} e$?

কাজ :

- ১। একটি নিরপেক্ষ ছক্কা নিষ্ক্ষেপ করা হলো, নিম্নলিখিত সম্ভাবনাগুলো বের কর।
 - (i) 4 আসা (ii) বিজোড় সংখ্যা আসা (iii) 4 অথবা 4 এর বেশি সংখ্যা আসা
 - (v) 5 এর কম সংখ্যা আসা
- ২। একটি থলেতে একই ধরনের 6টি কালো, 5টি লাল, 8টি সাদা মার্বেল আছে। থলে হতে একটি মার্বেল দৈবভাবে নির্বাচন করা হলো। নির্বাচিত মার্বেলটি- (i) লাল (ii) কালো (iii) হলুদ (iv) কালো নয়- সম্ভাবনাগুলো নির্ণয় কর।

১৪.৪ তথ্যভিত্তিক সম্ভাবনা নির্ণয়

যুক্তিভিত্তিক সম্ভাবনা নির্ণয়ে ফলাফলগুলো সমসম্ভাব্য হতে হয়। বাস্তবে সকল ক্ষেত্রে ফলাফলগুলো সমসম্ভাব্য হয় না। তাছাড়া অনেক ক্ষেত্রে সম্ভাবনার যুক্তিভিত্তিক সংজ্ঞার মত কিছু গণনা করা যায় না। যেমন আবহাওয়ার পূর্বাভাসে বলা হচ্ছে আজ বৃষ্টি হবার সম্ভাবনা 30%। বিশ্বকাপ ফুটবলে ব্রাজিলের জয়ী হওয়ার সম্ভাবনা 40%। এশিয়াকাপ ক্রিকেটে বাংলাদেশের জয়ী হওয়ার সম্ভাবনা 60%। এসব সিদ্ধান্ত নেয়া হয় অতীতের পরিসংখ্যান হতে এবং এটাই হচ্ছে তথ্যভিত্তিক সম্ভাবনার ধারণা।

ধরা যাক একটা মুদ্রা 1000 বার নিষ্ক্ষেপ করায় 523 বার হেড পাওয়া গেল। এ ক্ষেত্রে হেডের আপেক্ষিক গণসংখ্যা $\frac{523}{1000} = 0.523$ । ধরা যাক মুদ্রাটিকে 2000 বার নিষ্ক্ষেপ করাতে 1030 বার হেড আসে। তাহলে 2000 বারের

মধ্যে H এর আপেক্ষিক গণসংখ্যা $\frac{1030}{2000} = 0.515$ । এখান থেকে বোঝা যায় যে, পরীক্ষাটি ক্রমাগত চালিয়ে গেলে (পরীক্ষাটি যতবেশি বার করা যাবে) আপেক্ষিক গণসংখ্যার মানটি এমন একটি সংখ্যার কাছাকাছি হবে যাকে মুদ্রাটি একবার নিষ্ক্ষেপ করলে হেড আসার সম্ভাবনা হবে। একেই তথ্যভিত্তিক সম্ভাবনা বলা হয়।

উদাহরণ ৪। আবহাওয়া দপ্তর থেকে পাওয়া রিপোর্ট অনুযায়ী জুলাই মাসে ঢাকা শহরে 21 দিন বৃষ্টি হয়েছে। তাহলে 8ই জুলাই বৃষ্টি হওয়ার সম্ভাবনা কত ?

সমাধান : যেহেতু জুলাই মাস 31 দিন এবং জুলাই মাসে 21 দিন বৃষ্টি হয়েছে। তাহলে যেকোনো একদিন বৃষ্টি হওয়ার সম্ভাবনা $\frac{21}{31}$ । অতএব 8 জুলাই বৃষ্টি হওয়ার সম্ভাবনা $\frac{21}{31}$ ।

উদাহরণ ৫। কোনো একটি নির্দিষ্ট এলাকায় জরীপে দেখা গেল 65 জন প্রথম আলো, 40 জন ভোরের কাগজ, 45 জন জনকণ্ঠ, 52 জন যুগান্তর পত্রিকা পড়ে। এদের মধ্য হতে একজনকে দৈবভাবে নির্বাচন করলে তিনি যুগান্তর পত্রিকা পড়েন এর সম্ভাবনা কত ? তিনি প্রথম আলো পড়েন না এর সম্ভাবনাও কত ?

সমাধান : এখানে পত্রিকা পড়েন মোট $(65 + 40 + 45 + 52) = 202$ জন।

যুগান্তর পত্রিকা পড়েন 52 জন।

সুতরাং ঐ ব্যক্তির যুগান্তর পত্রিকা পড়ার সম্ভাবনা $\frac{52}{202}$.

প্রথম আলো পত্রিকা পড়েন 65 জন। সুতরাং প্রথম আলো পত্রিকা পড়েন না $(202 - 65) = 137$ জন।

\therefore প্রথম আলো পত্রিকা পড়েন না এর সম্ভাবনা = $\frac{137}{202}$.

কাজ :

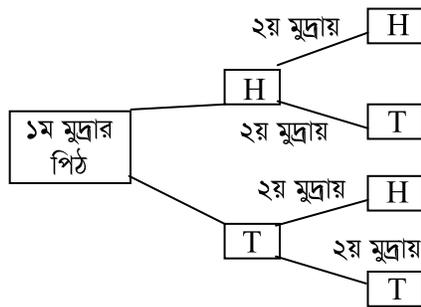
একটি জরীপে দেখা গেল কোনো এক বিশ্ববিদ্যালয়ে 1ম বর্ষে 284 জন ছাত্র অর্থনীতিতে, 106 জন ছাত্র ইতিহাসে, 253 জন ছাত্র সমাজবিজ্ঞানে, 169 জন ছাত্র ইংরেজিতে ভর্তি হয়েছে। একজন ছাত্রকে দৈবভাবে নির্বাচিত করলে নির্বাচিত ছাত্রটি সমাজবিজ্ঞানের ছাত্র হবে না এর সম্ভাবনা কত ?

১৪.৫ নমুনা ক্ষেত্র এবং সম্ভাবনা Tree দ্বারা সম্ভাবনা নির্ণয়

আগেই বলা হয়েছে কোনো পরীক্ষায় সম্ভাব্য ফলাফলগুলো নিয়ে যে ক্ষেত্র তৈরি হয় তাকে নমুনা ক্ষেত্র বলে। অনেক পরীক্ষায় নমুনা ক্ষেত্রের আকার বেশ বড় হয়। এসব ক্ষেত্রে নমুনা বিন্দু গণনা করা ও নমুনা ক্ষেত্র তৈরি করা সময় সাপেক্ষ এমন কি ভুল হওয়ার সম্ভাবনাও থাকে। সে ক্ষেত্রে আমরা সম্ভাবনা tree (probability tree) এর সাহায্যে নমুনা ক্ষেত্র তৈরি করতে পারি ও বিভিন্ন ঘটনার সম্ভাবনাও বের করতে পারি।

উদাহরণ ৬। মনে করি, দুইটি নিরপেক্ষ মুদ্রা একসাথে একবার নিক্ষেপ করা হলো। নমুনা ক্ষেত্রটি তৈরি করতে হবে। প্রথম মুদ্রায় H এবং দ্বিতীয় মুদ্রায় T আসার সম্ভাবনা নির্ণয় কর।

সমাধান : দুইটি মুদ্রা নিক্ষেপ পরীক্ষাকে দুইধাপ হিসেবে বিবেচনা করা যায়। প্রথম ধাপে একটা মুদ্রা নিক্ষেপে 2টি ফলাফল H অথবা T আসতে পারে। দ্বিতীয় ধাপে অপর মুদ্রা নিক্ষেপেও 2টি ফলাফল H অথবা T আসতে পারে। তাই পরীক্ষার মোট ফলাফলকে Probability tree এর সাহায্যে নিম্নভাবে দেখানো হয়।



সম্ভাব্য নমুনা বিন্দুগুলো HH, HT, TH, TT.

তাহলে নমুনা ক্ষেত্রটি হবে $\{HH, HT, TH, TT\}$. এখানে নমুনা বিন্দুর সংখ্যা 4 এবং প্রতিটি নমুনা বিন্দুর আসার সম্ভাবনা $\frac{1}{4}$ । তাই প্রথম মুদ্রায় H ও দ্বিতীয় মুদ্রায় T আসার সম্ভাবনা হবে $P(HT) = \frac{1}{4}$.

উদাহরণ ৭। মনে করি একটি মুদ্রা তিনবার নিক্ষেপ করা হলো। তিন নিরপেক্ষ মুদ্রা একসাথে একবার নিক্ষেপ করা হলে, Probability tree তৈরি করে নমুনা ক্ষেত্রটি দেখাও। তা হতে নিচের ঘটনাগুলোর সম্ভাবনা নির্ণয় কর।

(i) কেবল একটা টেল (ii) তিনটাই হেড (iii) কমপক্ষে একটা টেল পাওয়ার সম্ভাবনা বের কর ।

সমাধান : প্রথমে মুদ্রা তিনটিকে নিধাপ হিসেবে বিবেচনা করা এবং প্রতিধাপে 2টি ফলাফল H অথবা T আসতে পারে । সুতরাং মোট ফলাফলকে Probabilty tree সাহায্যে নিম্নভাবে দেখানো যায় :

তাহলে নমুনা ক্ষেত্রটি হবে : {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

এখানে মোট নমুনা বিন্দু 8টি এবং এদের যেকোনো একটি ঘটনা ঘটার সম্ভাবনা $\frac{1}{8}$.

(i) একটা টেল পাওয়ার অনুকূল ঘটনাগুলো {THH, HHT, HTH} = 3টি

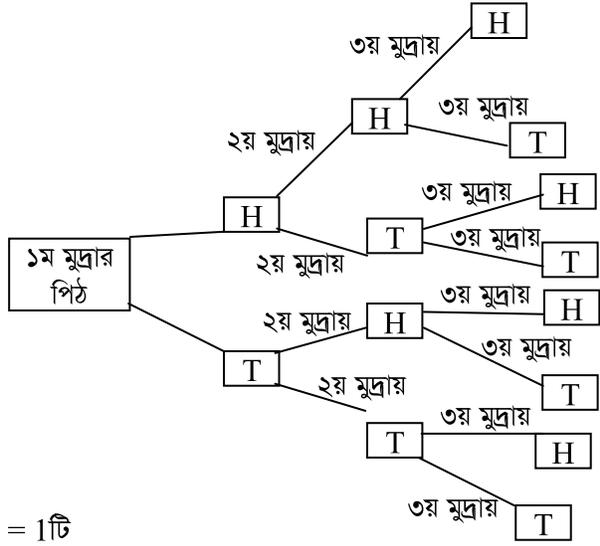
$\therefore P(1T) = \frac{3}{8}$ (কেননা প্রতিটি নমুনা বিন্দুর ঘটার সম্ভাবনা $\frac{1}{8}$)

(ii) তিনটাই হেড (H) পাওয়ার অনুকূল ঘটনা {HHH} = 1টি

$\therefore P(HHH) = \frac{1}{8}$.

(iii) কমপক্ষে 1T পাওয়ার অনুকূল ঘটনাগুলো {HHT, HTH, THH, HTT, TTH, TTT} = 7টি

$\therefore P[\text{কমপক্ষে } 1T] = \frac{7}{8}$

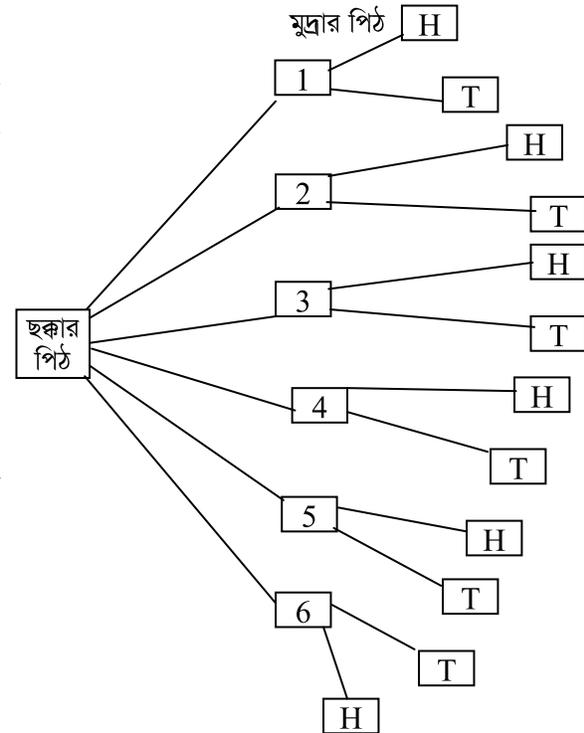


উদাহরণ ৮। একটি নিরপেক্ষ ছক্কা ও একটি মুদ্রা একবার নিষ্ক্ষেপ করা হলো । Probability tree তৈরি করে নমুনা ক্ষেত্রটি লিখ :

সমাধান : একটি ছক্কা ও একটি মুদ্রা নিষ্ক্ষেপ পরীক্ষাকে দুইধাপ হিসেবে বিবেচনা করি । প্রথম ধাপে ছক্কা নিষ্ক্ষেপে 6টি ফলাফল {1, 2, 3, 4, 5, 6} আসতে পারে । দ্বিতীয় ধাপে মুদ্রা নিষ্ক্ষেপে 2টি ফলাফল {H অথবা T} আসতে পারে । তাই পরীক্ষায় মোট ফলাফলকে Probability tree এর সাহায্যে নিম্নভাবে দেখানো যাবে ।

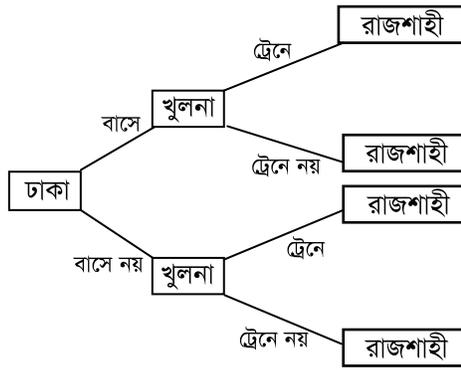
তাহলে নমুনা ক্ষেত্রটি হবে : {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T}

এখানে মোট নমুনা বিন্দু 12টি ।

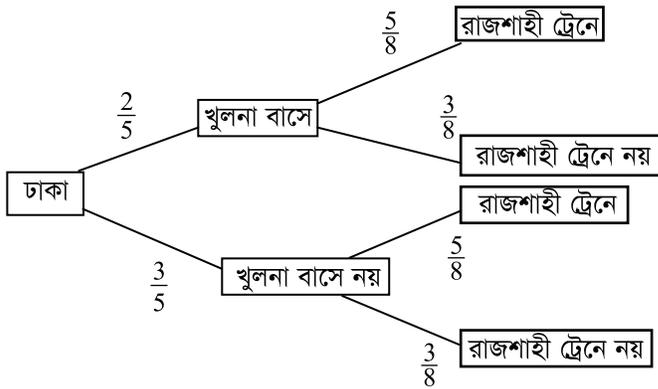


সুতরাং ছক্কায় 5 এবং মুদ্রায় H আসার সম্ভাবনা $P(5H) = \frac{1}{12}$.

উদাহরণ ৯। একজন লোক ঢাকা হতে খুলনায় বাসে যাওয়ার সম্ভাবনা $\frac{2}{5}$ এবং খুলনা হতে রাজশাহী ট্রেনে যাওয়ার সম্ভাবনা $\frac{5}{8}$ । লোকটি খুলনায় বাসে এবং রাজশাহী ট্রেনে না যাওয়ার সম্ভাবনা কত? Probability tree ব্যবহার করে দেখাও।



সম্ভাবনার মাধ্যমে Probability tree হবে



সুতরাং লোকটির খুলনায় বাসে এবং রাজশাহীতে ট্রেনে না যাওয়ার সম্ভাবনা

$$P[\text{খুলনা বাস, রাজশাহী ট্রেনে নয়}] = \frac{2}{5} \times \frac{3}{8} = \frac{6}{40} = \frac{3}{20}.$$

কাজ :

- ১। Probability tree এর সাহায্যে তিনবার মুদ্রা নিক্ষেপে সকল সম্ভাব্য ফলাফল লেখ এবং নমুনা ক্ষেত্রটি তৈরি কর। এখান হতে (i) মুদ্রা 3টিতে একই ফলাফল (ii) কমপক্ষে 2T (iii) বড়জোড় 2T আসার সম্ভাবনা নির্ণয় কর।
- ২। একটি ছক্কা ও 2টি মুদ্রা নিক্ষেপ ঘটনার Probability tree তৈরি কর।

অনুশীলনী ১৪

১। একটি ছক্কা মারলে 3 উঠার সম্ভাবনা কোনটি?

ক. $\frac{1}{6}$

খ. $\frac{1}{3}$

গ. $\frac{2}{3}$

ঘ. $\frac{1}{2}$

নিচের তথ্য থেকে (২-৩) নম্বর প্রশ্নের উত্তর দাও:

একটি থলিতে নীল বল 12টি, সাদা বল 16টি এবং কালো বল 20 টি আছে। দৈবভাবে একটা বল নেওয়া হলো।

২। বলটি নীল হওয়ার সম্ভাবনা কত?

ক. $\frac{1}{16}$

খ. $\frac{1}{12}$

গ. $\frac{1}{8}$

ঘ. $\frac{1}{4}$

৩। বলটি সাদা না হওয়ার সম্ভাবনা কত?

ক. $\frac{1}{3}$

খ. $\frac{2}{3}$

গ. $\frac{1}{16}$

ঘ. $\frac{1}{48}$

নিম্নের তথ্য থেকে (৫-৬) নম্বর প্রশ্নের উত্তর দাও:

একটি মুদ্রাকে তিনবার নিক্ষেপ করা হলো।

৪। সর্বাধিক বার H আসার সম্ভাবনা কত?

ক. 1 বার

খ. 2 বার

গ. 3 বার

ঘ. 4 বার

৫। সবচেয়ে কম সংখ্যক বার t আসার সম্ভাবনা কত?

ক. 0

খ. $\frac{1}{2}$

গ. 1

ঘ. 2

৬। চট্টগ্রাম আবহাওয়া অফিসের রিপোর্ট অনুযায়ী ২০১২ সালের জুলাই মাসের ১ম সপ্তাহে বৃষ্টি হয়েছে মোট 5 দিন। সোমবার বৃষ্টি না হওয়ার সম্ভাবনা কত?

ক. $\frac{1}{7}$

খ. $\frac{2}{7}$

গ. $\frac{5}{7}$

ঘ. 1

৭। 30টি টিকেটে 1 থেকে 30 পর্যন্ত ক্রমিক নম্বর দেয়া আছে। টিকেটগুলো ভালভাবে মিশিয়ে একটি টিকেট দৈবভাবে নেয়া হলো। টিকেটটি (i) জোড় সংখ্যা (ii) চার দ্বারা বিভাজ্য (iii) 8 এর চেয়ে ছোট (iv) 22 এর চেয়ে বড়- হওয়ার সম্ভাবনাগুলো নির্ণয় কর।

৮। কোনো একটি লটারিতে 570টি টিকেট বিক্রি হয়েছে। রহিম 15টি টিকেট কিনেছে। টিকেটগুলো ভালোভাবে মিশিয়ে একটি টিকেট দৈবভাবে প্রথম পুরস্কারের জন্য তোলা হলো। রহিমের প্রথম পুরস্কার পাওয়ার সম্ভাবনা কত?

৯। একটা ছক্কা একবার নিক্ষেপ করা হলে জোড় সংখ্যা অথবা তিন দ্বারা বিভাজ্য সংখ্যা উঠার সম্ভাবনা কত ?

- ১০। কোনো একটি স্বাস্থ্য কেন্দ্রের রিপোর্ট অনুযায়ী 155 শিশু কম ওজনের, 386 শিশু স্বাভাবিক ওজনের এবং 98টি শিশু বেশি ওজনের জন্ম নেয়। এখন হতে একটি শিশু দৈবভাবে নির্বাচন করলে নির্বাচিত শিশুটি বেশি ওজনের হবে এর সম্ভাবনা কত ?
- ১১। দুই হাজার লাইসেন্স প্রাপ্ত ড্রাইভার এক বছরে নিম্নলিখিত সংখ্যক ট্রাফিক আইন ভঙ্গ করে।

ট্রাফিক আইন ভঙ্গের সংখ্যা	ড্রাইভারের সংখ্যা
0	1910
1	46
2	18
3	12
4	9
5 বা তার অধিক	5

- একজন ড্রাইভারকে দৈবভাবে নির্বাচন করলে ড্রাইভারটির 1টি আইন ভঙ্গ করার সম্ভাবনা কত ? ড্রাইভারটির 4এর অধিক আইন ভঙ্গ করার সম্ভাবনা কত ?
- ১২। কোনো একটি ফ্যাক্টরীতে নিয়োগকৃত লোকদের কাজের ধরণ অনুযায়ী নিম্নভাবে শ্রেণিকৃত করা যায় :

শ্রেণি করণ	সংখ্যা
ব্যবস্থাপনায়	১৫৭
পরিদর্শক হিসেবে	৫২
উৎপাদন কাজে	১৪৭৩
অফিসিয়াল কাজে	২১৫

- একজনকে দৈবভাবে নির্বাচন করলে লোকটি ব্যবস্থাপনায় নিয়োজিত এর সম্ভাবনা কত ? লোকটি ব্যবস্থাপনায় অথবা উৎপাদন কাজে নিয়োজিত এর সম্ভাবনা কত ?
- লোকটি উৎপাদন কাজে নিয়োজিত নয় এর সম্ভাবনা কত ?

- ১৩। 1টি মুদ্রা ও 1টি ছক্কা নিক্ষেপ ঘটনায় Probability tree তৈরি কর।

- ১৪। Probability tree এর সাহায্যে নিচের ছকটি পূরণ কর :

মুদ্রা নিক্ষেপ	সকল সম্ভাব্য ফলাফল	সম্ভাবনা
একবার মুদ্রা নিক্ষেপ		$P(T) =$
দুইবার মুদ্রা নিক্ষেপ		$P(1H) =$ $P(HT) =$
তিনবার মুদ্রা নিক্ষেপ		$P(HHT) =$ $P(2H) =$

- ১৫। কোনো একজন লোকের ঢাকা হতে রাজশাহী ট্রেনে যাওয়ার সম্ভাবনা $\frac{5}{9}$ এবং রাজশাহী হতে খুলনা বাসে যাওয়ার সম্ভাবনা $\frac{2}{7}$. Probability tree ব্যবহার করে লোকটি ঢাকা হতে রাজশাহী ট্রেনে নয় এবং রাজশাহী হতে খুলনা বাসে যাওয়ার সম্ভাবনা কত বের কর। লোকটি রাজশাহী ট্রেনে কিন্তু খুলনা বাসে না যাওয়ার সম্ভাবনা বের কর।
- ১৬। একজন লোক ঢাকা হতে রাজশাহী ট্রেনে যাওয়ার সম্ভাবনা $\frac{2}{9}$, বাসে যাওয়ার সম্ভাবনা $\frac{3}{7}$ প্লেনে যাওয়ার সম্ভাবনা $\frac{1}{9}$ । লোকটির রাজশাহী হতে খুলনায় বাসে যাওয়ার সম্ভাবনা $\frac{2}{5}$ এবং ট্রেনে যাওয়ার সম্ভাবনা $\frac{3}{7}$ । Probability tree ব্যবহার করে লোকটি রাজশাহী ট্রেনে এবং খুলনায় বাসে যাওয়ার সম্ভাবনা বের কর।
- ১৭। একটি দুই টাকার মুদ্রা চার বার নিক্ষেপ করা হলো। (এর শাপলার পিঠকে L এবং প্রাথমিক শিক্ষার শিশুর পিঠকে C বিবেচনা কর)
- ক. যদি মুদ্রাটিকে চারবারের পরিবর্তে দুইবার নিক্ষেপ করা হয় তবে একটি L আসার সম্ভাবনা এবং একটি C না আসার সম্ভাবনা কত?
- খ. সম্ভাব্য ঘটনার Probability tree অঙ্কন কর। এবং নমুনা ক্ষেত্রটি লিখ।
- গ. দেখাও যে, মুদ্রাটি n সংখ্যক বার নিক্ষেপ করলে সংঘটিত ঘটনা 2^n কে সমর্থন করে।

উত্তরমালা

অনুশীলনী ১.১

১ – ৪ নিজে কর :

$$৫। (a) A = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

$$B = \{3, 5, 7, 9, 11, 13, 17, 19\}$$

$$(b) C = \{3, 5, 7, 9, 11, 13, 17, 19\}$$

$$D = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

৬। (a) 8 (b) 56 এবং 24

৭। (a) 12 (b) 21, 21

৮। 7, 4 ৯। 0, 3 ১৮। ২ জন ১৯। (a) 4, (b) 6 (c) 28

২০। (a) 4 (b) 6 (c) 28 ২১। (a) 4 (b) 16, 7 (c) 9

২৩। (a) $A \cap B = \{x : 2 < x < 3, x \in R\}$

(b) $A \cap B = \{x : 1 \leq x \leq 3, x \in R\}$

২৪। (a) $A' \cap B = \{x : 4 < x < 6\}$

(b) $A \cap B' = \{x : 1 < x < 3\}$

(c) $A' \cap B' = \{x : x \leq 4 \text{ অথবা } x \geq 6\}$ ২৬। (i) 10% (ii) 50%

অনুশীলনী-১.২

৭। ক। (a) ডোম $S = \{1, 2, 3, 4\}$, রেঞ্জ $S = \{5, 10, 15, 20\}$

$$S^{-1} = \{(5, 1), (10, 2), (15, 3), (20, 4)\},$$

(b) S ও S^{-1} প্রত্যেক ফাংশন

(c) এক একক ফাংশন

খ। (a) ডোম $S = \{-3, -2, -1, 0, 1, 2, 3\}$, রেঞ্জ $S = \{-1, 0, 3, 8\}$

$$S^{-1} = \{(8, -3), (3, -2), (0, -1), (-1, 0), (0, 1), (3, 2), (8, 3)\},$$

(b) S ফাংশন; S^{-1} ফাংশন নয়, কেননা $((0, 1), \text{ এবং } (0, -1), \leftarrow S^{-1}$

(c) এক একক ফাংশন

গ। (a) ডোম $S = \{\frac{1}{2}, 1, \frac{5}{2}\}$, রেঞ্জ $S = \{-2, -1, 0, 2\}$

$$S^{-1} = \{(0, \frac{1}{2}), (1, 1), (-1, 1), (2, \frac{5}{2}), (-2, \frac{5}{2}),$$

(b) S ফাংশন নয়; কেননা $((1, 1),$ এবং $(1, -1), \leftarrow S^{-1}$ ফাংশন

(c) এক একক ফাংশন

ঘ। (a) ডোম $S = \{-3, 1, 0, 3\}$, রেঞ্জ $S = \{-3, -1, 0, 3\}$

$$S^{-1} = S$$

(b) S, S^{-1} ফাংশন

(c) এক একক ফাংশন নয়

ঙ। (a) ডোম $S = \{2\}$, রেঞ্জ $S = \{1, 2, 3\}$

$$S^{-1} = \{(1, 2), (2, 2), (3, 2)\}$$

(b) S ফাংশন নয়

(c) এক একক ফাংশন নয়

চ। (ক) ডোম $F = \mathbb{R}$, এক-এক

(খ) ডোম $F = \mathbb{R}$, এক-এক নয়

(গ) ডোম $F = \{x \in \mathbb{R} : x \geq 1\}$, এক-এক

[কেননা $\sqrt{x-1}$ লিখলে অঋণাত্মক বর্গ মূলকেই বোঝায়]

(ঘ) ডোম $F = \mathbb{R} / \{2\}$, এক-এক

(ঙ) ডোম $F = \mathbb{R}$, এক-এক নয়

(চ) ডোম $F = \mathbb{R}$, এক-এক

৯। (ক) 0, 2, 3 (খ) [a]

(গ) 26 (ঘ) $1 + y^2$

১০। (ক) 3, 1, 0, 1, 3 (খ) ± 4

(গ) 0 (ঘ) y

১১। (ক) ডোম $F = \mathbb{R}$, (খ) রেঞ্জ $F = \mathbb{R}$,

(গ) $F^{-1} = \mathbb{R} \rightarrow \mathbb{R}$ (ঘ) $F^{-1}(x) = \sqrt{x}$

১৯। (ক) $P = \{(-2, 2), (-1, 3), (1, 5), (1, 7), (-2, 5)\}$

(ক) $S = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$,

(b) S ও S^{-1} প্রত্যেক ফাংশন

(c) এক একক ফাংশন

ডোম $S = \{-2, -1, 0, 1, 2\}$,

রেঞ্জ $S = \{0, 1, 4\}$

২০। (ক) $F(x+1) = 2x+1$, $F\left(\frac{1}{2}\right) = 0$

(খ) এক-এক

২১। (ক) \mathbb{R}_+ একধনাত্মক বাস্তব সংখ্যার সেট ডোম $F = \mathbb{R}_+$

(খ) কোন ফাংশনের অধীনে তার ডোমেনের ভিন্ন ভিন্ন সদস্যের ছবি সর্বদা ভিন্ন হয়। $F(x)$ ফাংশন নয়

(গ) রেঞ্জ $F = \mathbb{R}_+$, $x = 10$

অনুশীলনী ২

৬। (ক) $Q(x) = x^{n-1} + ax^{n-2} + a^2x^{n-3} + a^3x^{n-4} + \dots + a^{n-1}$

(খ) $Q(x) = x^{n-1} - ax^{n-2} + a^2x^{n-3} - a^3x^{n-4} + \dots + (-1)^{n-1}a^{n-1}$

৭। $Q(x) = x^{n-1} - ax^{n-2} + a^2x^{n-3} - a^3x^{n-4} + \dots + (-1)^{n-1}a^{n-1}$

৯। (i) $(x+1)^2(x+2)(x+3)$

(ii) $(2a-1)(a+1)(a+2)(2a+1)$

(iii) $(x+1)(x^2+x+1)$

(iv) $(x+y+z)(xy+yz+zx)$

(v) $-(x-y)(y-z)(z-x)$

(vi) $-(a-b)(b-c)(c-a)(a+b)(b+c)(c+a)$

১২। (a) 1 (b) $\frac{x}{(x-a)(x-b)(x-c)}$ (c) 0 (d) $\frac{1}{x-1}$

১৩। (a) $\frac{2}{x} + \frac{3}{x+2}$ (b) $\frac{6}{x-4} - \frac{5}{x-3}$ (c) $\frac{1}{x} - \frac{2}{x-2} + \frac{2}{x+3}$

(d) $\frac{3x+1}{x^2+4} - \frac{2}{x+1}$ (e) $\frac{4}{10(2x+1)} - \frac{12}{25(x+3)} - \frac{9}{5(x+3)^2}$

অনুশীলনী ৫.১

$$১। 3, -\frac{3}{2} \quad ২। -2-\sqrt{7}, -2+\sqrt{7} \quad ৩। 2-\sqrt{3}, 2+\sqrt{3}$$

$$৪। \frac{1}{4}(5-\sqrt{33}), \frac{1}{4}(5+\sqrt{33}) \quad ৫। \frac{1}{6}(-7-\sqrt{37}), \frac{1}{6}(-7+\sqrt{37})$$

$$৬। \frac{1}{6}(9-\sqrt{105}), \frac{1}{6}(9+\sqrt{105}) \quad ৭। 4, 4 \quad ৮। \frac{1}{4}(-7-\sqrt{57}), \frac{1}{4}(-7+\sqrt{57})$$

$$৯। \frac{1}{3}, 2$$

অনুশীলনী ৫.২

$$১। 13 \quad ২। \frac{6}{5} \quad ৩। 9 \quad ৪। 5 \quad ৫। 5 \quad ৬। \frac{5}{2}, \frac{13}{2}, \quad ৭। 1, 5$$

$$৮। 2, -\frac{9}{2}, \quad ৯। \frac{25}{7}, -\frac{1}{7} \quad ১০। -\frac{8}{11}, -\frac{3}{2}$$

অনুশীলনী ৫.৩

$$১। 2 \quad ২। \frac{7}{3} \quad ৩। 6 \quad ৪। 5 \quad ৫। 2 \quad ৬। \frac{5}{2}, \quad ৭। 3 \quad ৮। 0,$$

$$৯। 0, 2 \quad ১০। -1, 0 \quad ১১। -\frac{1}{21}, \frac{1}{2} \quad ১২। 2, 3$$

অনুশীলনী ৫.৪

$$১। (2, 3), \left(\frac{15}{2}, \frac{16}{9}\right) \quad ২। (3, 4), \left(-6, \frac{5}{8}\right) \quad ৩। (0, 0), (13, 13), (3, -2), (-2, 3)$$

$$৪। (0, 0), (5, 5), (2, -1), (-1, 2) \quad ৫। \left(\frac{1}{5}, 5\right), \left(\frac{4}{5}, 20\right) \quad ৬। \left(3, -\frac{5}{3}\right), \left(\frac{16}{9}, -\frac{3}{4}\right)$$

$$৭। (1, 2), (-1, -2) \quad ৮। (7, 5), (7, -5) \quad ৯। (3, 4), (4, 3), (-3, -4), (-4, -3)$$

$$\left(\sqrt{2}, -6\sqrt{2}\right), \left(-\sqrt{2}, 6\sqrt{2}\right)$$

$$১০। (2, 1), 2, -1), (-2, 1), (-2, -1) \quad ১১। (1, -2), (2, -1), -1, 2), (-2, 1)$$

$$১২। (1, 3), (-1, -3), \left(\frac{13}{\sqrt{21}}, \frac{2}{\sqrt{21}}\right), \left(\frac{-13}{\sqrt{21}}, \frac{-2}{\sqrt{21}}\right)$$

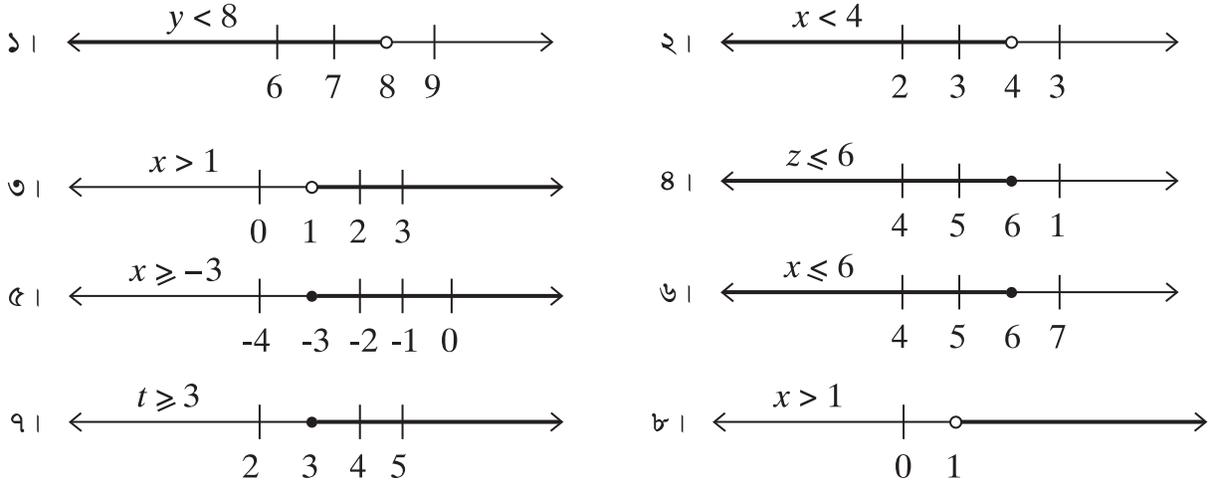
অনুশীলনী ৫.৫

১। 16 মিটার, 15 মিটার ২। 13, 9 ৩। দৈর্ঘ্য 8 মিটার, প্রস্থ 6 মিটার ৪। 19 ৫। দৈর্ঘ্য 6 মিটার, প্রস্থ 4 মিটার অথবা দৈর্ঘ্য 16 মিটার, প্রস্থ $1\frac{1}{2}$ মিটার ৬। দৈর্ঘ্য 25 মিটার, প্রস্থ 24 মিটার ৭। দৈর্ঘ্য 8 মিটার, প্রস্থ 6 মিটার ৮। 36 ৯। $8\sqrt{3}$ মিটার ১০। দৈর্ঘ্য 20 মিটার, প্রস্থ 15 মিটার।

অনুশীলনী ৫.৬

১। (2, 3) ২। (2, 1), $(-\frac{1}{2}, -\frac{1}{4})$ ৩। (4, 0) ৪। (1, 2) ৫। (3, 3)
 ৬। $(2, \pm 2)$, $(-2, \pm \frac{1}{2})$ ৭। $(2, \pm 2)$, $(-2, \pm \frac{1}{2})$ ৮। $(1, 2)$ $(-\frac{1}{3}, \frac{2}{3})$
 ৯। $(2, \pm 2)$, $(-2, \pm \frac{1}{2})$

অনুশীলনী ৬.১



অনুশীলনী ৬.২

১। $3x + \frac{x+2}{2} < 29, 0 < x < 8$ ২। $4x + x - 3 \leq 40, 0 < x < \frac{43}{5}$
 ৩। $0x + 20x < 500, 0 < x < 5$ ৪। $\frac{x+x+120}{9} \leq 100, 0 < x \leq 390$
 ৫। $5x < 40, 5 < x < 8$ ৬। পিতার বয়স ≤ 42 বছর
 ৭। জেনির বর্তমান বয়স x বছর হলে, $14 < x < 17$ ৮। সময় t সেকেন্ড হলে, $t \geq 50$

৯। উড্ডয়নের সময় t ঘণ্টা হলে, $t \geq 6\frac{1}{4}$

১০। উড্ডয়নের সময় t ঘণ্টা হলে, $t \geq 5\frac{5}{8}$

১১। সংখ্যাটি x হলে, $0 < x < 5$

অনুশীলনী ৭

৮। (ক) 20, 30, $2r$ (খ) $5, \frac{15}{2}, \frac{r}{2}$ (গ) $\frac{1}{110}, \frac{1}{240}, \frac{1}{r(r+1)}$

(ঘ) 1, 0, 1 (r জোড় হলে) এবং 0 (r বিজোড় হলে)

(ঙ) $\frac{5}{3^9}, \frac{5}{3^{14}}, \frac{5}{3^{r-1}}$ (চ) $0, 1, \frac{1-(-1)^{3r}}{2}$

৯। (ক) $n > 10^5$ (খ) $n < 10^5$ (গ) 0

১১। (ক) 2 (খ) $\frac{1}{7}$ (গ) $\frac{32}{3}$ (ঘ) সমষ্টি নেই (ঙ) $\frac{1}{3}$

১২। (ক) $\frac{70}{81}(10^n - 1) - \frac{7n}{9}$ (খ) $\frac{50}{81}(10^n - 1) - \frac{5n}{9}$

১৩। শর্ত $x < -2$ অথবা $x > 0$; সমষ্টি = $\frac{1}{x}$

১৪। (ক) $\frac{3}{11}$ (খ) $\frac{2303}{999}$ (গ) $\frac{41}{3330}$ (ঘ) $3\frac{403}{9990}$

অনুশীলনী ৮.১

১। (ক) (i) $75^\circ 30'$ (ii) $55^\circ 54' 53''$ (iii) $33^\circ 22' 11''$

(খ) (i) $110^\circ 46' 9 \cdot 23''$ (ii) $75^\circ 29' 54 \cdot 5''$ (iii) $55^\circ 54' 53 \cdot 35''$

৩। 0.2626 মি. (প্রায়) ৪। 57 কি.মি./ঘণ্টা (প্রায়) ৫। $\frac{\pi}{5}$ রেডিয়ান, $\frac{\pi}{2}$ রেডিয়ান

৬। $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$ ৭। 562 কি.মি. (প্রায়) ৮। 1,135.4 কি.মি. (প্রায়)

৯। 4.78 মি./সে. (প্রায়) ১০। 1 কি.মি. (প্রায়) ১১। 1.833 রেডিয়ান (প্রায়)

১২। 114.59 মিটার (প্রায়) ১৩। 1745 মি. (প্রায়) বা 1.75 মি. (প্রায়)

অনুশীলনী ৮.২

- ১। (i) $\frac{1}{\sqrt{6}}$ (ii) $1 + \frac{1}{\sqrt{2}}$ ২। $\tan \theta = \frac{3}{4}, \sin \theta = -\frac{3}{5}$
- ৩। $\sin A = -\frac{1}{\sqrt{5}}, \tan A = -2$ ৪। $\sin A = -\frac{\sqrt{3}}{2}, \tan A = \sqrt{3}$
- ৫। $\sin A = -\frac{5}{13}, \cos A = \frac{12}{13}$ ৬। $\frac{x^2 + y^2}{x^2 - y^2}$
- ১২। (i) $\frac{27}{4}$ (ii) $\frac{17}{12}$ (iii) $\frac{3}{4}$ (iv) $\frac{5\sqrt{3}}{6}$ ১৩। ২

অনুশীলনী ৮.৩

- ৭। (i) ০ (ii) ০ (iii) অসংজ্ঞায়িত (iv) $\frac{1}{\sqrt{3}}$ (v) $\frac{2}{\sqrt{3}}$ (vi) অসংজ্ঞায়িত
- (vii) $-\frac{1}{2}$ (viii) $\frac{\sqrt{3}}{2}$
- ৯। (i) ০ (ii) ১ (iii) ২ (iv) ২ (v) ২
- ১১। (i) $\frac{5\pi}{3}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{4\pi}{3}$ (iv) $\frac{7\pi}{4}$
- ১২। (i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{\pi}{4}$ (v) $\frac{\pi}{6}$
- ১৩। (i) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (ii) $\frac{\pi}{6}, \frac{5\pi}{6}$ (iii) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
- (iv) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (v) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
- (vi) $\frac{\pi}{3}, \frac{5\pi}{3}$ (vii) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

অনুশীলনী ৯.১

- ৫। (ক) x (খ) $\frac{\sqrt{a}}{B}$ (গ) $\frac{a^2 - b^2}{ab}$ (ঘ) ১ (ঙ) ১ (চ) $\left(\frac{a}{b}\right)^{a+b}$
- ৮। (ক) ১ (খ) ০ (গ) ৩ : ২
- ৯। (ক) ০ (খ) $x = 1, y = 1$ (গ) $x = 2, y = -2$ (ঘ) $x = -1, y = 1$

অনুশীলনী ৯.২

৮। (ক) 1.01302 (খ) 19995.62 ৯। (ক) 9.2104 (খ) -4.90779 (গ) 230.76

১১। (ক) $x = \log(1-y)$, $\log a < y < 1$ (খ) $x = 10^y$, $-a < y < a$

(গ) $x = \sqrt{y}$, $0 < y < a$

১২। $D_f = (2, \infty)$, $R_f = R$

১৪। (ক) $D_f = [-5, 5]$, $R_f = [0, 5]$ (খ) $D_f = [-2, 2]$, $R_f = [0, 4]$

(গ) $D_f = (-5, 5)$, $R_f = R$

অনুশীলনী ১০.১

১। $1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$

(i) $1 - 5y + 10y^2 - 10y^3 + 5y^4 - y^5$

(ii) $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

২। (a) $1 + 4x + 240x^2 + 1280x^3 + \dots$

(b) $1 - 21x + 189x^2 - 945x^3 + \dots$

৩। (a) $1 + 8x + 28x^4 + 56x^6 + \dots$ এবং 1.082856

৪। (a) $1 - 10x + 40x^2 - \dots$

(b) $1 + 27x + 324x^2 + \dots$

(c) $1 + 17x + 94x^2 + \dots$

৫। (a) $1 - 14x^2 + 84x^4 - 280x^6 + \dots$

(b) $1 + \frac{16}{x} + \frac{112}{x^2} + \frac{448}{x^3} + \dots$

(c) $1 - \frac{7}{2} \cdot \frac{1}{x} + \frac{21}{4} \cdot \frac{1}{x^2} - \frac{35}{8} \cdot \frac{1}{x^3} + \dots$

৬। (a) $1 - 6x + 15x^2 - 20x^3 + \dots$

(b) $1 + 12x + 604x^2 + 160x^3 + \dots$

(c) $1 + 6x + 3x^2 - 40x^3 + \dots$

অনুশীলনী ১০.২

১০। (a) $32 + 80x^2 + 80x^4 + 40x^6 + 10x^8 + x^{10}$

(b) $64 - \frac{96}{x} + \frac{60}{x^2} - \frac{20}{x^3} + \frac{15}{4x^4} - \frac{3}{8x^5} + \frac{1}{64x^6}$

১১। (a) $64 + 576x + 2160x^2 + 4320x^3 + \dots$

(b) $1024 - \frac{640}{x} + \frac{160}{x^2} - \frac{20}{x^3} + \dots$

১২। $p = 2$, $r = 64$, $s = 60$ ১৩। 7 ১৪। $64 + 48x + 15x^2 + \frac{5}{2}x^3 + \dots$

১৫। 31.2080 ১৬। $n = 8$, পদসংখ্যা 9 ও মধ্যপদ $\frac{35}{128}$ ১৭। (a) $x = \pm 6$ (b) $k = 2$

অনুশীলনী ১১.১

- ১। (i) $\sqrt{13}$ একক (ii) $4\sqrt{2}$ একক (iii) $(a-b)\sqrt{2}$ একক (iv) 1 একক (v) $\sqrt{13}$ একক
 ৫। $k = -5, 5$ ৬। 16.971 (প্রায়) ৯। B নিকটবর্তী, A দূরবর্তী

অনুশীলনী ১১.২

- ১। (i) 7 একক, $4\sqrt{2}$ একক, 5 একক, $12 + 4\sqrt{2}$ একক (ii) 14 বর্গ একক
 ২। (i) 6 বর্গ একক (ii) 24 বর্গ একক
 ৩। $\sqrt{58}$ একক, $\sqrt{10}$ একক, 16.971 একক ৪। a^2 বর্গ একক
 ৫। 10 একক, 10 একক, 40 বর্গ একক
 ৬। $a = 5$, হলে $\frac{119}{2}$ বর্গ একক ৭। $a = 2$, $5\frac{1}{3}$ $a = 15$, হলে $\frac{109}{2}$ বর্গ একক

$a=2$ হলে, ABC ত্রিভুজটি সমকোণী AC অতিভুজ এবং $\angle BAC$ সমকোণ

- ৮। (i) 21 বর্গ একক (ii) 24 বর্গ একক (iii) 15 বর্গ একক ১০। $P = 20$

অনুশীলনী ১১.৩

- ১। (ক) -1 (খ) $\frac{7}{4}$ (গ) 1 (ঘ) 2 ২। 5 ৪। $\frac{1}{2}$ ৫। 1, 2

অনুশীলনী ১১.৪

- ১০। $y = 2x - 5$ ১১। (a) $y = -x + 6$ (b) $y = x - 3$ (c) $y = 3x - 3a$
 ১২। (a) $y = 3x - 5$ (b) $y = -3x - 5$ (c) $y = 3x + 5$ (d) $y = -3x + 5$
 ১৩। (a) (1, 0); (0, 3) (b) $\left(-\frac{6}{5}, 0\right); (0, 3)$ (c) $\left(-\frac{4}{3}, 0\right); (0, -1)$
 ১৪। $y = k(x - k); k = 2, 3$ ১৫। $y = \frac{1}{k}(x + k); k = -1, 2$ ১৬। $k = \frac{11}{2}$

অনুশীলনী- ১৩

- ৭। 636 বর্গ মি., 20.5 মি., 864 ঘন মি. ৮। 1 ঘন মি., 7.8 বর্গ মি. ৯। 300 বর্গ সে. মি. (প্রায়) ১০। 8.75 মি., 3.2 মি. ১১। 188.5 বর্গ সে. মি. (প্রায়), 301.6 ঘন সে. মি. (প্রায়), ১২। 25 সে. মি. (প্রায়) ১৩। 91.63 ঘন সে. মি. (প্রায়) ১৪। 452.39 বর্গ সে. মি. (প্রায়), 904.8 ঘন সে. মি. (প্রায়) ১৫। 1 সে. মি. ১৬। 11.37 সে. মি. (প্রায়) ১৭। 1.06 সে. মি. (প্রায়) ১৮। ৪ টি ১৯। 1308.82 ঘন সে. মি. (প্রায়) ২০। 78.5 বর্গ সে. মি. (প্রায়) ২১। 7.48 বর্গ মি. (প্রায়) ২২। 83800 টি ২৩। 2 সে. মি., 12 সে. মি., 12 সে. মি. ২৪। 2086.49 বর্গ মি. (প্রায়) ২৫। 798 বর্গ সে. মি., 1550 ঘন সে. মি. ২৬। 203.14 বর্গ সে. মি., 207.85 ঘন সে. মি. ২৭। 296.38 বর্গ সে. মি., 311.77 ঘন সে. মি. ২৮। 110.85 বর্গ সে. মি., 60.34 ঘন সে. মি. ২৯। 40.65 বর্গ সে. মি., 16 ঘন সে. মি. ৩০। 4662.86 ঘন সে. মি.

অনুশীলনী ১৪

- ৭। (i), $\frac{1}{2}$ (ii), $\frac{7}{30}$ (iii), $\frac{7}{30}$ (iv) $\frac{4}{15}$ ৮। $\frac{1}{38}$ ৯। $\frac{2}{3}$ ১০। $\frac{98}{639}$
 ১১। (i) $\frac{23}{1000}$ (ii) $\frac{1}{400}$ ১২। (i) $\frac{157}{1897}$ (ii) $\frac{1630}{1897}$ (iii) $\frac{424}{1897}$ ১৫। (i) $\frac{8}{63}$ (ii) $\frac{25}{63}$ ১৬। $\frac{4}{45}$



সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর

- মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

জ্ঞান মানুষের অন্তরকে আলোকিত করে



২০১০ শিক্ষাবর্ষ থেকে সরকার কর্তৃক বিনামূল্যে বিতরণের জন্য

মুদ্রণে :